Loughborough University

STRUCTURAL DYNAMICS AND EARTHQUAKE ENGINEERING
(21CVD020)

Semester 1 2021-22 (1b) Exam paper

This is a (1b) online examination, meaning you have a total of **2 hours plus additional 30 minutes** to complete and submit this paper. The additional 30 minutes are for downloading the paper and uploading your answers when you have finished. If you have extra time or rest breaks as part of a Reasonable Adjustment, you will have further additional time as indicated on your exam timetable.

It is your responsibility to submit your work by the deadline for this examination. You must make sure you leave yourself enough time to do so.

It is also your responsibility to check that you have submitted the correct file.

Exam Help If you are experiencing difficulties in accessing or uploading files during the exam period you should contact the exam helpdesk. For urgent queries please call **01509 222900**. For other queries email examhelp@lboro.ac.uk

You may handwrite and/or word process your answers, as you see fit.

You may use any calculator (not just those on the University’s approved list).

ANSWER **TWO** questions from Section A – “Structural Dynamics”

AND **ONE** question from Section B – “Earthquake Engineering”:

Each question in Section A is worth **25 marks**; each question in Section B is worth **50 marks**.

A formula sheet for the dynamics of SDoF (single-degree-of-freedom) oscillators is attached.

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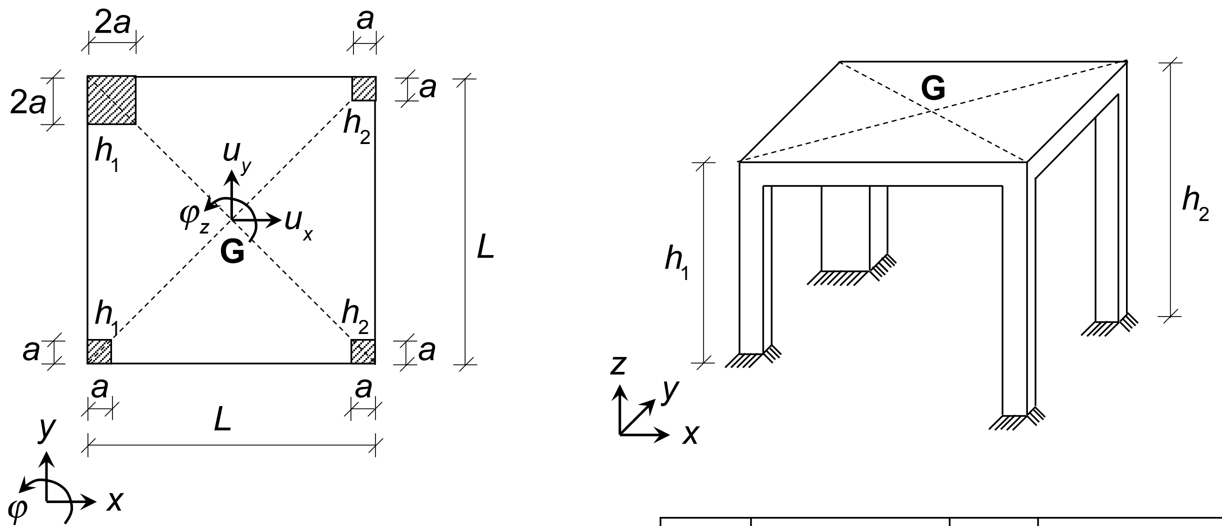
SECTION A – "Structural Dynamics"
(ANSWER **TWO** questions from Section A)

1. Figure Q1(a) shows a plan and a 3D view of a spatial RC (reinforced concrete) frame, in which four columns support a thick flat slab. The columns have different heights and square cross sections of different sizes, as specified within Figure Q1. The total mass density is $\rho = 5{,}550\ \text{kg/m}^2$ (uniformly distributed over the area of the slab) and the Young's modulus of the concrete is $E = 30\ \text{GPa}$.

   (a) For the three degrees of freedom, u_x, u_y and φ_z, identified in Figure Q1(a), <u>calculate</u> the mass matrix and the stiffness matrix of the frame, applying reasonable assumptions to simplify the hand calculations. [13 marks]

   (b) <u>State</u> explicitly <u>up to three</u> assumptions made in answering Question 1(a) above, and <u>comment</u> on whether each of these assumptions is likely to introduce significant inaccuracies in the dynamic analysis of the frame. [**Note:** Two marks are allocated for each assumption correctly stated and briefly discussed, up to six marks] [6 marks]

   (c) Figure Q1(b) shows the moduli of the frequency response functions (FRFs) of the frame before (—— solid line) and after (- - - dashed line) a seismic event has occurred. These FRFs have been obtained considering the same gravitational load. Their comparison reveals that the frame has been damaged by the earthquake. <u>Explain</u> which two main changes in the FRF support the diagnosis that the frame is damaged. [**Note:** Three marks are allocated for each variation correctly stated and briefly justified, up to six marks] [6 marks]



a =	30 cm	h_1 =	3.50 m
L =	6.00 m	h_2 =	4.50 m
E =	30 GPa	ρ =	5,500 kg/m²

Figure Q1(a). 3D reinforced concrete (RC) frame

Question 1 continues/…

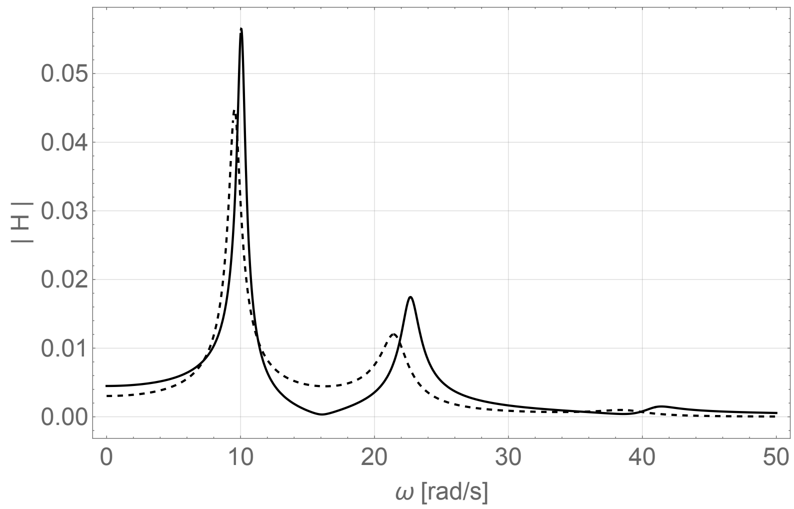


Figure Q1(b). FRF moduli before (——) and after (---) a seismic event

2. Figure Q2 shows a simplified planar model for an existing three-storey steel frame subjected to lateral vibrations. The lateral stiffness is provided by the flexural stiffness of the columns, EI_c, and the axial stiffness of the brace at the ground floor, EA_b. Gravitational loads w, Young's modulus of the steel E, columns' second moment of area I_c, brace's cross-sectional area A_b, inter-storey heights h_i and span widths b_i are provided in the same figure. Note the presence of three pinned connections in the frame and a reduced gravitational load on the roofing beams.

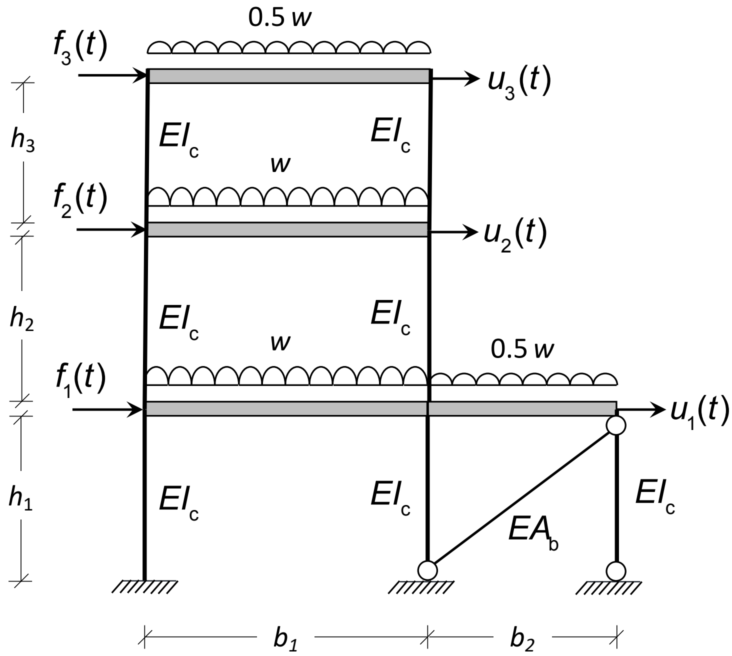
   (a) Calculate the mass matrix **M** and the stiffness matrix **K**. You can assume that the masses are concentrated at the level of each storey and the beams are rigid in bending (i.e. shear-type model). [8 marks]

   (b) Knowing that the fundamental period of the frame is T_1= 0.402 s, find and draw the corresponding modal shape ϕ_1 (the latter can be normalised in such a way that the largest term in the vector ϕ_1 is set to 1). [7 marks]

   (c) Knowing that the second and third modal circular frequencies are ω_2= 42.59 rad/s and ω_3= 52.25 rad/s, calculate the Rayleigh's damping parameters α_M and α_K to have ζ_0=5% of viscous damping ratio in the first two modes of vibration. Subsequently, find the resulting damping ratio ζ_3 for the third mode. [6 marks]

   (d) Comment on the appropriateness, e.g., the conservatism or lack thereof, of using the Rayleigh's damping model for the third mode of vibration when the model is calibrated against the first two modes. [**Note:** The answer to this question is not expected to exceed 70 words] [4 marks]

Question 2 continues/…



h_1= h_2=	3.50 m	w=	40.0 kN/m
h_3=	3.00 m	E=	210 GPa
b_1=	7.00 m	I_c=	15,500 cm^4
b_2=	4.50 m	A_b=	10 cm^2
		ζ_0=	0.05

Figure Q2. Planar MDoF frame subjected to lateral vibrations

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3. Figure Q3(a,left) shows a three-storey moment resisting frame subjected to the dynamic forces $f_i(t)$ acting at the position of the frame's floors. The sway displacements of the frame, $u_i(t)$, are the only dynamically significant degrees of freedom, ordinately collected in the vector $\underline{\mathbf{u}}(t) = \{u_1(t), u_2(t), u_3(t)\}^T$.

The modal analysis of the frame has been carried out, i.e., the eigenproblem $\underline{\mathbf{K}} \cdot \underline{\phi}_i = \omega_i^2 \, \underline{\mathbf{M}} \cdot \underline{\phi}_i$ has been solved. The resulting modal matrix $\underline{\mathbf{\Phi}}$ is:

$$\underline{\mathbf{\Phi}} = \left[\begin{array}{c|c|c} \underline{\phi}_1 & \underline{\phi}_2 & \underline{\phi}_3 \end{array} \right] = \left[\begin{array}{c|c|c} -0.081 & -0.119 & 0.021 \\ -0.112 & 0.055 & -0.112 \\ -0.118 & 0.110 & 0.173 \end{array} \right]. \qquad (3.1)$$

Each column of the modal matrix $\underline{\mathbf{\Phi}}$ represents one of the structure's modal shapes, normalised with respect to the mass matrix, i.e., $M_i = \{\underline{\phi}_i\}^T \cdot \underline{\mathbf{M}} \cdot \underline{\phi}_i = 1$.

The dynamic forces are caused by an overpressure $p(t)$ on the left side of the building, which in turn is the result of a nearby explosion. The Friedlander's waveform of Figure Q3(a,right) has been used to model the blast loading. This plot shows that the dynamic action is impulsive, as the overpressure vanishes after 0.5 s.

Figure Q3(b) shows the time histories $q_i(t)$ of the first two modal coordinates due to the blast loading.

(a) Comparing the two plots of Figure Q3(b), <u>identify</u> the time history of the first modal coordinate, $q_1(t)$. You should unambiguously explain why either the solid line (——) or the dashed line (- - -) represents the first modal response.
[**Note:** You may use some simple calculations to support your answer, if you wish to do so] [5 marks]

(b) The maximum displacement of the top floor, $u_{3,max} = u_3(t_{max})$, is known to occur at time $t_{max} = 0.410$ s. The values of the first two modal coordinates at $t = t_{max}$ are offered in Figure Q3(b). <u>Calculate</u> the maximum displacement $u_{3,max}$ using the first two modes of vibration. [5 marks]

(c) Based on the calculations done to answer Question Q3(b) above, <u>comment</u> on whether a SDoF (single-degree-of-freedom) approximation would be suitable to evaluate the top displacement of the frame for this particular loading condition.
[**Note:** The answer to this question is not expected to exceed 50 words]
[4 marks]

(d) Assuming that the mass matrix $\underline{\mathbf{M}}$ and the stiffness matrix $\underline{\mathbf{K}}$ of the frame are known, <u>list</u> the steps required to derive the state-space equation of motion for the first modal coordinate $q_1(t)$. [6 marks]

Question 3 continues/…

(e) It is brought to your attention that the stiffness matrix of the frame, **K**, has been evaluated using a "shear-type" model. Discuss the limitations of this model and explain which alternative approach could be used to overcome them.
[**Note:** The answer to this question is not expected to exceed 70 words]

[5 marks]

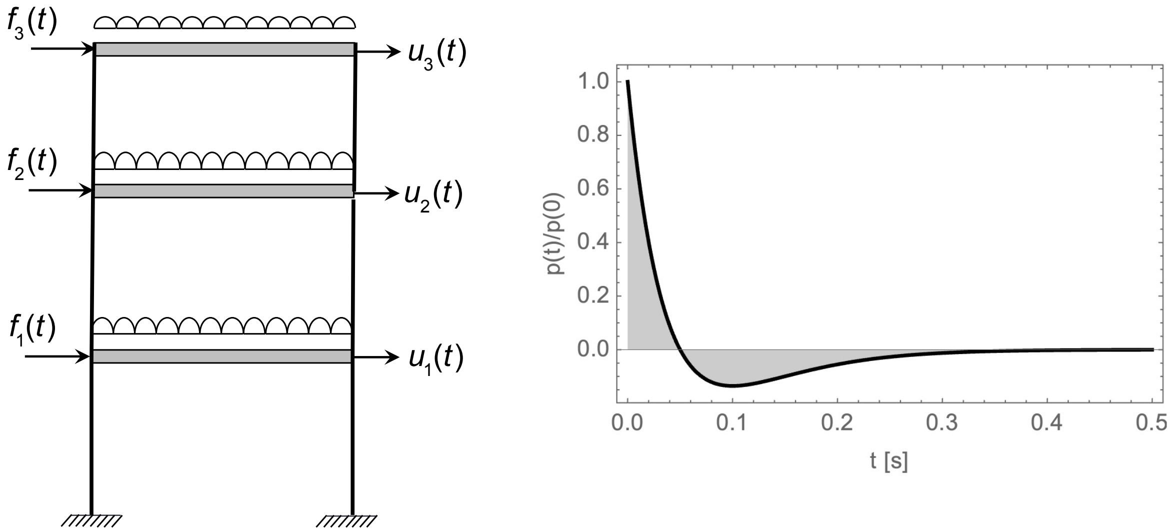

Figure Q3(a). MDoF frame (left) and time history of the overpressure acting on it (right)

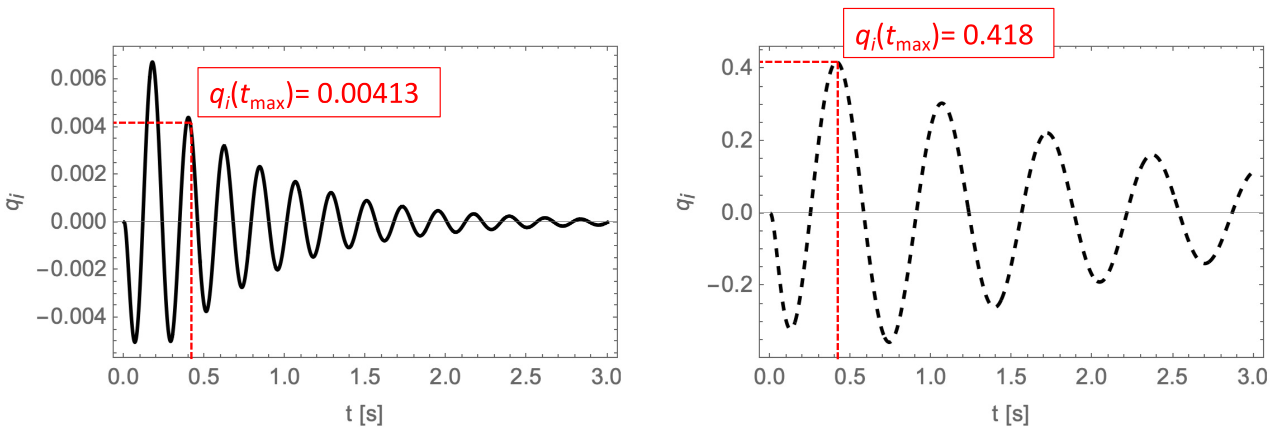

Figure Q3(b). Time histories of the first two modal coordinates

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SECTION B - "Earthquake Engineering"
(ANSWER **ONE** question from Section B)

4. Figure Q4(a) shows the elevation of a 3-bay, 3-storey steel frame, with a one-side setback at the first floor. The geometrical dimensions h_i and b_i (i = 1,2,3) of the frame are given in the same figure, along with the gravitational loads w, the Young's modulus E, and the cross-sectional area A of the diagonal braces.

The frame has been erected with simple connections, and for this reason it can be assumed that the lateral stiffness and strength is only provided by the diagonal braces.

The seismic response of the frame needs to be quantified using the lateral force method, as formulated in the Eurocode 8 (EC8). Both damage limitation requirement (DLR, with return period T_R= 95 years) and no-collapse requirement (NCR, with T_R= 475 years) have to be considered. Figure Q4(b) offers the mathematical expressions of the four-branch spectra for both levels of the seismic action, along with the values of the defining parameters a_g, S, T_B, T_C, T_D and β.

One can assume that:

- The beams are rigid, implying a shear-type behaviour for the frame;
- The fundamental period of vibration of the frame is T_1= 0.38 s;
- The equivalent viscous damping ratio for the DLR is ζ_0= 0.03;
- The behaviour factor for the NCR is q= 4.

(a) Calculate the three lateral forces $f_i^{(DLR)}$ acting on the frame for the DLR. [14 marks]

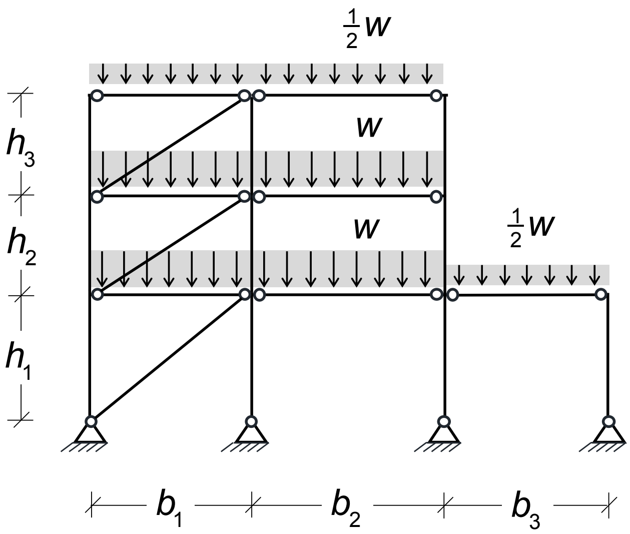
(b) Calculate the resulting inter-storey drifts $\Delta_i^{(DLR)}$. [12 marks]

(c) Calculate the axial forces $N_i^{(NCR)}$ in the three diagonal braces for the NCR. [10 marks]

(d) Explain why the ordinates of the elastic design spectrum $S_e(T)$ for the DLR tend to be higher than those of the elasto-plastic design spectrum $S_d(T)$ for the NCR, even if the peak ground acceleration (PGA) tends to increase with the return period T_R. [**Note:** The answer to this question is not expected to exceed 100 words] [6 marks]

(e) Comment on the appropriateness of the lateral force method for the seismic analysis of the frame in Figure Q4(a). Which alternative method would you suggest, and why? [**Note:** The answer to this question is not expected to exceed 100 words] [8 marks]

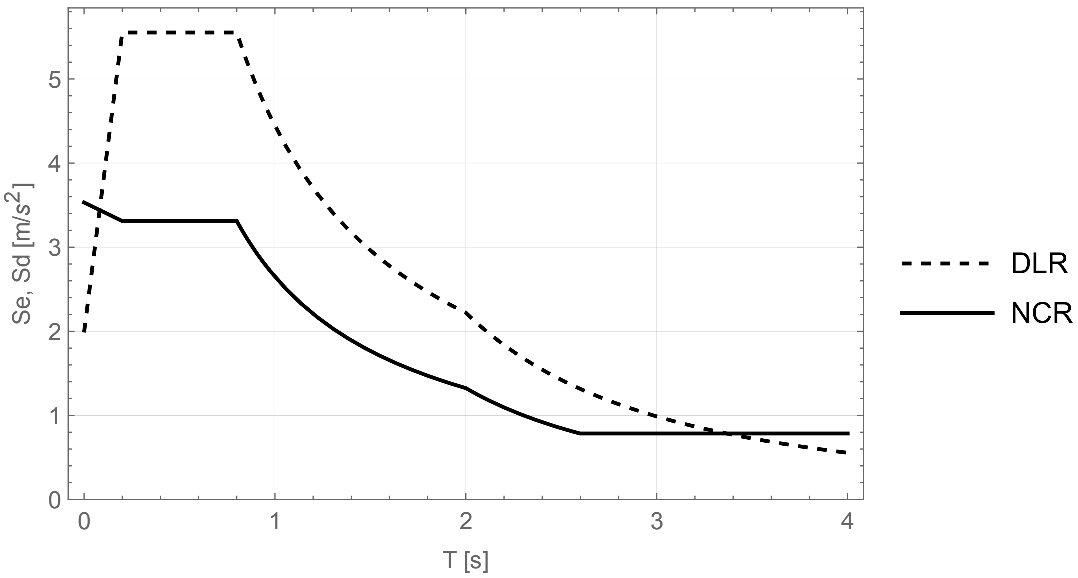
Question 4 continues/…



h_1=	4.00 m	w=	50.0 kN/m
h_2= h_3=	3.00 m	E=	210 GPa
b_1= b_3=	5.00 m	A=	30 cm^2
b_2=	7.00 m	T_1=	0.38 s
ζ_0=	0.03	q=	4

Figure Q4(a). Steel frame with simple connections

Question 4 continues/…



Elastic design spectrum for the DLR
(T_R= 95 years)

$$S_e(T) = \begin{cases} a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2.5 - 1)\right] , & 0 \le T < T_B \\ a_g \cdot S \cdot \eta \cdot 2.5 , & T_B \le T \le T_C \\ a_g \cdot S \cdot \eta \cdot 2.5 \cdot \left[\frac{T_C}{T}\right] , & T_C < T \le T_D \\ a_g \cdot S \cdot \eta \cdot 2.5 \cdot \left[\frac{T_C \cdot T_D}{T^2}\right] , & T_D < T \le 4 \text{ s} \end{cases}$$

where $a_g^{(DLR)}$ = 0.15 g , and:

$$\eta = \sqrt{\frac{10}{5 + 100\,\zeta}} \quad \ge \quad 0.55$$

Elasto-plastic design spectrum for the NCR
(T_R= 475 years)

$$S_d(T) = \begin{cases} a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right] , & 0 \le T < T_B \\ a_g \cdot S \cdot \frac{2.5}{q} , & T_B \le T \le T_C \\ a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C}{T}\right] \ \ge \ \beta \cdot a_g , & T_C < T \le T_D \\ a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C \cdot T_D}{T^2}\right] \ \ge \ \beta \cdot a_g , & T > T_D \end{cases}$$

where $a_g^{(NCR)}$ = 0.40 g , and:

$$\beta = 0.2$$

Other design parameters which are in common between the two spectra are:

- S= 1.35
- T_B= 0.20 s
- T_C= 0.80 s
- T_D= 2.00 s

Figure Q4(b). Definition of the seismic spectra

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5. (a) As a structural consultant, which method of structural analysis allowed by Eurocode 8 (EC8) would you recommend for the following cases, and why?

- Three-dimensional pin-jointed truss used as a transmission tower for the electricity network;
- Low-rise brick-masonry building, with regular structural layout both in plan and in elevation;
- Medium-span suspension bridge;
- Existing multi-storey moment-resisting frame (MRF) frame, designed without any special earthquake engineering detailing;
- Adjacent mid-rise buildings exposed to pounding risk during a seismic event.

[**Note:** Two marks are allocated for each bullet point above. At least one justifying argument is expected for each case] [10 marks]

(b) Explain why the geological and geotechnical information about the soil strata beneath the foundation is critically important to quantify the design seismic action on a structure. [**Note:** Marks will only be awarded if a well-reasoned justification is provided. The answer to this question is not expected to exceed 70 words.] [5 marks]

(c) The construction of a medium-rise building is planned in an earthquake-prone area. The structural consultant is considering the use of base isolators to mitigate the effects of the seismic action. Alluvial sediments characterise the site's geology, and there are soft clay strata underneath the ground level. Do you envisage any problem with the use of base isolators in this circumstance? [**Note:** Marks will only be awarded if a well-reasoned justification is provided. The answer to this question is not expected to exceed 50 words] [5 marks]

(d) Figure Q5(a) shows two alternative detailing designs of beam-to-column connections in reinforced concrete (RC) frames. In this figure, *c* is the dimension of the column's cross-section and *d* is the beam's effective depth (see Sections 1-1 to 4-4 at the bottom of Figure Q5(a)). Which one of the two designs will offer better seismic performance, and why? [**Note:** Marks will only be awarded if a well-reasoned justification is provided. The answer to this question is not expected to exceed 100 words] [5 marks]

(e) Timber structures have increasingly been considered as a viable alternative to reinforced concrete (RC), steel and masonry structures. As a consultant, would you recommend timber as the main material for the load-bearing elements in earthquake-resistant structures? [**Note:** Marks will only be awarded if a well-reasoned justification is provided. The answer to this question is not expected to exceed 50 words] [5 marks]

(f) For each of the five cases illustrated in Figure Q5(b), briefly explain the reasons why the seismic performance of these structures might not be satisfactory. [**Note:** Two marks are allocated for each case, and at least one justifying argument is expected for each of them] [10 marks]

Question 5 continues/…

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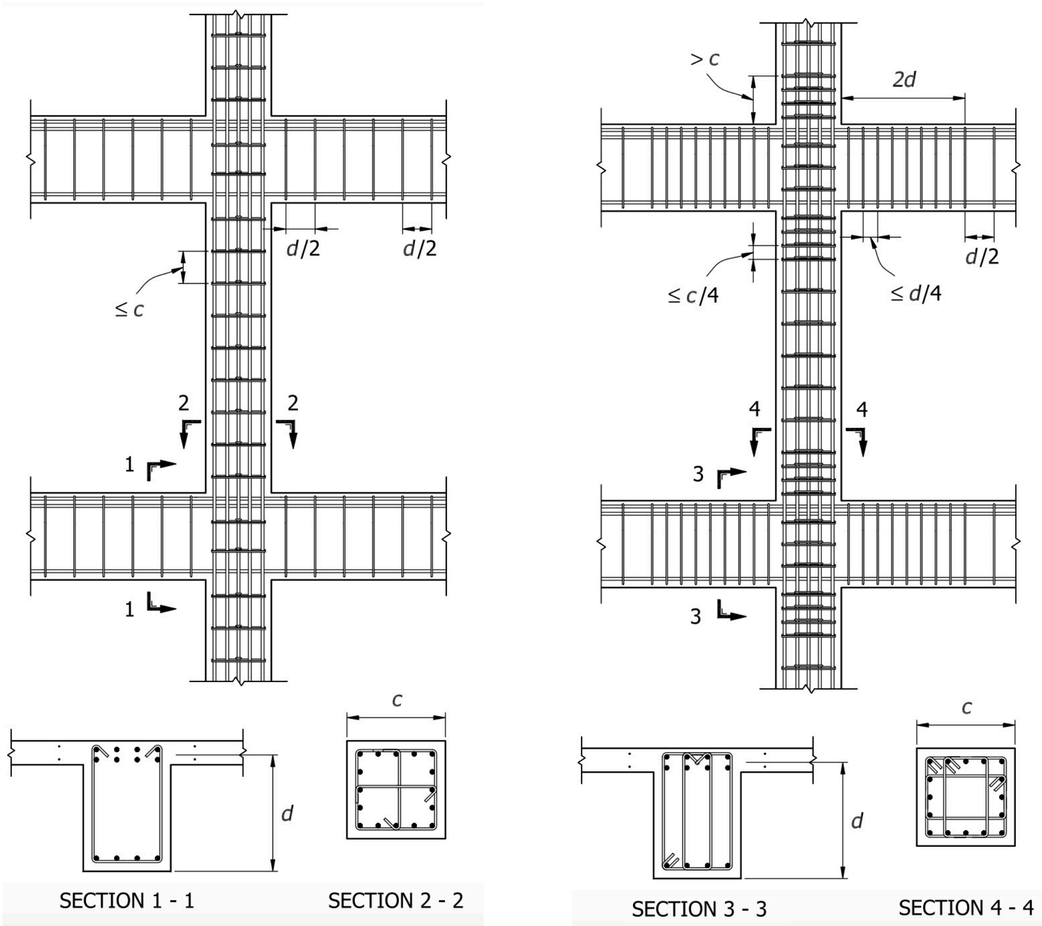

Figure Q5(a). Alternative RC detailing designs

(g) Metal dampers are typically characterised by large and stable hysteretic cycles when subjected to seismic action. For this reason, they are widely adopted in the design of earthquake-resistant frames. Why is it essential that the hysteretic cycles are both "large" (i.e., relatively great in size) and "stable" (i.e., not deteriorating over time)? [**Note:** Marks will only be awarded if a well-reasoned justification is provided. The answer to this question is not expected to exceed 50 words] [5 marks]

(h) In the application of the lateral force method, Eurocode 8 allows the distribution of the overall seismic force F_d according to the following formula:

$$f_i = F_\mathrm{d} \frac{m_i z_i}{\sum_{j=1}^{n} m_j z_j} \quad , \qquad (5.1)$$

where m_i is the mass of the ith floor and z_i is its height with respect to the foundation level; n is the number of storeys. Explain why both m_i and z_i appear in the numerator of the fraction in Eq. (5.1) above. [**Note:** Marks will only be awarded if a well-reasoned justification is provided. The answer to this question is not expected to exceed 50 words] [5 marks]

Question 5 continues/…

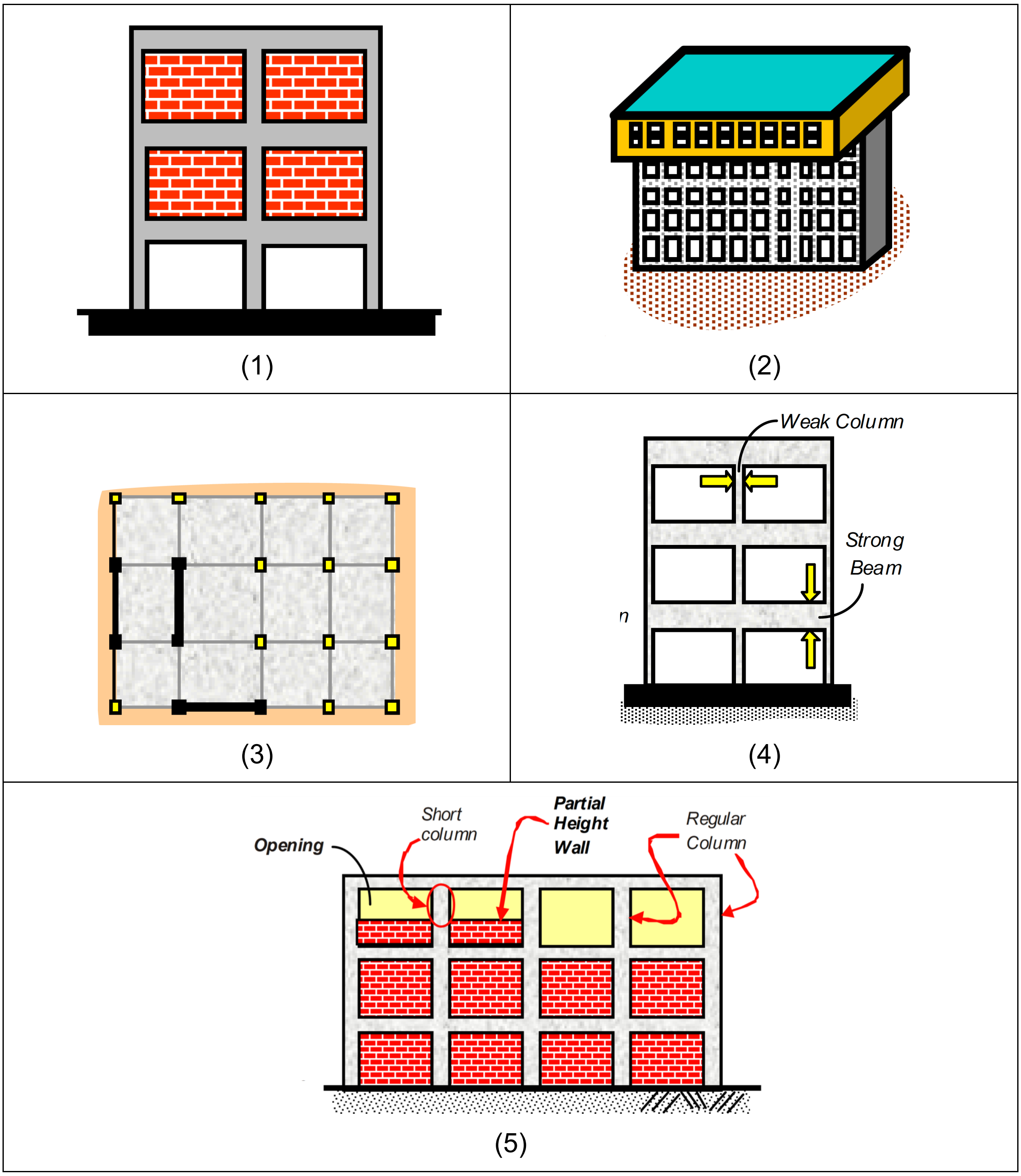

Figure Q5(b). Frame structures with possible unsatisfactory seismic behaviour

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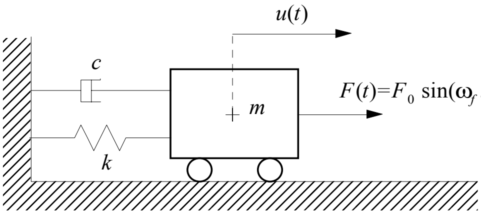
<u>Formula Sheet</u>: Dynamics of SDoF Oscillators



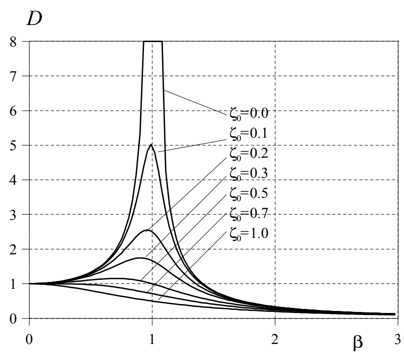
$$m\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = F_0 \sin\left(\omega_\mathrm{f}\, t\right)$$

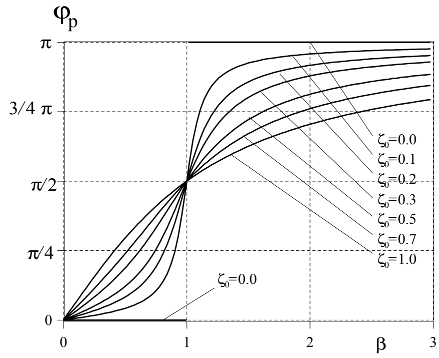
$$u(t) = u_\mathrm{h}(t) + u_\mathrm{p}(t)$$

$$u_\mathrm{h}(t) = \exp\left(-\zeta_0\, \omega_0\, t\right)\left[\bar{C}_1 \cos\left(\bar{\omega}_0\, t\right) + \bar{C}_2 \sin\left(\bar{\omega}_0\, t\right)\right]$$

$$u_\mathrm{p}(t) = u_\mathrm{stat}\, D(\beta) \sin\left(\omega_\mathrm{f}\, t - \varphi_\mathrm{p}\right)$$

$$\beta = \frac{\omega_\mathrm{f}}{\omega_0} \qquad D(\beta) = \frac{1}{\sqrt{\left(1-\beta^2\right)^2 + \left(2\,\zeta_0\,\beta\right)^2}} \qquad \tan(\varphi_\mathrm{p}) = \frac{2\,\zeta_0\,\beta}{1-\beta^2} \qquad \left\{0 \le \varphi_\mathrm{p} < \pi\right\}$$





Complete quadratic combination (CQC) coefficients

$$\rho_{ij} = \frac{8\,\zeta^2\left(1+\beta_{ij}\right)\beta_{ij}^{1.5}}{\left(1-\beta_{ij}^2\right)^2 + 4\,\zeta^2\,\beta_{ij}\left(1+\beta_{ij}\right)^2}\ , \quad \text{with } \beta_{ij} = \min\left\{\frac{T_i}{T_j}, \frac{T_j}{T_i}\right\} \quad \left(0 < \beta_{ij} \le 1\right)$$