

## **Heat Transfer**

### **21CGA006**

Semester 2 2021/22

In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

#### Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Attempt **THREE** questions in total. Each question carries 25 marks.

Candidates should show full working for all calculations and derivations.

1. (a) An electric heater is turned on to raise the air temperature in a house from 50°F to 70°F.
- (i) If the house is air-tight and thus no air escapes during the heating process, provide the problem statement, and then determine the amount of energy transferred to the air and the cost of this heat. [7 marks]
  - (ii) If instead some air escapes through the cracks as the heated air in the house expands at constant pressure, determine the amount of energy transferred to the air and the cost of this heat. [4 marks]
  - (iii) State any assumption(s) made. [3 marks]

Relevant Data and Equations

The floor area of the house is: 2000 ft<sup>2</sup>

The average height of the house is: 9 ft

The elevation of the house is: 5000 ft

The local atmospheric pressure is: 12.2 psia

The cost of electricity in that area is: \$0.075 per kWh

1 kWh = 3412 Btu

Ideal gas constant  $R = 0.3704 \text{ psia ft}^3 \text{ lbm}^{-1} \text{ R}^{-1}$

$$m = \frac{PV}{RT}$$

$$\Delta U = mC_v\Delta T$$

$$\Delta H = mC_p\Delta T$$

where the symbols have their usual meanings.

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Table 1.1 (a)

Molar mass, gas constant, and ideal-gas specific heats of some substances						
Substance	Molar mass M, lbm/lbmol	Gas Constant R*		Specific Heat Data at 77 °F		
		Btu/ lbm · R	psia · ft <sup>3</sup> / lbm · R	$C_p$ , Btu/lbm · R	$C_v$ , Btu/lbm · R	$k = C_p/C_v$
Air	28.97	0.06855	0.3704	0.2400	0.1715	1.400
Ammonia, NH <sub>3</sub>	17.03	0.1166	0.6301	0.4999	0.3834	1.304
Argon, Ar	39.95	0.04970	0.2686	0.1243	0.07457	1.667
Bromine, Br <sub>2</sub>	159.81	0.01242	0.06714	0.0538	0.04137	1.300
Isobutane, C <sub>4</sub> H <sub>10</sub>	58.12	0.03415	0.1846	0.3972	0.3631	1.094
n-Butane, C <sub>4</sub> H <sub>10</sub>	58.12	0.03415	0.1846	0.4046	0.3705	1.092
Carbon dioxide, CO <sub>2</sub>	44.01	0.04512	0.2438	0.2016	0.1564	1.288
Carbon monoxide, CO	28.01	0.07089	0.3831	0.2482	0.1772	1.400
Chlorine, Cl <sub>2</sub>	70.905	0.02802	0.1514	0.1142	0.08618	1.325
Chlorodifluoromethane (R-22), CHClF <sub>2</sub>	86.47	0.02297	0.1241	0.1552	0.1322	1.174

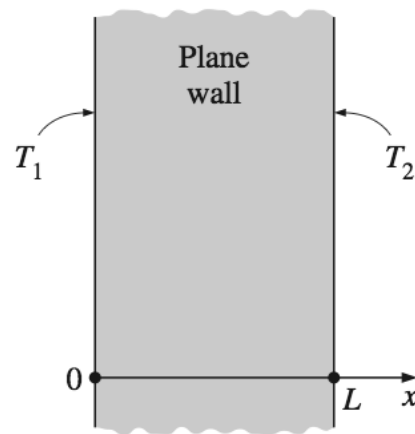
Table 1.2 (a)

Properties of air at 1 atm pressure							
Temp. $T$ , °F	Density $\rho$ , lbm/ft <sup>3</sup>	Specific Heat $C_p$ , Btu/lbm · R	Thermal Conductivity $k$ , Btu/h · ft · R	Thermal Diffusivity $\alpha$ , ft <sup>2</sup> /s	Dynamic Viscosity $\mu$ , lbm/ft · s	Kinematic Viscosity $\nu$ , ft <sup>2</sup> /s	Prandtl Number Pr
-50	0.09683	0.2389	0.01164	$1.397 \times 10^{-4}$	$1.006 \times 10^{-5}$	$1.039 \times 10^{-4}$	0.7439
0	0.08630	0.2401	0.01288	$1.726 \times 10^{-4}$	$1.102 \times 10^{-5}$	$1.278 \times 10^{-4}$	0.7403
10	0.08446	0.2402	0.01312	$1.797 \times 10^{-4}$	$1.121 \times 10^{-5}$	$1.328 \times 10^{-4}$	0.7391
20	0.08270	0.2403	0.01336	$1.868 \times 10^{-4}$	$1.140 \times 10^{-5}$	$1.379 \times 10^{-4}$	0.7378
30	0.08101	0.2403	0.01361	$1.942 \times 10^{-4}$	$1.158 \times 10^{-5}$	$1.430 \times 10^{-4}$	0.7365
40	0.07939	0.2404	0.01385	$2.016 \times 10^{-4}$	$1.176 \times 10^{-5}$	$1.482 \times 10^{-4}$	0.7350
50	0.07783	0.2404	0.01409	$2.092 \times 10^{-4}$	$1.194 \times 10^{-5}$	$1.535 \times 10^{-4}$	0.7336
60	0.07633	0.2404	0.01433	$2.169 \times 10^{-4}$	$1.212 \times 10^{-5}$	$1.588 \times 10^{-4}$	0.7321
70	0.07489	0.2404	0.01457	$2.248 \times 10^{-4}$	$1.230 \times 10^{-5}$	$1.643 \times 10^{-4}$	0.7306
80	0.07350	0.2404	0.01481	$2.328 \times 10^{-4}$	$1.247 \times 10^{-5}$	$1.697 \times 10^{-4}$	0.7290
90	0.07217	0.2404	0.01505	$2.409 \times 10^{-4}$	$1.265 \times 10^{-5}$	$1.753 \times 10^{-4}$	0.7275

(b) As shown in Figure 1 (b), two sides of a large plane wall are maintained at constant temperatures of  $T_1 = 120^\circ\text{C}$  and  $T_2 = 50^\circ\text{C}$ , respectively.

- (i) Provide the problem statement, and then determine the variation of temperature within the wall and the value of temperature at  $x = 0.1$  m. [5 marks]
- (ii) Determine the rate of heat conduction through the wall under steady conditions. [2 marks]
- (iii) State any assumption(s) made. [4 marks]

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**Figure 1 (b)**

Relevant Data and Equations

Thickness of the plane wall:  $L = 0.2 \text{ m}$

Surface area of the plane wall:  $A = 15 \text{ m}^2$

Thermal conductivity of the plane wall:  $1.2 \text{ W m}^{-1} \text{ K}^{-1}$

The differential equation for heat conduction in the wall can be formulated as:

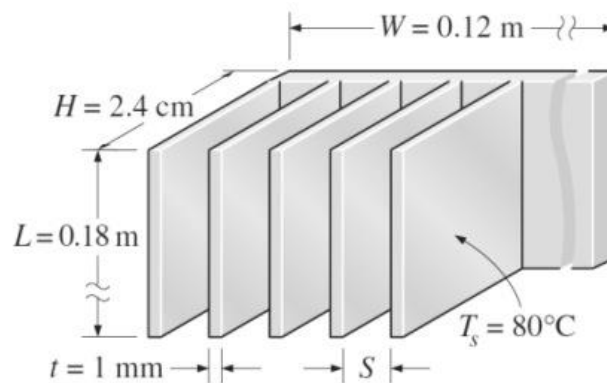
$$\frac{d^2T}{dx^2} = 0$$

Rate of heat conduction based on Fourier's law:

$$\dot{Q} = -kA \frac{dT}{dx}$$

where the symbols have their usual meanings.

2. (a) As shown in Figure 2 (a), a vertical hot surface in air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base.
- (i) Provide the problem statement, and then determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink. [10 marks]
- (ii) State any assumption(s) made. [6 marks]



**Figure 2 (a)**

### Relevant Data and Equations

The width of the vertical hot surface: 12 cm

The height of the vertical hot surface: 18 cm

The air temperature is: 30°C

The base temperature is: 80°C

$$\beta = \frac{1}{T_f}$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr$$

$$S_{opt} = 2.714 \frac{L}{Ra_L^{0.25}}$$

$$Nu = \frac{hS_{opt}}{k} = 1.307$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

where the symbols have their usual meanings.

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Table 2 (a)

Properties of air at 1 atm pressure							
Temp. $T, ^\circ\text{C}$	Density $\rho, \text{kg/m}^3$	Specific Heat $C_p, \text{J/kg} \cdot \text{K}$	Thermal Conductivity $k, \text{W/m} \cdot \text{K}$	Thermal Diffusivity $\alpha, \text{m}^2/\text{s}$	Dynamic Viscosity $\mu, \text{kg/m} \cdot \text{s}$	Kinematic Viscosity $\nu, \text{m}^2/\text{s}$	Prandtl Number Pr
-150	2.866	983	0.01171	$4.158 \times 10^{-6}$	$8.636 \times 10^{-6}$	$3.013 \times 10^{-6}$	0.7246
-100	2.038	966	0.01582	$8.036 \times 10^{-6}$	$1.189 \times 10^{-5}$	$5.837 \times 10^{-6}$	0.7263
-50	1.582	999	0.01979	$1.252 \times 10^{-5}$	$1.474 \times 10^{-5}$	$9.319 \times 10^{-6}$	0.7440
-40	1.514	1002	0.02057	$1.356 \times 10^{-5}$	$1.527 \times 10^{-5}$	$1.008 \times 10^{-5}$	0.7436
-30	1.451	1004	0.02134	$1.465 \times 10^{-5}$	$1.579 \times 10^{-5}$	$1.087 \times 10^{-5}$	0.7425
-20	1.394	1005	0.02211	$1.578 \times 10^{-5}$	$1.630 \times 10^{-5}$	$1.169 \times 10^{-5}$	0.7408
-10	1.341	1006	0.02288	$1.696 \times 10^{-5}$	$1.680 \times 10^{-5}$	$1.252 \times 10^{-5}$	0.7387
0	1.292	1006	0.02364	$1.818 \times 10^{-5}$	$1.729 \times 10^{-5}$	$1.338 \times 10^{-5}$	0.7362
5	1.269	1006	0.02401	$1.880 \times 10^{-5}$	$1.754 \times 10^{-5}$	$1.382 \times 10^{-5}$	0.7350
10	1.246	1006	0.02439	$1.944 \times 10^{-5}$	$1.778 \times 10^{-5}$	$1.426 \times 10^{-5}$	0.7336
15	1.225	1007	0.02476	$2.009 \times 10^{-5}$	$1.802 \times 10^{-5}$	$1.470 \times 10^{-5}$	0.7323
20	1.204	1007	0.02514	$2.074 \times 10^{-5}$	$1.825 \times 10^{-5}$	$1.516 \times 10^{-5}$	0.7309
25	1.184	1007	0.02551	$2.141 \times 10^{-5}$	$1.849 \times 10^{-5}$	$1.562 \times 10^{-5}$	0.7296
30	1.164	1007	0.02588	$2.208 \times 10^{-5}$	$1.872 \times 10^{-5}$	$1.608 \times 10^{-5}$	0.7282
35	1.145	1007	0.02625	$2.277 \times 10^{-5}$	$1.895 \times 10^{-5}$	$1.655 \times 10^{-5}$	0.7268
40	1.127	1007	0.02662	$2.346 \times 10^{-5}$	$1.918 \times 10^{-5}$	$1.702 \times 10^{-5}$	0.7255
45	1.109	1007	0.02699	$2.416 \times 10^{-5}$	$1.941 \times 10^{-5}$	$1.750 \times 10^{-5}$	0.7241
50	1.092	1007	0.02735	$2.487 \times 10^{-5}$	$1.963 \times 10^{-5}$	$1.798 \times 10^{-5}$	0.7228
60	1.059	1007	0.02808	$2.632 \times 10^{-5}$	$2.008 \times 10^{-5}$	$1.896 \times 10^{-5}$	0.7202
70	1.028	1007	0.02881	$2.780 \times 10^{-5}$	$2.052 \times 10^{-5}$	$1.995 \times 10^{-5}$	0.7177
80	0.9994	1008	0.02953	$2.931 \times 10^{-5}$	$2.096 \times 10^{-5}$	$2.097 \times 10^{-5}$	0.7154

(b) You are working as a forensic doctor to investigate a person who died in a room; and the body temperature is measured to be  $25^\circ\text{C}$  when found. The body can be modelled as a 30 cm diameter and 1.70 m long cylinder.

(i) Provide the problem statement, and then estimate the time of death of that person.

[6 marks]

(ii) State any assumption(s) made.

[3 marks]

Continued/...

Q2 Continued/...

### Relevant Data and Equations

The room temperature is: 20°C

The heat transfer coefficient,  $h = 8 \text{ W m}^{-2} \text{ K}^{-1}$

$$L_c = \frac{V}{A_s}$$

$$\text{Bi} = \frac{hL_c}{k}$$

$$b = \frac{h}{\rho C_p L_c}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

where the symbols have their usual meanings.

**Table 2 (b)**

<b>Properties of saturated water</b>													
Temp. $T, ^\circ\text{C}$	Saturation Pressure $P_{\text{sat}}, \text{kPa}$	Density $\rho, \text{kg/m}^3$		Enthalpy of Vaporization $h_{fg}, \text{kJ/kg}$	Specific Heat $C_p, \text{J/kg} \cdot \text{K}$		Thermal Conductivity $k, \text{W/m} \cdot \text{K}$		Dynamic Viscosity $\mu, \text{kg/m} \cdot \text{s}$		Prandtl Number Pr		Volume Expansion Coefficient $\beta, 1/\text{K}$
		Liquid	Vapor		Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid	Vapor	Liquid
0.01	0.6113	999.8	0.0048	2501	4217	1854	0.561	0.0171	$1.792 \times 10^{-3}$	$0.922 \times 10^{-5}$	13.5	1.00	$-0.068 \times 10^{-3}$
5	0.8721	999.9	0.0068	2490	4205	1857	0.571	0.0173	$1.519 \times 10^{-3}$	$0.934 \times 10^{-5}$	11.2	1.00	$0.015 \times 10^{-3}$
10	1.2276	999.7	0.0094	2478	4194	1862	0.580	0.0176	$1.307 \times 10^{-3}$	$0.946 \times 10^{-5}$	9.45	1.00	$0.733 \times 10^{-3}$
15	1.7051	999.1	0.0128	2466	4185	1863	0.589	0.0179	$1.138 \times 10^{-3}$	$0.959 \times 10^{-5}$	8.09	1.00	$0.138 \times 10^{-3}$
20	2.339	998.0	0.0173	2454	4182	1867	0.598	0.0182	$1.002 \times 10^{-3}$	$0.973 \times 10^{-5}$	7.01	1.00	$0.195 \times 10^{-3}$
25	3.169	997.0	0.0231	2442	4180	1870	0.607	0.0186	$0.891 \times 10^{-3}$	$0.987 \times 10^{-5}$	6.14	1.00	$0.247 \times 10^{-3}$
30	4.246	996.0	0.0304	2431	4178	1875	0.615	0.0189	$0.798 \times 10^{-3}$	$1.001 \times 10^{-5}$	5.42	1.00	$0.294 \times 10^{-3}$
35	5.628	994.0	0.0397	2419	4178	1880	0.623	0.0192	$0.720 \times 10^{-3}$	$1.016 \times 10^{-5}$	4.83	1.00	$0.337 \times 10^{-3}$
40	7.384	992.1	0.0512	2407	4179	1885	0.631	0.0196	$0.653 \times 10^{-3}$	$1.031 \times 10^{-5}$	4.32	1.00	$0.377 \times 10^{-3}$
45	9.593	990.1	0.0655	2395	4180	1892	0.637	0.0200	$0.596 \times 10^{-3}$	$1.046 \times 10^{-5}$	3.91	1.00	$0.415 \times 10^{-3}$

3. A heat exchanger has been designed to have an overall heat transfer coefficient of  $1750 \text{ W m}^{-2} \text{ K}^{-1}$  with three different configurations: (i) double pipe co-current flow, (ii) double pipe counter-current flow, and (iii) shell-and-tube with one shell pass and 4-tube passes (oil in the shell side). The heat exchanger will be used to heat water from  $20^\circ\text{C}$  to  $80^\circ\text{C}$  by oil entering the exchanger at  $150^\circ\text{C}$ . The mass flow rate of oil and water are  $6.75 \text{ kg s}^{-1}$  and  $2.25 \text{ kg s}^{-1}$ , respectively.
- (a) State all assumptions made. [3 marks]
- (b) Determine the final temperature of oil leaving the heat exchanger. [2 marks]
- (c) Calculate the log mean temperature difference and sketch the temperature profile for each configuration of the heat exchanger. [6 marks]
- (d) Determine the heat transfer surface area required for each configuration of the heat exchanger. [6 marks]
- (e) If the double-pipe counter-current heat exchanger is subjected to a fouling resistance of  $1 \times 10^{-4} \text{ m}^2 \text{ K W}^{-1}$ , calculate the new oil flow rate required to guarantee the same heat duty (i.e., to heat water from  $20^\circ\text{C}$  to  $80^\circ\text{C}$  flowing at  $2.25 \text{ kg s}^{-1}$ ). [8 marks]

#### Relevant Data and Equations

Specific heat capacity of water =  $4180 \text{ J kg}^{-1} \text{ K}^{-1}$

Specific heat capacity of oil =  $2200 \text{ J kg}^{-1} \text{ K}^{-1}$

The log-mean temperature difference for the heat exchanger is given by:

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

The overall heat transfer coefficient for the dirty heat exchanger is given by:

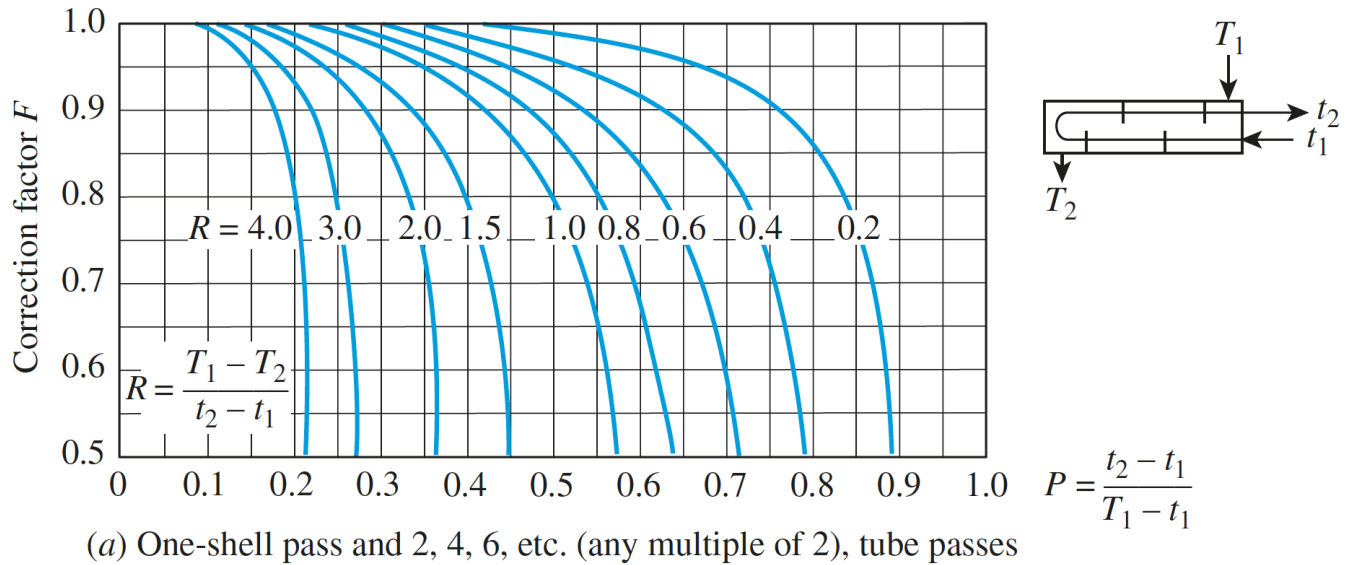
$$\frac{1}{U_{Dirty}} = \frac{1}{U_{Clean}} + R_f$$

where the symbols have their usual meanings.

Continued/...



Q3 Continued/...



**Figure 3.** Correction factor ( $F$ ) chart for counter-flow, multi-pass heat exchangers.  $T$  and  $t$  represent the shell- and tube-side temperatures, respectively.

4. (a) Derive an expression for the net radiation heat transfer between two grey, opaque, diffuse concentric spheres of surface emissivities,  $\varepsilon_1$  and  $\varepsilon_2$ , maintained at uniform temperatures,  $T_1$  and  $T_2$ , to show that

$$\dot{Q}_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1 - \varepsilon_2}{\varepsilon_2} \left(\frac{r_1}{r_2}\right)^2}$$

where the symbols have their usual meanings. Draw the thermal resistance network of the system. [8 marks]

- (b) Liquid nitrogen is stored in a thin-walled, spherical container of 0.6 m in diameter, which is enclosed within a second thin-walled, spherical container of 1.2 m in diameter. The opaque, diffuse, grey container surfaces have emissivities of  $\varepsilon_1 = 0.02$ ,  $\varepsilon_2 = 0.05$ , and are separated by an evacuated space. The outer surface is at 8°C and the inner surface is at -196°C.

Continued/...

Q4 Continued/...

- (i) If the latent heat of vaporisation of nitrogen is  $1.99 \times 10^5 \text{ J kg}^{-1}$ , what is the mass rate of nitrogen loss due to evaporation? [4 marks]
- (ii) If a thin radiation shield of 0.8 m diameter with different surface emissivities of  $\varepsilon_{3,1} = 0.03$  and  $\varepsilon_{3,2} = 0.06$  is inserted midway between the inner and outer surfaces of the container, calculate the percentage reduction in the rate of nitrogen loss due to the shield. [8 marks]
- (iii) Draw the thermal resistance network of the system. [2 marks]
- (iv) State all assumptions made. [3 marks]

Relevant Data and Equations

Stefan-Boltzmann constant,  $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

The surface resistance for a surface  $i$  is given by  $\frac{1-\varepsilon_i}{A_i \varepsilon_i}$

The space resistance between surfaces  $i$  and  $j$  is given by  $\frac{1}{A_i F_{ij}}$

where symbols have their usual meanings.

END OF PAPER

Dr A Islam, Dr T Sun