22CGB012
Mass Transfer and Separations

Semester 2 2022/23 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Attempt **FOUR** questions in total — **TWO** from **EACH SECTION.**

Each question carries 25 marks.

Candidates should show full working for all calculations and derivations.

Formulae and Nomenclature provided at the end of the paper.

Section A

Attempt **TWO** questions

1. (a) Aspirin (acetylsalicylic acid) is used extensively to relieve pain, reduce fever, and relieve redness and swelling caused by arthritis. It is also used to reduce the formation of blood clots that may cause heart attack and stroke. More recently, aspirin has been found to inhibit the growth of small tumours and to prevent deaths from digestive tract cancers. The major disadvantage of aspirin is that it can cause stomach upsets when present in high concentrations in the stomach, especially in those people with sensitive stomachs.

   Using your knowledge related to mass transfer operations suggest a means of preventing stomach upsets for these people. [3 marks]

   (b) A gas stream containing 1.8% acetone is passed through a packed tower to remove 95% of acetone using pure water. The gas mass flux, G_y, is 0.82 kg m^{-2} s^{-1} and the film volumetric mass transfer coefficients for the gas and liquid phases are $k_y a = 0.048$ and $k_x a = 0.266$ kmol s^{-1} m^{-3} mol fraction, respectively. If the water flow rate is 20% in excess of the minimum calculate the following (all compositions are given in mass %):

   (i) The actual water phase mass flux, G_x; [4 marks]

   (ii) The mole fraction of acetone in the exit water stream; [2 marks]

   (iii) $K_y a$, H_{Oy}, H_y, and H_x; (all symbols have their usual meaning) [5 marks]

   (iv) The height of the packing. [5 marks]

   Relevant Data

   The equilibrium relationship is $y^* = 2.53\,x$;

   The molar mass of air is 28.8 g mol^{-1};

   The molar mass of water is 18 g mol^{-1}.

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(c) Through the accidental opening of a valve, water has spilled on the floor of an industrial plant. Estimate the time required to evaporate the water into the surrounding air through a gas film which is 5 mm thick. The water layer is 1 mm thick, the temperature of both air and water may be assumed to be 24°C and the air is at atmospheric pressure (1.013×10^2 kN m^{-2}). [6 marks]

Relevant Data

Diffusion coefficient for water vapour in air is 2.59×10^{-5} m^{-2} s^{-1};

Mole fraction of water vapour in saturated air, $y_{A1,}$ is 0.0295;

Mole fraction of water vapour in main air stream, $y_{A2,}$ is 0.0032;

R is 8.314 J mol^{-1} K^{-1}.

2. (a) Considering equilibrium and mass transfer of the processes explain the following:

(i) Look at a saturated sugar solution in contact with sugar crystals. All that can be seen are sugar crystals in a clear solution. Initially, no changes are apparent, but over time the shape of the crystals can be observed to change. Explain what is taking place. [2 marks]

(ii) What is the fastest way to make a tasty cup of soup from bouillon cubes? Briefly justify your answer. [2 marks]

(b) Consider evaporation in the two different systems (i) and (ii) and calculate the rate of evaporation.

(i) Water in the bottom of a narrow tube is held at a constant temperature of 293 K. The total pressure of air (assumed dry) is 1.01325×10^5 Pa (1 atm) and the temperature is 293 K. Water evaporates and diffuses through the air in the tube and the diffusion path, z_2-z_1, is 0.1524 m long. Calculate the rate of evaporation at steady state in kg mol s^{-1} m^{-2}. [5 marks]

(ii) A sphere of naphthalene having a radius of 2 mm and at a temperature of 318 K is suspended in a large volume of still air at 318 K and 1.01325×10^5 Pa. Calculate the rate of evaporation (sublimation) of naphthalene from the surface. [6 marks]

<u>Relevant Data</u>

Vapour pressure of water at 293 K (p_{A1}) is 2.341×10^3 Pa;

Diffusivity of water vapour at 293 K and 1 atm is 0.25×10^{-4} m^2 s^{-1};

R = 8314 Pa m^3 $kmol^{-1}$ K^{-1};

D_{AB} of naphthalene at 318 K is 6.92×10^{-6} m^2 s^{-1}.

(c) A gas stream containing a low concentration of component A is passed through a packed bed column to remove 95% of the A into pure liquid. The liquid flow rate is 70% in excess of the minimum and the height of transfer unit based on the overall mass transfer coefficient for the gas phase, H_{Oy}, is 0.95 m. Estimate the height, h_T, of packing required. [10 marks]

3. (a) Heart/lung machines have been developed to take the place of the heart and lungs during the course of certain heart operations. The device is an oxygenator that supplies oxygen to the red blood cells and removes carbon dioxide from the blood.

Suggest an effective means of oxygenating blood, assuming the process is mainly mass transfer controlled. [4 marks]

(b) Water is flowing past a flat surface of solid benzoic acid at 0.08 m s^{-1} and 25°C. If the characteristic length of the surface is 200 mm, calculate the mass transfer coefficient and the flux. [5 marks]

Relevant Data

Diffusivity of benzoic acid in water at 25 °C, 1.25x10^{-9} m^2 s^{-1};

Solubility of benzoic acid in water, 29.5 $molm^{-3}$;

Density of liquid water at 25 °C, 0.997 g cm^{-3};

Viscosity of liquid water at 25 °C, 0.8937x10^{-3} Pa s;

$\mathrm{Sc} = \frac{\mu}{\rho D_{AB}}$; $\mathrm{Re} = \frac{Lu\rho}{\mu}$

$J_D = 0.99\mathrm{Re}^{-0.5}$

$J_D = \frac{k_c}{u}\mathrm{Sc}^{2/3}$

All symbols have their usual meaning

Continued/…

(c) Ammonia is to be absorbed from air at 20°C and atmospheric pressure into pure water in a counter-current packed column. The inlet gas rate is 43.6 m^3 h^{-1} and the ammonia free water rate is 34 kg h^{-1}. The volumetric mass transfer coefficients remain constant throughout the height of the column and have the following values:

$K_x a$=0.15 kmol s^{-1} m^{-3} mol fraction; $k_y a$=0.042 kmol s^{-1} m^{-3} mol fraction. The equilibrium constant for the system is 1.1 and the wetted packing has a specific surface area of 120 m^{-1}.

(i) If the ammonia concentration is reduced from 3.52% to 1.29%, determine the inlet and outlet gas concentrations in molar ratios. [2 marks]

(ii) What are the solute free water and gas flow rates expressed as kmol h^{-1}? [3 marks]

(iii) What is the mole fraction of ammonia in the outlet water stream? [2 marks]

(iv) Determine the overall mass transfer coefficient K_y in kmol s^{-1} m^{-2}. [2 marks]

(v) What is the percentage resistance on the gas side? [1 mark]

(vi) Determine the mass transfer flux, N (kmol s^{-1} m^{-2}.), at the bottom of the column. [2 marks]

(vii) What are the interfacial compositions at the bottom of the column? [2 marks]

(viii) What is the mass transfer flux, N, at the top of the column? [2 marks]

(all compositions are given in mass %).

Section B

Attempt **TWO** questions

4. An absorption column is required to scrub a nitrogen stream containing 3.0 vol% ethylene oxide, where the nitrogen flowrate is 280 kg h^{-1}. The column operating pressure is set at 20 bara and it can be assumed to be operating isothermally at 30°C. The scrubbing liquid is water which contains 0.1 wt% ethylene oxide when it enters the column. The incoming gas stream can be considered to be saturated with water so no evaporation of water occurs.

   (a) For a water flowrate of 235 kg h^{-1}, calculate the number of ideal trays required to remove 94% of the ethylene oxide from the gas stream. [10 marks]

   (b) What would the theoretical minimum flowrate of water (in kg h^{-1}) be? [6 marks]

   (c) After the column has been constructed (i.e. keeping the same number of trays), it is now required that 97.5% of the ethylene oxide is removed from the gas stream. One option is to change the liquid flowrate. If the column is to be operated at the same temperature and pressure, what would the new liquid flowrate have to be (to the nearest kmol h^{-1}) to meet the new specification? [9 marks]

   <u>Data</u>

   Henry Law constant for ethylene oxide in water at 30 °C = 24 bar;
   Molecular masses:

   water = 18 kg $kmol^{-1}$
   nitrogen = 28 kg $kmol^{-1}$
   ethylene oxide = 44 kg $kmol^{-1}$

   The Kremser equation is:

$$N+1=\frac{\ln\left(\frac{A-\phi}{1-\phi}\right)}{\ln A}\ , \qquad \text{where } A=\frac{L}{mG}\ ,\phi=\frac{Y_B-Y_T}{Y_B-Y_T^*}$$

   All the terms have their usual meaning.

5. A binary mixture of ethanol and butanol is separated using an existing plate distillation column containing a total condenser and a partial reboiler. The feed flow rate is 1000 kmol h^{-1} and is supplied as a two-phase mixture. The column operates at a constant pressure of 1 bara and recovers 42.4% of the feed ethanol in the distillate product. In addition, 280 kmol h^{-1} of liquid is to be withdrawn as a side stream. All the plates in the column have a constant Murphree vapour phase efficiency. Constant molar overflow can be assumed. A rating calculation has been carried out on the column, as shown in Figure Q5 overleaf (McCabe-Thiele diagram). Using this diagram, answer <u>all</u> the following parts:

(a) Find the mole fractions of the three product streams, and the feed stream mole fraction and thermodynamic quality. [6 marks]

(b) Estimate the Murphree vapour phase efficiency (use a stage with a large degree of separation) and find the actual number of plates in the column. Comment on the Murphree vapour phase efficiency of the reboiler. [4 marks]

(c) Find the plate number of the feed and side streams (counting from the bottom of the column). Comment on the position of the feed and side stream plates and the effect these have on the recovery of ethanol in the distillate. [3 marks]

(d) Find the flow rates of the side stream, distillate and bottoms product streams. [5 marks]

(e) From the diagram, show that the reflux ratio is $R = 8.3$, and hence find the vapour and liquid flow rates in the bottom section of the column. [7 marks]

<u>Equations</u>

Binary distillation, top operating line

$$y_n = \frac{R}{R+1}x_{n+1} + \frac{x_D}{R+1}$$

Binary distillation, q-line

$$y = \left(\frac{-q}{1-q}\right)x + \frac{z}{1-q}$$

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Q5 Continued/…

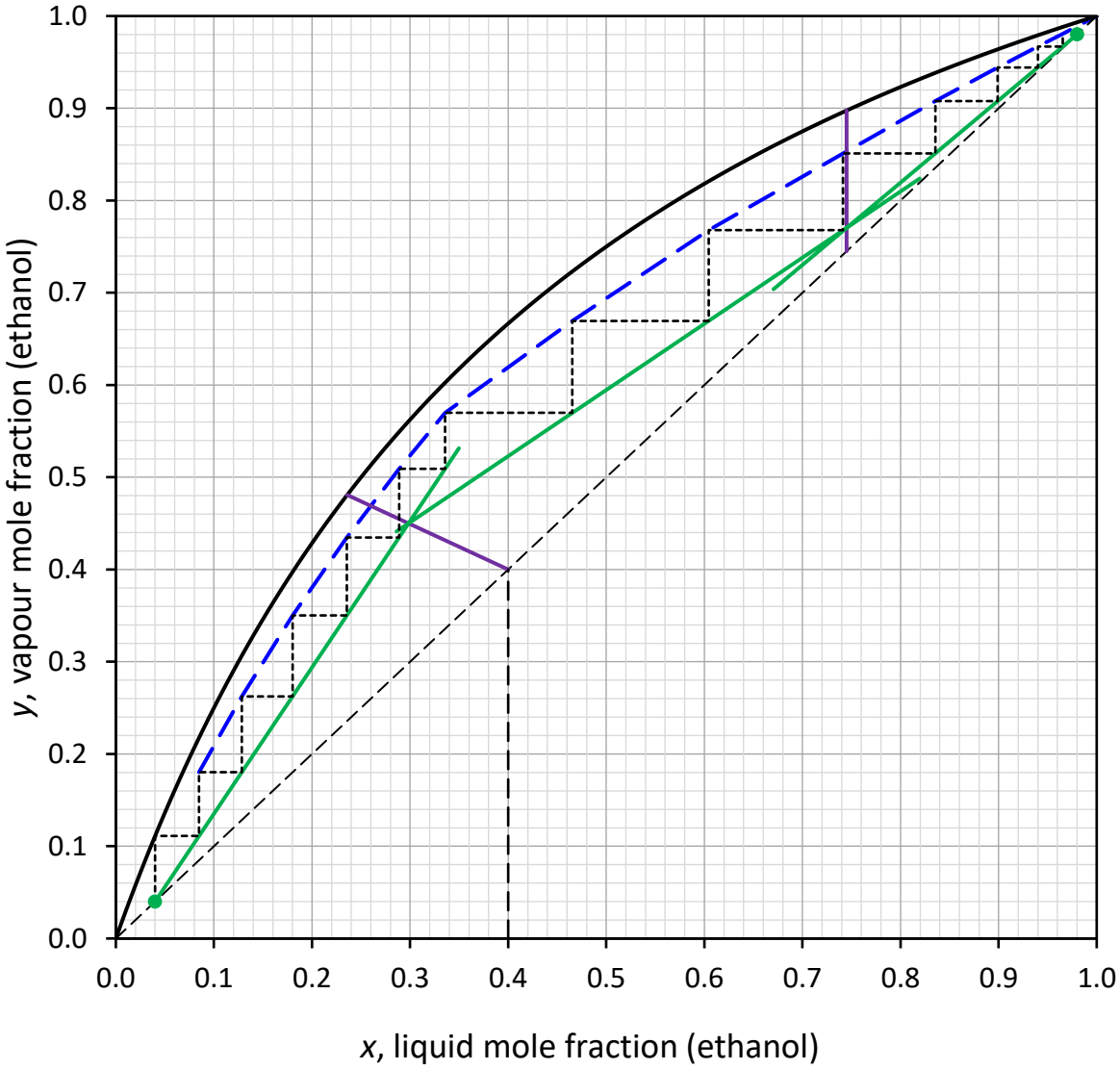

Figure Q5: McCabe-Thiele construction (1 bar)

6. A gaseous mixture with the flow rates shown below in Table Q6, is fed to a partial condenser, which operates at 4.0 bara. The nitrogen may be assumed to be an incondensable gas at the condenser pressure and temperature. The partial condenser acts like a flash separation stage, producing gas and liquid streams of composition y_i and x_i, respectively, in equilibrium with each other according to $y_i = K_i x_i$.

(a) Sketch a flow diagram of the flash unit, labelling the streams with nomenclature to show their flow rates and mole fractions. [2 marks]

(b) By carrying out mass balances and using the equilibrium relationship, show that the mole fractions of the hydrocarbons (HC) in the liquid condensate are given by:

$$x_i = \frac{z_i}{1+(K_i-1)\phi}$$

where ϕ is the vapour to feed molar flow rate ratio and z_i is the feed mole fraction. [5 marks]

(c) Show that the dew point of the feed mixture is approximately 69 °C. [5 marks]

(d) Show that 83.1% of the hydrocarbons in the feed may be recovered as liquid product, if the flash unit is operated at 20°C. [8 marks]

(e) Hence determine the mole fraction compositions of the vapour and liquid streams leaving the partial condenser if this is operated at 20°C. [5 marks]

Table Q6: Feed flow rates

Component	propane	n-butane	n-pentane	n-hexane	nitrogen
Feed (kmol h^{-1})	15	30	30	10	15

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Q6 Continued/…

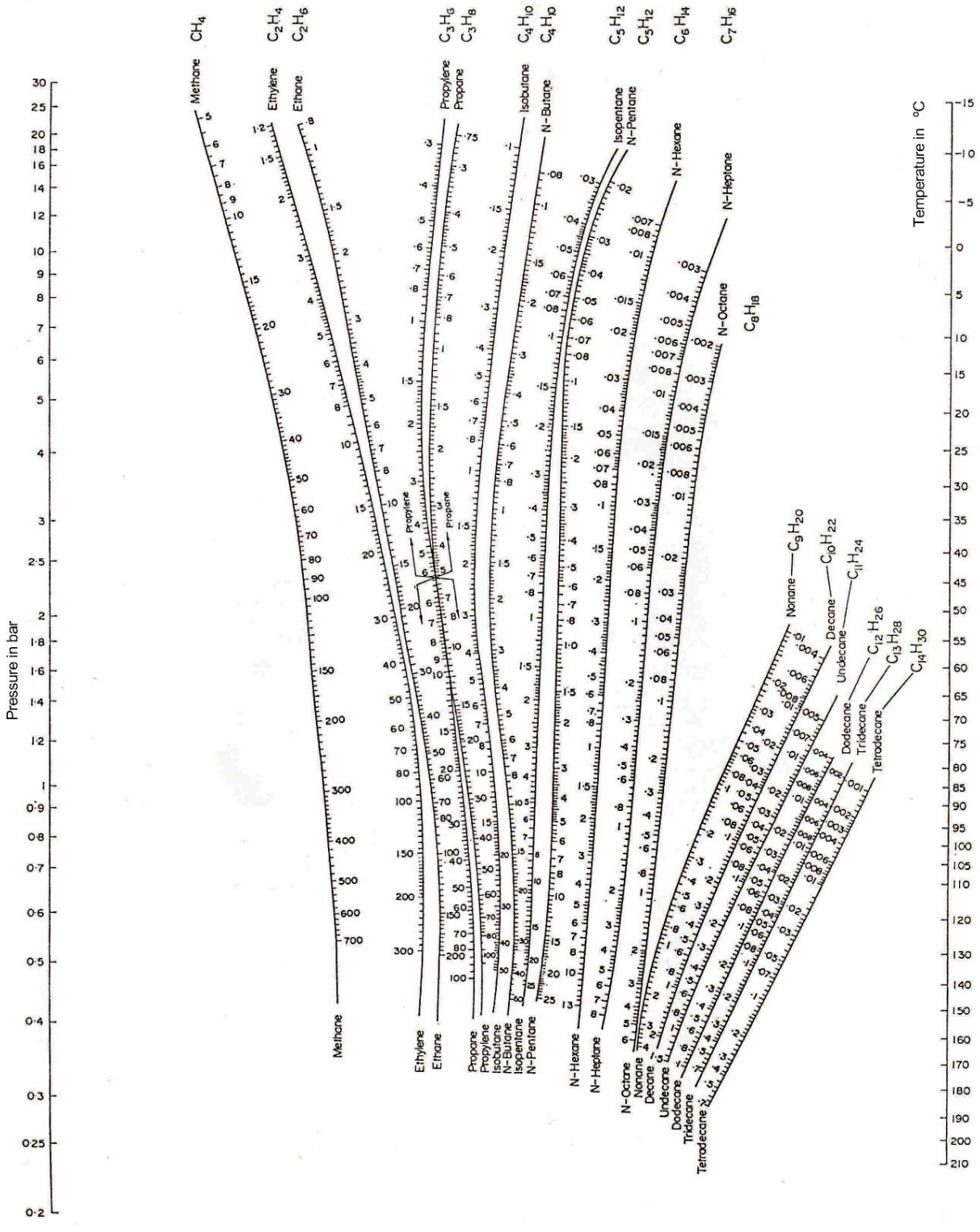

Figure Q6: K-factor nomogram for selected hydrocarbons

END OF PAPER

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Mass Transfer - Formulae and Nomenclature

CGB012

1. Transport phenomena

- General transport equation

$$\text{rate of transport process} = \frac{\text{driving force}}{\text{resistance}}$$

$$\psi_z = -\delta \frac{d\Gamma}{dz} \tag{1}$$

$$\psi_z \int_{z_1}^{z_2} dz = -\delta \int_{\Gamma_1}^{\Gamma_2} d\Gamma \tag{2}$$

$$\psi_z = \frac{\delta(\Gamma_1 - \Gamma_2)}{z_2 - z_1} \tag{3}$$

(rate of property in) = (rate of property out) + (rate of accumulation of property) (4)

$$\psi_{z|z} = \psi_{z|z+\Delta z} + \frac{\partial \Gamma}{\partial t} \Delta z \tag{5}$$

$$\frac{\partial \Gamma}{\partial t} = \delta \frac{\partial^2 \Gamma}{\partial z^2} \tag{6}$$

- Molecular diffusion equations for momentum, heat and mass transfer

$$\tau_{zx} = -\nu \frac{d(\upsilon_x \rho)}{dz} \tag{7}$$

$$\frac{q_z}{A} = -\alpha \frac{d(\rho C_p T)}{dz} \tag{8}$$

Fick's law for molecular transport of mass in a fluid or solid

$$J_{Az}^{*} = -D_{AB}\frac{dc_A}{dz} \tag{9}$$

2. Mass Transfer

- Fick's law

$$J_{Az}^{*} = -cD_{AB}\frac{dx_A}{dz} \tag{10}$$

$$J_{Az}^{*} = -D_{AB}\frac{dc_A}{dz} \tag{11}$$

Binary systems – concentrations.

ρ_A : mass concentration (density) of A (kg m^{-3})
$\rho = \rho_A+\rho_B$: total mass concentration of A+B mixture (kg m^{-3})
$\omega_A = x_A M_A/(x_A M_A+x_B M_B) = \rho_A/\rho$: mass fraction of A
$\omega_A+\omega_B = 1$
$c_A = n_A/V = \rho_A/M_A$: molar concentration of A (kmol m^{-3})
$c = c_A+c_B$: total molar concentration of A+B mixture (kmol m^{-3})
$x_A = c_A/c$: mole fraction of A in liquid phase (y_A in gas phase **)
$x_A+x_B = 1$
M_A : relative molecular mass (molecular weight) of A (kg kmol^{-1})
$M = x_A M_A+x_B M_B = \rho/c$: mass(A+B)/mole(A+B)

** For gases, using the ideal gas law $p_A V = n_A RT$

$$c_A = \frac{p_A}{RT} \qquad y_A = \frac{c_A}{c} = \frac{p_A}{P}$$

p_A : partial pressure of A (Pa)
T : absolute temperature (K)
R : gas constant (Pa m^3 mol^{-1} K^{-1})
V : gas volume (m^3)
P : total pressure (Pa)

- Equimolar counter-diffusion

$$J_A^{*} = -J_B^{*} \rightarrow -D_{AB}\frac{dc_A}{dz} = D_{BA}\frac{dc_B}{dz} \tag{12}$$

$$c = c_A + c_B \rightarrow dc_A = -dc_B \tag{13}$$

3. Diffusion coefficient

The Prandtl number:	$\mathrm{Pr} = \frac{\nu}{\alpha}$
The Schmidt number:	$\mathrm{Sc} = \frac{\nu}{D_{AB}}$
The Lewis number:	$\mathrm{Le} = \frac{\alpha}{D_{AB}}$

- Diffusion coefficients in gases

Chapman-Enskog kinetic theory

$$D_{AB} = \frac{1.86\times10^{-3}T^{3/2}\left[\frac{1}{M_A}+\frac{1}{M_B}\right]^{1/2}}{p\sigma_{AB}^2\Omega_{D,AB}} \tag{14}$$

$$\sigma_{AB} = \tfrac{1}{2}(\sigma_A + \sigma_B) \tag{15}$$

$$\varepsilon_{AB}/\mathrm{k_B} = \sqrt{(\varepsilon_A/\mathrm{k_B})(\varepsilon_B/\mathrm{k_B})} \tag{16}$$

$$D_{AB} = 10^{-3}\frac{T^{1.75}\left[\frac{1}{M_A}+\frac{1}{M_B}\right]^{1/2}}{p\left[\left(\sum_i V_{iA}\right)^{1/3}+\left(\sum_i V_{iB}\right)^{1/3}\right]} \tag{17}$$

- Diffusion coefficients in liquids

Nernst-Einstein equation

$$D_{AB} = \mathrm{k_B}T\left(\upsilon_A / F_A\right) \tag{18}$$

$$\frac{\upsilon_A}{F_A} = \left(\frac{3\mu_B + R_A\beta_{AB}}{2\mu_B + R_A\beta_{AB}}\right)\frac{1}{6\pi\mu_B R_A} \tag{19}$$

Non-slip condition $\beta_{AB} = \infty$:

$$\frac{D_{AB}\mu_B}{k_B T} = \frac{1}{6\pi R_A} \tag{20}$$

Complete slip condition $\beta_{AB} = 0$:

$$\frac{D_{AB}\mu_B}{k_B T} = \frac{1}{4\pi R_A} \tag{21}$$

$$D_{AB} = 7.4\times10^{-8}\frac{\sqrt{\psi_B M_B}T}{\mu U_A^{0.6}} \tag{22}$$

- Diffusion coefficients in solids

$$D_{AB} = D_0 e^{-E/RT} \tag{23}$$

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4. Diffusion of gases A & B plus convection

$$J_A^* \text{ (kmol A m}^{-2}\text{ s}^{-1}) = \upsilon_{Ad} \text{ (m s}^{-1}) \, c_A \text{ (kmol A m}^{-3}) \tag{24}$$

Binary systems – velocities.

υ_A : velocity of A relative to stationary point (m s^{-1})

υ_B : velocity of B relative to stationary point (m s^{-1})

$\upsilon = \omega_A\upsilon_A + \omega_B\upsilon_B = (\rho_A\upsilon_A + \rho_B\upsilon_B)/\rho$: mass average velocity of the bulk stream relative to stationary point (m s^{-1})

$\upsilon^* = x_A\upsilon_A + x_B\upsilon_B = (c_A\upsilon_A + c_B\upsilon_B)/c$: molar average velocity of the bulk stream relative to stationary point (m s^{-1})

$\upsilon_{Ad} = \upsilon_A - \upsilon^*$: diffusion velocity of A relative to the molar average velocity of the bulk stream (m s^{-1})

$\upsilon_{Bd} = \upsilon_B - \upsilon^*$: diffusion velocity of B relative to the molar average velocity of the bulk stream (m s^{-1})

$$\upsilon_A = \upsilon_{Ad} + \upsilon^* \tag{25}$$

$$c_A \upsilon_A = c_A \upsilon_{Ad} + c_A \upsilon^* \tag{26}$$

$$N_A = J_A^* + c_A \upsilon^* \tag{27}$$

$$N = c\upsilon^* = N_A + N_B \rightarrow \upsilon^* = \frac{N_A + N_B}{c} \tag{28}$$

$$N_A = J_A^* + \frac{c_A}{c}\left(N_A + N_B\right) \tag{29}$$

$$N_A = -cD_{AB}\frac{dx_A}{dz} + \frac{c_A}{c}\left(N_A + N_B\right) \tag{30}$$

$$N_B = -cD_{BA}\frac{dx_B}{dz} + \frac{c_B}{c}\left(N_A + N_B\right) \tag{31}$$

$$N_A = J_A^* = -N_B = -J_B^* \tag{32}$$

- Diffusion of A through stagnant non-diffusing B

$$N_A = -cD_{AB}\frac{dx_A}{dz} + \frac{c_A}{c}N_A \tag{33}$$

$$N_A = -\frac{D_{AB}}{RT}\frac{dp_A}{dz} + \frac{p_A}{P}N_A \tag{34}$$

$$N_A = \frac{D_{AB}P}{RT\left(z_2 - z_1\right)}\ln\frac{P - p_{A2}}{P - p_{A1}} \tag{35}$$

the log mean value of the inert B

$$p_{BM} = \frac{p_{B2} - p_{B1}}{\ln\left(p_{B2}/p_{B1}\right)} = \frac{p_{A1} - p_{A2}}{\ln\left[\left(P - p_{A2}\right)/\left(P - p_{A1}\right)\right]} \tag{36}$$

$$N_A = \frac{D_{AB}P}{RT\left(z_2 - z_1\right)p_{BM}}\left(p_{A1} - p_{A2}\right) \tag{37}$$

5. Mass Transfer coefficient

- Definition of mass transfer coefficient

$$\left(\begin{array}{c}\text{Rate of mass}\\ \text{transferred}\end{array}\right) = k \left(\text{Interfacial area}\right) \left(\begin{array}{c}\text{Concentration}\\ \text{difference}\end{array}\right)$$

$$N_{A0} = k(c_{A0} - c_A) \tag{38}$$

- Other definitions of mass transfer coefficients

Common definitions of mass transfer coefficients.

Basic equation	**Typical units of *k***	**Remarks**
$N = k\,\Delta c$	cm s^{-1}	Common in the older literature
$N = k_p\,\Delta p$	mol cm^{-2} Pa^{-1} s^{-1}	Common in gas absorption
$N = k_x\,\Delta x$ $N = k_y\,\Delta y$	mol cm^{-2} s^{-1}	Preferred for practical calculations, especially in gases

- Correlations of mass transfer coefficients

Dimensionless numbers.

Sherwood number: $\mathrm{Sh} = \dfrac{kL}{D}$

Stanton number: $\mathrm{St} = \dfrac{k}{u}$

Schmidt number: $\mathrm{Sc} = \dfrac{\nu}{D}$

Lewis number: $\mathrm{Le} = \dfrac{\alpha}{D}$

Prandtl number: $\mathrm{Pr} = \dfrac{\nu}{\alpha}$

Reynolds number: $\mathrm{Re} = \dfrac{Lu}{\nu}$

Péclet number: $\mathrm{Pe} = \dfrac{Lu}{D}$

k: mass transfer coefficient
L : characteristic length
D : diffusion coefficient
u : fluid velocity
ν : kinematic viscosity
α : thermal diffusivity

6. Interface mass transfer

$$k \propto u^{2/3} D^{1/2} \tag{39}$$

$$N_{A0} = k(c_{A0} - c_A) \tag{40}$$

$$N_{A0} = \frac{D_{AB}}{l}(c_{A0} - c_A) \tag{41}$$

$$k = \frac{D_{AB}}{l} \tag{42}$$

$$N_{A0} = 2\sqrt{D_{AB}u_{\max}/\pi L}(c_{A0} - c_A) \tag{43}$$

$$k = 2\sqrt{D_{AB}u_{\max}/\pi L} \tag{44}$$

$$N_{A0} = \sqrt{D_{AB}/\tau}(c_{A0} - c_A) \tag{45}$$

$$k = \sqrt{D_{AB}/\tau} \tag{46}$$

$$k_{avg} = 0.664\,\frac{D_{AB}}{L}\,\mathrm{Re}^{0.5}\,\mathrm{Sc}^{1/3} \tag{47}$$

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7. Mass transfer between phases (Interface mass transfer)

Rault's law

$$p_A = x_A P_A \tag{48}$$

Dalton's law

$$p_A = y_A P \tag{49}$$

Raoult-Dalton law

$$x_A p_A = y_A P \tag{50}$$

Henry's law

$$p_A = H x_A \tag{51}$$

Distribution law

$$c_{A,\text{liquid}\,1} = K c_{A,\text{liquid}\,2} \tag{52}$$

- Interface mass transfer

$$y_{A0} = f(x_{A0}) \tag{53}$$

$$N_{A0,G} = N_{A0,L} = N_{A0} = k_y(y_{AG} - y_{A0}) = k_x(x_{A0} - x_{AL}) \tag{54}$$

$$-\frac{k_x}{k_y} = \frac{y_{AG} - y_{A0}}{x_{AL} - x_{A0}} \tag{55}$$

the overall mass transfer coefficients

$$N_{A0} = K_y(y_{AG} - y_A^*) = K_x(x_A^* - x_{AL}) \tag{56}$$

$$y_{AG} - y_A^* = (y_{AG} - y_{A0}) + (y_{A0} - y_A^*) \tag{57}$$

$$m_y = \frac{y_{A0} - y_A^*}{x_{A0} - x_{AL}} \tag{58}$$

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m_y}{k_x} \tag{59}$$

$$\frac{1}{K_x} = \frac{1}{m_x k_y} + \frac{1}{k_x} \tag{60}$$

$$\frac{1}{K_y} \cong \frac{1}{k_y} \tag{61}$$

$$\frac{1}{K_x} \cong \frac{1}{k_x} \tag{62}$$

$$N_{A0} = k_p(p_{AG} - p_{A0}) = k_x(x_{A0} - x_{AL}) \tag{63}$$

$$N_{A0} = K_p(p_{AG} - p_A^*) = K_x(x_A^* - x_{AL}) \tag{64}$$

$$\frac{1}{K_x} = \frac{1}{Hk_p} + \frac{1}{k_x} \tag{65}$$

$$\frac{1}{K_p} = \frac{1}{k_p} + \frac{H}{k_x} \tag{66}$$

Useful conversions between mass transfer coefficients.

Gases: $N_{A0} = k_c(c_{A0} - c_A) = k_p(p_A - p_{A0}) = k_y(y_A - y_{A0})$

$$k_c c = k_c \frac{P}{RT} = k_p P = k_y$$

Liquids: $N_{A0} = k_c(c_{A0} - c_A) = k_x(x_{A0} - x_A)$

$$k_c c = k_c \frac{\rho}{M} = k_x$$

Units: k_c (m s^{-1}); k_p (kmol m^{-2} Pa^{-1} s^{-1}); k_x, k_y (kmol m^{-2} s^{-1})

8. Gas Absorption

the capacity parameter

$$C_s F_p^{0.5} \nu^{0.05} = u_y \left[\rho_y / \left(\rho_x - \rho_y\right)\right]^{0.5} F_p^{0.5} \nu^{0.05} \tag{67}$$

the flow parameter

$$\frac{G_x}{G_y}\left(\frac{\rho_y}{\rho_x}\right)^{0.5} \tag{68}$$

the flood pressure drop

$$\Delta P_{flood} = 0.12 F_p^{0.7} \tag{69}$$

- Material balances and operating line

  $L_{M\text{-}in}\ x_{in} + G_{M\text{-}in}\ y_{in} = L_{M\text{-}out}\ x_{out} + G_{M\text{-}out}\ y_{out}$ (70)

$$L_M x_{in}+G_M y=L_M x+G_M y_{out} \tag{71}$$

the operating line equation

$$y = \frac{L_M}{G_M}(x - x_{in}) + y_{out} \tag{72}$$

$$y^* = m\,x \tag{73}$$

- Mass transfer coefficients for packed towers

$$r = k_y a(y - y_0) = k_x a(x_0 - x) \tag{74}$$

$$\frac{y - y_0}{x - x_0} = -\frac{k_x a}{k_y a} \tag{75}$$

$$r = K_y a(y - y^*) = K_x a(x^* - x) \tag{76}$$

$$\frac{1}{K_x a} = \frac{1}{k_x a} + \frac{1}{k_y a}\left(\frac{x^* - x_0}{y - y_0}\right) \tag{77}$$

$$\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{1}{k_x a}\left(\frac{y_0 - y^*}{x_0 - x}\right) \tag{78}$$

$$m = \frac{y_0 - y^*}{x_0 - x} = \frac{y - y_0}{x^* - x_0} \tag{79}$$

$$\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{m}{k_x a} \tag{80}$$

$$\frac{1}{K_x a} = \frac{1}{k_x a} + \frac{1}{m k_y a} \tag{81}$$

- Calculation of packed tower height

(amount absorbed in section *dh*) = (absorption rate) · (differential volume)

$$-G_M S dy = K_y a(y - y^*) S dh \tag{82}$$

$$\frac{K_y a}{G_M}\int dh = \frac{K_y a\, h_T}{G_M} = \int_{y_{out}}^{y_{in}} \frac{dy}{y - y^*} \tag{83}$$

$$h_T = \frac{G_M}{K_y a}\int_{y_{out}}^{y_{in}} \frac{dy}{y - y^*} \tag{84}$$

$$h_T = H_{Oy} \cdot N_{Oy} \tag{85}$$

the *overall height of a transfer unit* (HTU

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$$H_{Oy} = \frac{G_M}{K_y a} \qquad\qquad H_{Oy} = \frac{G}{K_y a S}$$

$$N_{Oy} = \int_{y_{out}}^{y_{in}} \frac{dy}{y - y^*} \tag{87}$$

HTUs and NTUs based on the basic four types of individual and overall volumetric mass transfer coefficients.

Gas film:	$H_y = \frac{G_M}{k_y a}$	$N_y = \int \frac{dy}{y - y_0}$
Liquid film:	$H_x = \frac{L_M}{k_x a}$	$N_x = \int \frac{dx}{x_0 - x}$
Overall gas:	$H_{Oy} = \frac{G_M}{K_y a}$	$N_{Oy} = \int \frac{dy}{y - y^*}$
Overall liquid:	$H_{Ox} = \frac{L_M}{K_x a}$	$N_{Ox} = \int \frac{dx}{x^* - x}$

Straight operating and equilibrium lines

$$N_{Oy} = \frac{y_{in} - y_{out}}{(y - y^*)_M} \tag{88}$$

$$(y - y^*)_M = \frac{(y_{in} - y_{in}^*) - (y_{out} - y_{out}^*)}{\ln\left[(y_{in} - y_{in}^*)/(y_{out} - y_{out}^*)\right]} \tag{89}$$

$$N_{Ox} = \frac{x_{out} - x_{in}}{(x^* - x)_M} \tag{90}$$

$$(x^* - x)_M = \frac{(x^*_{out} - x_{out}) - (x^*_{in} - x_{in})}{\ln\left[(x^*_{out} - x_{out}) / (x^*_{in} - x_{in})\right]} \tag{91}$$

absorption factor, *A*

$$A = \frac{L_M}{mG_M} \tag{92}$$

$$N_{Oy} = \ln\left[\left(1 - \frac{1}{A}\right)\left(\frac{y_{in} - mx_{in}}{y_{out} - mx_{in}}\right) + \frac{1}{A}\right] \Big/ \left(1 - \frac{1}{A}\right) \tag{93}$$

$$N_{Oy} = N_t \ln\left(\frac{1}{A}\right) \Big/ \left(\frac{1 - A}{A}\right) \tag{94}$$

$$\text{HETP} = H_{Oy} \ln\left(\frac{1}{A}\right) \Big/ \left(\frac{1 - A}{A}\right) \tag{95}$$

Straight and parallel operating and equilibrium lines

$$N_t = N_{Oy} = \frac{y_{in} - y_{out}}{y - y^*} \tag{96}$$

$$\text{HETP} = \text{HTU} \tag{97}$$

- Alternative forms of mass transfer coefficients

$$k_g a = k_y a / P \qquad K_g a = K_y a / P \tag{98}$$

$$k_L a = k_x a \,/ c \qquad K_L a = K_x a / c \tag{99}$$

$$H_y = \frac{G_M}{k_g a P} \qquad H_{Oy} = \frac{G_M}{K_g a P} \tag{100}$$

$$H_x = \frac{G_x/\rho}{k_L a} \qquad H_{Ox} = \frac{G_x/\rho}{K_L a} \tag{101}$$

$$\frac{1}{K_y a} = \frac{1}{k_y a} + \frac{m}{k_x a} \rightarrow \frac{G_M}{K_y a} = \frac{G_M}{k_y a} + \frac{G_M m}{k_x a}\left(\frac{L_M}{L_M}\right) \tag{102}$$

$$H_{Oy} = H_y + \frac{G_M m}{L_M} H_x \rightarrow H_{Oy} = H_y + \frac{H_x}{A} \tag{103}$$

$$H_{Ox} = H_x + H_y A \tag{104}$$

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$$\frac{1}{N_{Oy}} = \frac{1}{N_y} + \frac{1}{AN_x} \qquad \frac{1}{N_{Ox}} = \frac{1}{N_x} + \frac{A}{N_y}$$

- Absorption of concentrated mixtures in packed towers:

$$L' = L(1-x) \tag{106}$$

$$G' = G(1-y) \tag{107}$$

$$d(Gy) = G' d\left(\frac{y}{1-y}\right) = G' \frac{dy}{(1-y)^2} = G \frac{dy}{(1-y)} \tag{108}$$

$$d(Lx) = L' d\left(\frac{x}{1-x}\right) = L' \frac{dx}{(1-x)^2} = L \frac{dx}{(1-x)} \tag{109}$$

$$h_T = \int_{y_{out}}^{y_{in}} \left(\frac{G}{K'_y aS}\right) \frac{dy}{(1-y)(y-y^*)} = \frac{G}{K'_y aS} \int_{y_{out}}^{y_{in}} \frac{dy}{(1-y)(y-y^*)} \tag{110}$$

$$h_T = \int_{x_{in}}^{x_{out}} \left(\frac{L}{K'_x aS}\right) \frac{dx}{(1-x)(x^*-x)} = \frac{L}{K'_x aS} \int_{x_{in}}^{x_{out}} \frac{dx}{(1-x)(x^*-x)} \tag{111}$$

$$K'_y = K_y/(1-y)_{*M} \qquad K'_x = K_x/(1-x)_{*M}$$

$$(1-y)_{*M} = \frac{(1-y)-(1-y^*)}{\ln[(1-y)/(1-y^*)]} \tag{112}$$

$$(1-x)_{*M} = \frac{(1-x)-(1-x^*)}{\ln\left[(1-x)/(1-x^*)\right]} \tag{113}$$

$$h_T = \int_{y_{out}}^{y_{in}} \left(\frac{G}{K'_y\, a(1-y)_{*M} S} \right) \frac{(1-y)_{*M}\, dy}{(1-y)(y-y^*)} \tag{114}$$

$$h_T = \int_{x_{in}}^{x_{out}} \left(\frac{L}{K'_x\, a(1-x)_{*M} S} \right) \frac{(1-x)_{*M}\, dx}{(1-x)(x^*-x)} \tag{115}$$

$$h_T = \frac{L}{K'_x\, a(1-x)_{*M} S} \int_{x_{in}}^{x_{out}} \frac{(1-x)_{*M}\, dx}{(1-x)(x^*-x)} \tag{116}$$

$$h_T = \frac{G}{K'_y\, a(1-y)_{*M} S} \int_{y_{out}}^{y_{in}} \frac{(1-y)_{*M}\, dy}{(1-y)(y-y^*)} \tag{117}$$

$$G + L_{in} = G_{out} + L \tag{118}$$

$$L_{in} x_{in} + Gy = Lx + G_{out} y_{out} \tag{119}$$

:

$$L_{in} = L(1-x) \tag{120}$$

$$y = \frac{G_{out} y_{out} + \left[L_{in} x/(1-x)\right]}{G_{out} + \left[L_{in} x/(1-x)\right]} \tag{121}$$

$$L' X_{in} + G' Y_{in} = L' X_{out} + G' Y_{out} \quad \rightarrow$$

$$L' \frac{x_{in}}{1-x_{in}} + G' \frac{y_{in}}{1-y_{in}} = L' \frac{x_{out}}{1-x_{out}} + G' \frac{y_{out}}{1-y_{out}} \tag{122}$$

$$L' X + G' Y_{out} = L' X_{in} + G' Y \quad \rightarrow$$

$$L' \frac{x}{1-x} + G' \frac{y_{out}}{1-y_{out}} = L' \frac{x_{in}}{1-x_{in}} + G' \frac{y}{1-y} \tag{123}$$

$$h_T = \int_{Y_{out}}^{Y_{in}} \left(\frac{G'}{K'_Y\, aS} \right) \frac{dY}{(Y-Y^*)} = \frac{G'}{K'_Y\, aS} \int_{Y_{out}}^{Y_{in}} \frac{dY}{(Y-Y^*)} \tag{124}$$

$$h_T = \int_{X_{in}}^{X_{out}} \left(\frac{L'}{K'_X\, aS} \right) \frac{dX}{(X^*-X)} = \frac{L'}{K'_X\, aS} \int_{X_{in}}^{X_{out}} \frac{dX}{(X^*-X)} \tag{125}$$

Nomenclature

A	Absorption factor
a	Area of interface per unit packed volume, $m^2\ m^{-3}$ (m^{-1}) or $ft^2\ ft^{-3}$ (ft^{-1})
c	Molar concentration, kmol m^{-3} ($c=n/V=\rho/M$)
C_S	C-factor, m s^{-1} or ft s^{-1}
D	Molecular diffusivity, $m^2\ s^{-1}$
F_p	Packing factor, ft^{-1}
G	Gas phase molar flow rate, kmol s^{-1} ($G=G_M\ S$)
G'	Gas phase molar flow rate on a solute-free basis, kmol s^{-1} [$G'=G(1-y)$]
G_M	Gas phase molar flux, kmol $m^{-2}\ s^{-1}$ or lb-mol $ft^{-2}\ h^{-1}$
G_x	Liquid phase mass flux, kg $m^{-2}\ s^{-1}$ or lb $ft^{-2}\ h^{-1}$ ($G_x=L_M\ M$)
G_y	Gas phase mass flux, kg $m^{-2}\ s^{-1}$ or lb $ft^{-2}\ h^{-1}$ ($G_y=G_M\ M$)
H	Height of transfer unit, m or ft H_{Ox}, overall based on liquid phase H_{Oy}, overall based on gas phase H_x, individual based on liquid phase H_y, individual based on liquid phase
HETP	Height equivalent to a theoretical (equilibrium) plate (stage)
j	Mass flux, kg $m^{-2}\ s^{-1}$
J^*	Diffusion flux, kmol $m^{-2}\ s^{-1}$
K'	Overall mass transfer coefficient including one-way diffusion, kmol $m^{-2}\ s^{-1}$ K'_x, based on liquid phase K'_y, based on gas phase
k_c	Mass transfer coefficient based on molar concentrations, m s^{-1}
$K_g\ a$	Overall volumetric mass transfer coefficient based on partial pressure difference, kmol $m^{-3}\ atm^{-1}\ s^{-1}$ ($K_g\ a=K_y\ a/P$)
$k_g\ a$	Individual volumetric mass transfer coefficient for gas phase based on partial pressure driving force, kmol $m^{-3}\ atm^{-1}\ s^{-1}$ ($k_g\ a=k_y\ a/P$)
$K_L\ a$	Overall volumetric mass transfer coefficient based on liquid phase concentration difference, s^{-1} ($K_L\ a=K_x\ a/c$)
$k_L\ a$	Individual volumetric mass transfer coefficient for liquid phase based on concentration difference, s^{-1} ($k_La=k_xa/c$)
k_p	Mass transfer coefficient based on partial pressures, kmol $m^{-2}\ Pa^{-1}\ s^{-1}$
k_x , k_y	Mass transfer coefficient based on mole fractions, kmol $m^{-2}\ s^{-1}$ k_x, liquid phase k_y, gas phase

$K_x\,a$	Overall volumetric mass transfer coefficient based on liquid phase mole fraction difference, kmol m^{-3} s^{-1}
$k_x\,a$	Individual volumetric mass transfer coefficient for liquid phase based on mole fraction difference, kmol m^{-3} s^{-1}
K_x, K_y	Overall mass transfer coefficient, based on mole fractions, kmol m^{-2} s^{-1} K_x, based on liquid phase K_y, based on gas phase
$K_y\,a$	Overall volumetric mass transfer coefficient based on gas phase mole fraction difference, kmol m^{-3} s^{-1}
$k_y\,a$	Individual volumetric mass transfer coefficient for gas phase based on mole fraction difference, kmol m^{-3} s^{-1}
L	Liquid phase molar flow rate, kmol s^{-1} ($L=L_M\,S$)
L'	Liquid phase molar flow rate on a solute-free basis, kmol s^{-1} [$L'=L(1-x)$]
L_M	Liquid phase molar flux, kmol m^{-2} s^{-1} or lb-mol ft^{-2} h^{-1}
M	Molecular weight, kg $kmol^{-1}$
m	Slope of equilibrium curve
N	Number of transfer units N_{Ox}, overall based on liquid phase N_{Oy}, overall based on gas phase N_x, individual based on liquid phase N_y, individual based on gas phase
N	Total convective flux relative to a stationary point, kmol m^{-2} s^{-1}
n	Number of moles
N_t	Number of equilibrium (theoretical) stages
S	Cross sectional area of tower, m^2 or ft^2
u	Superficial velocity, m s^{-1} or ft s^{-1}
X	Mole ratio, moles solute per mole of solute-free liquid [$X=x/(1-x)$]
x	Mole fraction of solute in liquid x^*, equilibrium concentration corresponding to gas phase composition y x_0, at the gas-liquid interface
Y	Mole ratio, moles solute per mole of solute-free gas [$Y=y/(1-y)$]
y	Mole fraction of solute in gas y^*, equilibrium concentration corresponding to liquid phase composition x y_0, at the gas-liquid interface

Greek letters

μ	Dynamic viscosity, Pa s or P (poise)
ν	Kinematic viscosity, $m^2\ s^{-1}$ or St (Stokes)
ρ	Mass concentration (density), kg m^{-3}
υ	Mass velocity relative to a stationary point, m s^{-1}
υ_d	Diffusion velocity relative to the molar average velocity of the bulk stream, m s^{-1}
$\upsilon*$	Molar velocity relative to a stationary point, m s^{-1}
ω	Mass fraction