# ADVANCED HEAT TRANSFER

21WSC801

January 2022 (1b) Exam paper

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This is a (1b) online examination, meaning you have a total of **2 hours plus an additional 30 minutes** to complete and submit this paper. The additional 30 minutes are for downloading the paper and uploading your answers when you have finished. If you have extra time or rest breaks as part of a Reasonable Adjustment, you will have further additional time as indicated on your exam timetable.

**It is your responsibility to submit your work by the deadline for this examination. You must make sure you leave yourself enough time to do so.**

**It is also your responsibility to check that you have submitted the correct file.**

Exam Help
If you are experiencing difficulties in accessing or uploading files during the exam period you should contact the exam helpdesk. For urgent queries please call **01509 222900**.
For other queries email examhelp@lboro.ac.uk

You may handwrite and/or word process your answers, as you see fit.

You may use any calculator (not just those on the University's approved list).

# ADVANCED HEAT TRANSFER

(21WSC801)

January 2022 2 Hours

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Answer **ALL QUESTIONS**

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/(m$^2$K$^4$)

Figures relating to questions can be found at the back of the paper.

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**1.** Shown in **Figure Q.1** is a cross-section of a long cylindrical bar. Heat is generated in this bar at a rate of $g$ W/m$^3$ and the temperature at the outer radius where $r = b$ is maintained at $T_2$ . In usual notations, where $k$ is thermal conductivity, the heat transfer in this situation is governed by

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dT}{dr}\right] + \frac{g}{k} = 0$$

a) Develop an expression for one dimensional radial steady state temperature distribution $T(r)$ and heat flux $q(r)$. [6 marks]

For the case, $b = 6$ cm, $T_2 = 200$ ºC, $g = 2 \times 10^6$ W/m$^3$ and $k = 50$ W/(m. ºC), the energy balance method is to be used to develop numerical solutions for radial temperature and heat flux.

b) Using the sector shown in **Figure Q.1** which consists of 4 equally spaced nodes, write a set of equations that could be solved to obtain temperatures at the nodes. [6 marks]

c) Solve the equations obtained in (b) and calculate temperatures at the nodes. [4 marks]

d) Compare your solution obtained for node 1 with the analytical temperature obtained from the expression for $T(r)$ developed in (a). Comment on the accuracy of the numerical solution. [2 marks]

e) Using the numerical solution calculate the heat flux at the outer radius $r = b$ and compare the numerical value with the analytical value of flux obtained using the expression for $q(r)$ developed in (a). Comment on the accuracy of the numerical solution. [2 marks]

**2.** Shown in **Figure Q.2(a)** and **Figure Q.2(b)** are configurations of a room where surface 1 is a radiator and surface 2 is a window. In each case the radiator is at a temperature of 340K, emissivity 0.8 and the window is at a temperature of 280 K, emissivity 0.7. All other surfaces in each configuration could be treated as one black surface (3) at temperature 300 K. It is required to calculate heat loss through the window for both cases to determine the most energy efficient location for the radiator. Assume the radiative heat transfer is the dominant heat transfer mechanism.

<u>Configuration 1</u>

a) Noting that, in configuration 1, surface 1 and surface 2 are on the same wall, draw an electrical network that could be used to calculate radiative heat transfer at each surface. [ 2 marks]

b) Calculate necessary configuration factors and resistances required for the circuit drawn in (a). [ 2 marks]

c) Use the network drawn in (a) to write nodal equations and solve them to calculate heat transfer at surfaces 1, 2 and 3. [3 marks]

d) Provide a heat balance to verify your answers obtained in (c). [1 mark]

<u>Configuration 2</u>

e) Draw an electrical network to calculate radiative heat transfer at surfaces of Configuration 2. [ 2 marks]

f) Calculate necessary configuration factors required for the circuit drawn in (e). [2 marks]

g) Use the network drawn in (e) to write nodal equations and solve them to calculate heat transfer at surfaces 1, 2 and 3. [6 marks]

h) Provide a heat balance to verify your answers in (g). [1 mark]

i) Based on your answers obtained in (c) and (g) what is the best configuration to minimise heat transfer to the window. What is the energy provided to the radiator in each case? [1 mark]

**Figure Q.2(c)** is to be used for configuration factor calculations.

Some useful configuration factor properties:

Reciprocal property

$$A_1 F_{1-2} = A_2 F_{2-1}$$

Summation rule: when a surface is completely enclosed by $n$ surfaces

$$\sum_{j=1}^{n} F_{i-j} = 1 , \quad i = 1,2,3,............n$$

For two surfaces, $i$ and $j$; if the surface $i$ is subdivided into areas $A_1$ and $A_2$, and the $j$ surface is subdivided into areas $A_3$ and $A_4$.

$$A_{12}F_{12-34} = A_1F_{1-3} + A_1F_{1-4} + A_2F_{2-3} + A_2F_{2-4}$$

**3.** Shown in **Figure Q.3(a)** is the configuration of a furnace geometry. Surface 1 is at temperature 900K, emissivity $\varepsilon_1 = 0.8$, Surface 2 is at a temperature of 500 K, emissivity $\varepsilon_2 = 0.7$ and the cylindrical surface is at a temperature of 400 K, emissivity $\varepsilon_3 = 0.8$. The radiosity-matrix method is to be used to calculate radiative heat transfer at surfaces in this situation.

In usual notations the Radiosity-matrix method uses the equation

$$\left[ J_i - (1-\varepsilon_i)\sum_j F_{ij}J_j \right] = \varepsilon_i E_{bi}$$

a) Using **Figures Q.3(b)** and other useful configuration factor properties calculate $F_{3-3}$, $F_{3-1}$, $F_{3-2}$, $F_{1-2}$ , $F_{1-3}$, $F_{2-1}$, $F_{2-3}$. [8 marks]

b) Use the radiosity-matrix method to write a set of equations that could be solved to obtain radiosities of the surfaces. Calculate all necessary numerical values required and show the set of equations in the matrix form. [8 marks]

c) If the solution to the matrix equation is given as $J_1 = 30.2376$ W/m$^2$, $J_2 = 3.6401$ kW/m$^2$, and $J_{31} = 2.3142$ kW/m$^2$ calculate heat transfer at each surface and verify your calculation using a heat balance. [4 marks]

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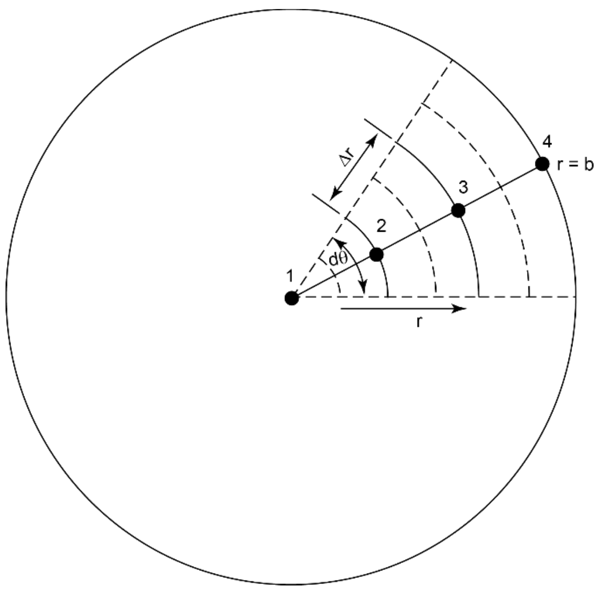



**Figure Q.1**

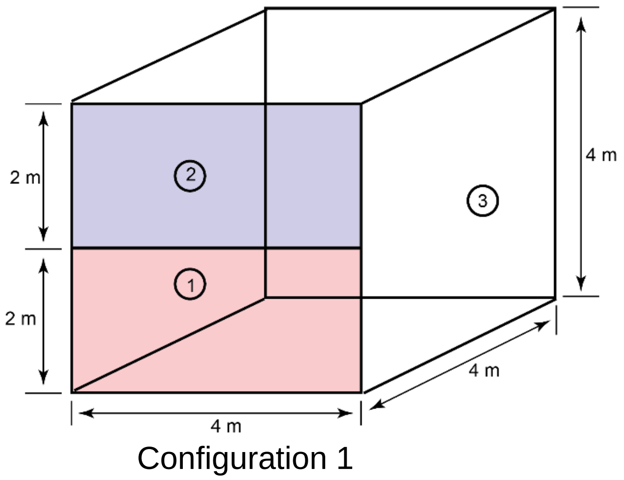



**Figure Q.2(a)**

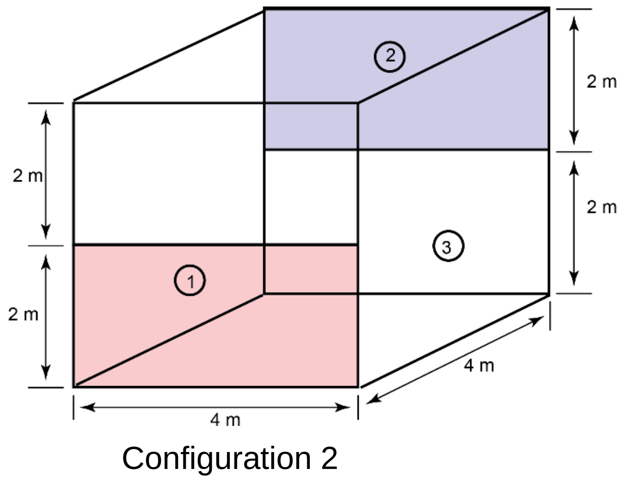



**Figure Q.2(b)**

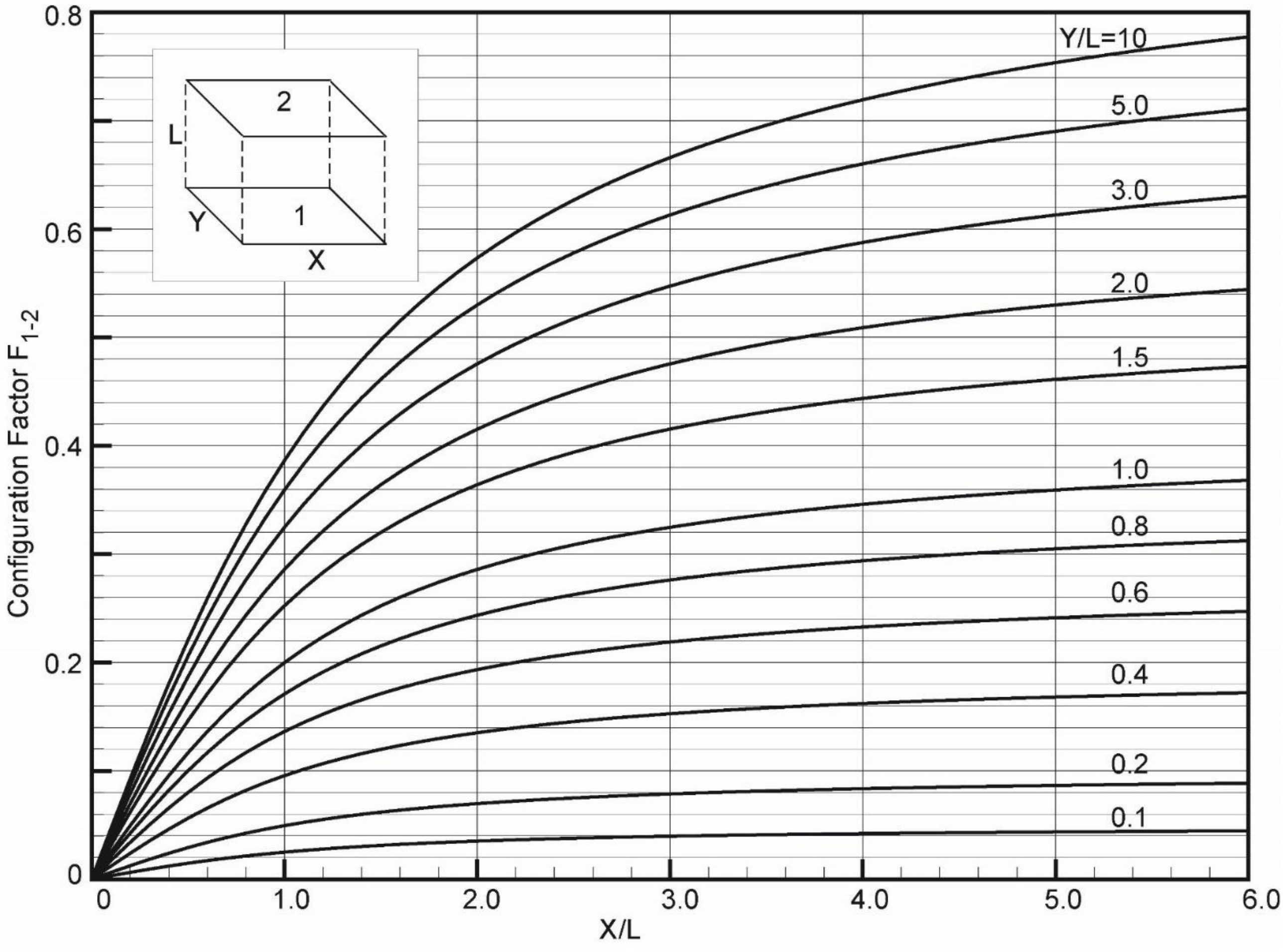



**Figure Q.2(c)**

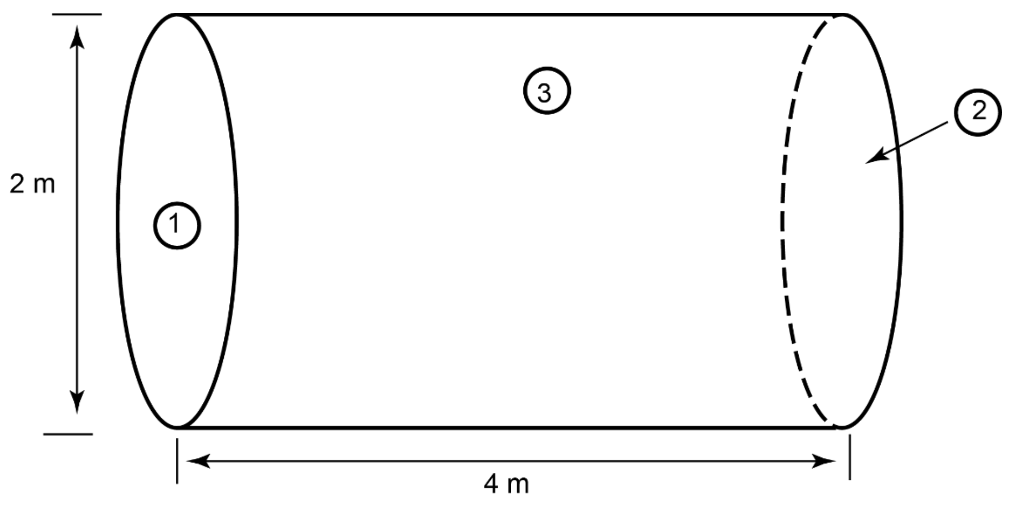
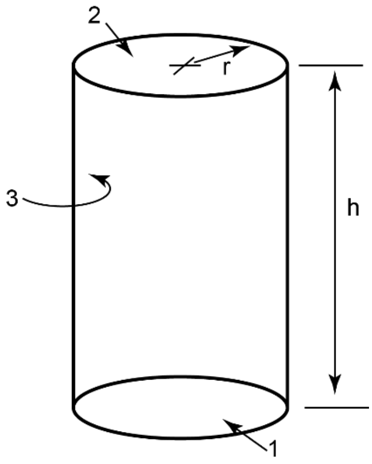



**Figure Q.3(a)**




Inside surface of right circular cylinder to itself

$$H = h/2r$$

$$F_{3-3} = 1 + H - \sqrt{1 + H^2}$$

**Figure Q.3(b)**