Loughborough University

Structural Forms and Stress Analysis

22CVA103

Semester 2 2023 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Answer **THREE** questions.

Answer **ONE** question in Section A, **ONE** question in Section B and **ONE** question in Section C.

All questions carry equal marks.

A two-page Aide-Mémoire formula sheet is attached.

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SECTION A
(Answer **ONE** question)

1. (a) A beam is a structural member subjected to bending, and it is probably the most common structural element that designers must cope with. Figures Q1(a,b) show two beams with different supports.

   i. Under the assumption that the material used is concrete, **explain** why steel reinforcement is needed. [4 marks]

   ii. The beams in Figures Q1(a,b) are subjected to the same load. **Sketch** how each beam deforms under the considered loading condition. [6 marks]

   iii. For each beam in Figures Q1(a,b), **identify** the tension and compression sides and **sketch** where the steel reinforcement is strictly needed. [6 marks]

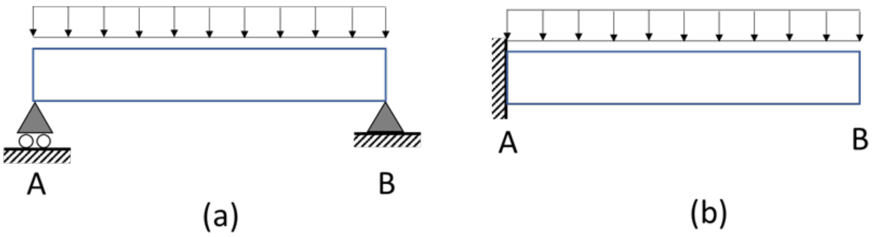

Figure Q1 – (a) Simply supported beam; (b) Cantilever beam.

(b) In some cases, it is easy to determine whether a truss member is in compression or tension simply by using qualitative analysis. This is an essential skill that should be developed to check the results of numerical calculations.

   i. For the pin-jointed structure shown in Figure Q1(c), **indicate by inspection** whether the perimeter members (AC, CE, BD, and DE) are in tension or compression. **No calculations are required.** [12 marks]

   ii. **Sketch** the load path showing how the load applied in E is transferred to the supports in A and B. [5 marks]

Note: Ignore the self-weight of the elements.

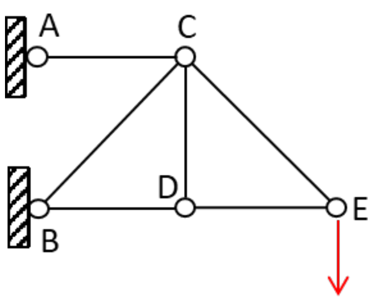

Figure Q1 – (c) Structural model of a truss.

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SECTION B
(Answer **ONE** question)

2. Figures Q2(a) depicts the elevation view of two steel columns with different cross-sections. Column *A* supports a compressive axial force *P* = 1200 kN generated from the top column. The axial force *P* is applied along the horizontal axis of symmetry of column *A* at 50 mm from its left side, as shown in Figure Q2(b).

   (a) For column *A*'s cross-section, shown in Figure Q2(b), **determine** the distance S_y of the centroid to the column's right edge, **calculate** the second moments of area about the centroidal axes *y* and *z*, and **evaluate** the load eccentricity, *e*.
   [9 marks]

   (b) **Determine** the magnitude of the maximum compressive and tensile stresses on column *A*'s cross-section. Clearly **indicate** the locations of the maximum tensile and compressive stresses on the cross-section.
   [9 marks]

   (c) **Determine** the change in the length for column *A* under the axial load *P* if the column's height is 6.0 m. Assume the Young's modulus *E* for the steel is 210 GPa. Consider the axial deformation due to the axial force *P* only and neglect any deformation due to the bending moment of the column's self-weight.
   [7 marks]

   (d) Assuming column *A* to be subjected to an additional shear force *V* = 300 kN acting along its horizontal axis of symmetry, *z*, **determine** the magnitude of the maximum shear stress (see Figure Q2(b)). Clearly **indicate** where the maximum shear stress is found in the cross-section.
   [8 marks]

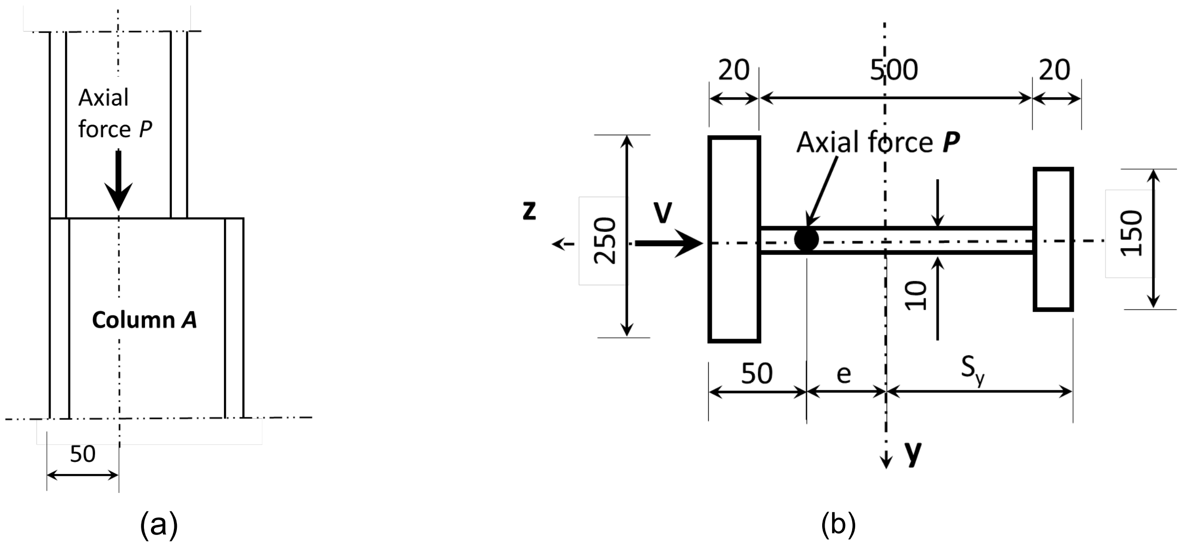

Figure Q2 – (a) Elevation view and (b) sectional view of column A (all dimensions are in mm)

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3. Figure Q3(a) shows a 400 mm wide and 500 mm deep rectangular hollow section beam, with web thickness t_w = 10 mm and flange thickness t_f = 20 mm. The cross-section is subjected to a compressive load P acting with eccentricity e_y = 240 mm with respect to the horizontal axis of symmetry and eccentricity e_z = 195 mm with respect to the vertical axis of symmetry.

   (a) Assuming the maximum admissible elastic normal stress to be σ_{adm} = 200 N/mm^2 (both in compression and tension), **determine** the magnitude of the maximum elastic axial force P that can be safely applied to the cross-section shown in Figure Q3(a).
   [10 marks]

   (b) **Determine** the change in the beam's length due to the calculated axial force P if its length is 8.0 m. Assume the Young's modulus E for the steel is 210 GPa. Consider the axial deformation due to the axial force P only and neglect any deformation due to the bending moment and the beam's self-weight.
   [7 marks]

   (c) Assuming an external torque T = 25 kN m is further applied to the cross-section shown in Figure Q3(a), **determine** the magnitude of the maximum shear stress. Clearly **indicate** where this is found in the cross-section.
   [6 marks]

   (d) Figure Q3(b) shows a similar cross-section, in which the right web has been cut in the middle (neglect the size of the cut). **Determine** the maximum shear stress in the cross-section of Figure Q3(b) for the effect of the same external torque T = 25 kNm. Clearly **indicate** where the maximum shear stress is found in this case.
   [10 marks]

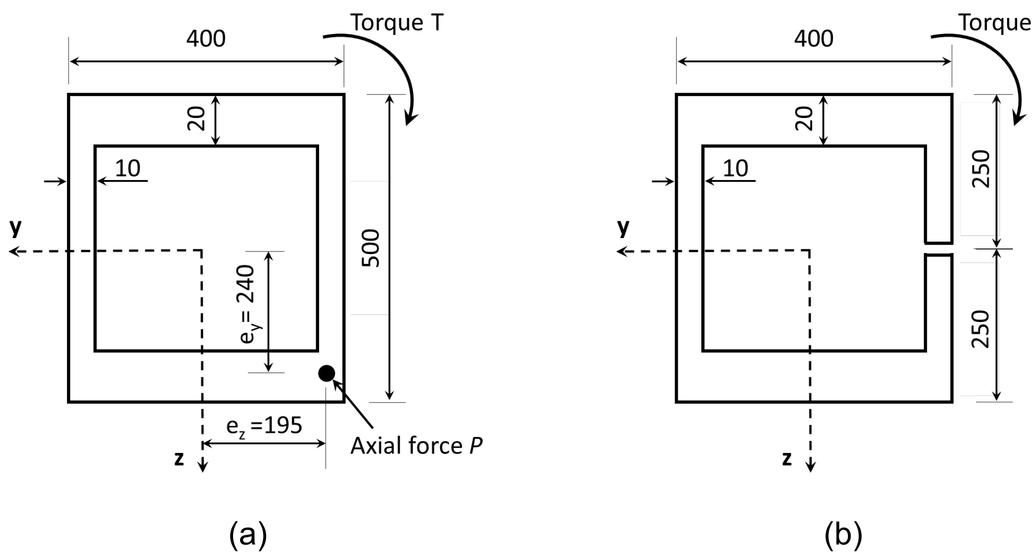

Figure Q3 – (a) Rectangular hollow section; (b) Open section due to cut on the right side (cut size considered negligible). All dimensions are in mm

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SECTION C
(Answer **ONE** question)

4. Figure Q4 shows a steel element (Young modulus E= 210 GPa; Poisson ratio ν= 0.3) subjected to planar state of stress with σ_x= 15 MPa, σ_z= 0, and τ_{xz}= 15 MPa.

(a) **Draw** the Mohr circle of complex stresses and **find** the magnitude of the principal stresses and the orientation of the corresponding principal directions with respect to the *xz* reference system.

[10 marks]

(b) **Represent** the principal stresses on a conveniently rotated material element.

[5 marks]

(c) **Draw** the Mohr circle of complex strains and **calculate** the maximum compressive strain, tensile strain, and shear strain.

[12 marks]

(d) **Calculate** the axial strain ε_m measured by the strain gauge shown in Figure Q4, knowing that the angle it forms with the horizontal direction *x* is α_{xm}= 30°.

[6 marks]

σ_x =	15 MPa
σ_z =	0
τ_{xz} =	15 MPa
E =	210 GPa
ν=	0.30
α_{xm} =	30°

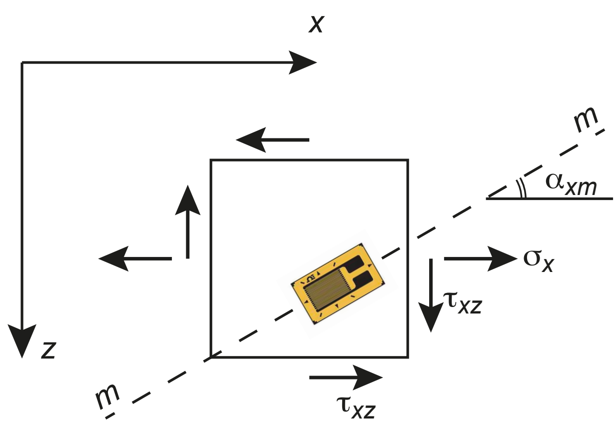

Figure Q4 –Steel element in planar stress conditions equipped with a strain gauge that measures the axial strain along the *m*-*m* direction.

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5. Figure Q5(a) shows a cantilevered steel column (Young modulus E = 210 GPa and yield strength σ_{yld} = 260 MPa) subjected (not simultaneously) to a compressive force, F_v, and a lateral force at the free end, F_h. The height of the column is h = 3.30 m, and its cross-section is the hollow square shown in Figure Q5(b), with outer dimension d = 120 mm and constant thickness t = 12 mm.

   (a) **Estimate** the capacity of the column in compression using the Rankine failure load and **calculate** the corresponding safety factor against the compressive load F_v = 250 kN.

   [16 marks]

   (b) **Calculate** the yield moment, M_{yld}, and the corresponding safety factor against the maximum bending moment, M_{max}, caused by the lateral force F_h = 10 kN.

   [11 marks]

   (c) Knowing that the shape factor of the cross-section is f = 1.24, **calculate** the magnitude of the lateral force $F_{h,pls}$ which causes the full plasticization of the cross-section at the base of the column.

   [6 marks]

Note: In your calculations, assume that vertical and horizontal forces are not applied simultaneously.

h =	3.30 m
d =	120 mm
t =	12 mm
E =	210 GPa
σ_{yld} =	260 MPa
F_v =	250 kN
F_h =	10 kN
f =	1.24

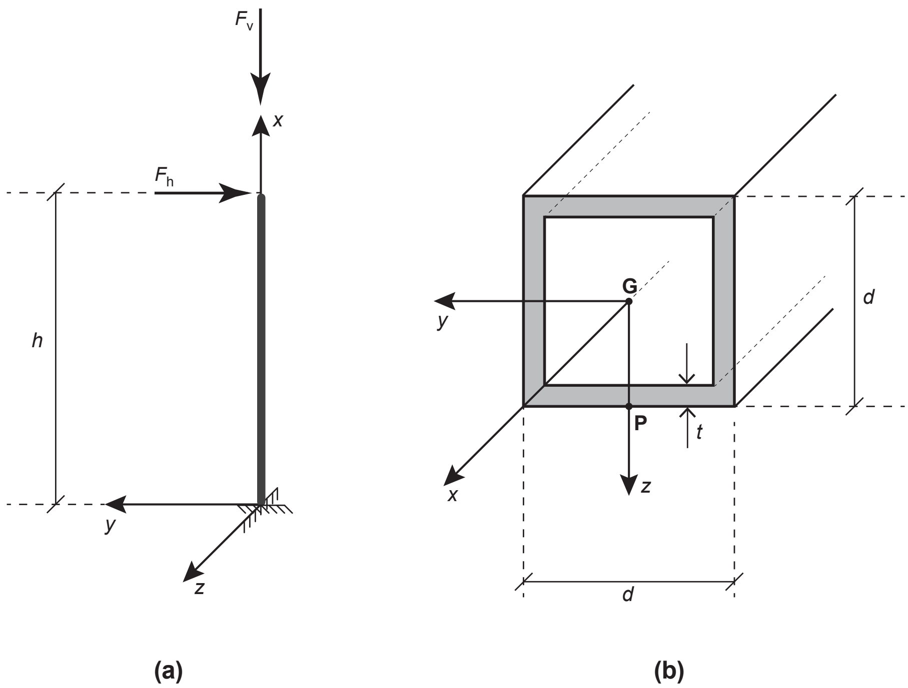

Figure Q5 – Cantilevered steel column subjected to non-simultaneous vertical and horizontal forces at the free end (a) and hollow square cross-section (b).

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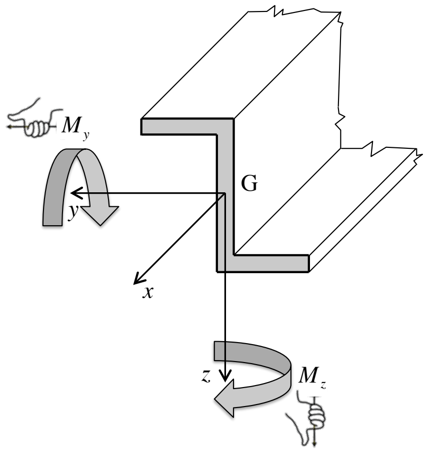
Aide-Mémoire Formula Sheet (Page 1 of 2)

Buckling Load

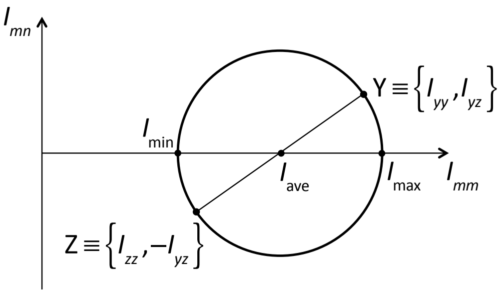
$$P_E = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 E}{\lambda^2} \quad ; \quad \lambda = \frac{L_e}{\rho}$$

$$P_R = \frac{P_E P_c}{P_E + P_c}$$

Unsymmetrical Bending



$$\sigma_x = \beta y + \gamma z \quad ; \quad \begin{cases} \beta = -\dfrac{M_z I_{yy} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2} \\ \gamma = \dfrac{M_y I_{zz} + M_z I_{yz}}{I_{yy} I_{zz} - I_{yz}^2} \end{cases}$$



Elastic Constitutive Law

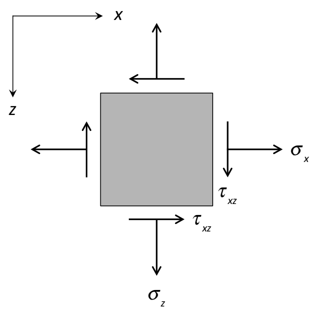
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right) \quad ;$$

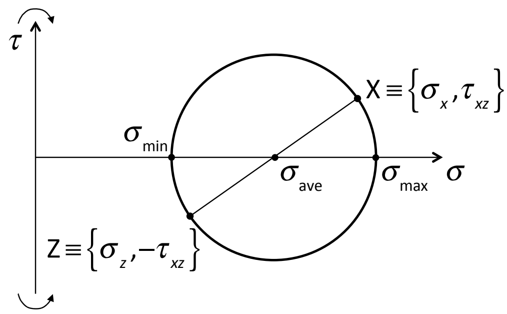
$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y - \nu\varepsilon_z\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad ; \quad \tau_{xy} = G\gamma_{xy}$$

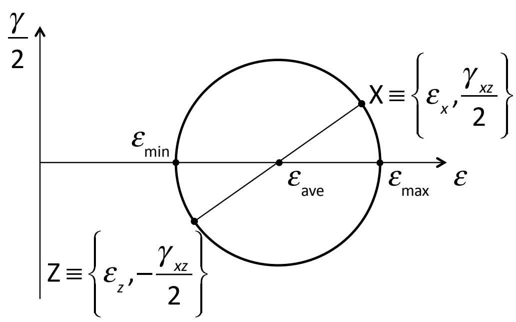
$$G = \frac{E}{2(1+\nu)}$$

Mohr's Circle for Stress State





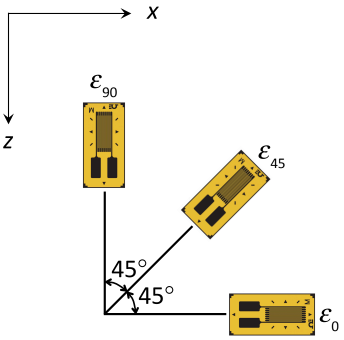
Mohr's Circle for Strain State



Aide-Mémoire Formula Sheet (Page 2 of 2)

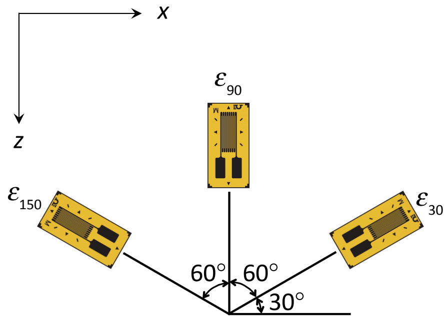
Strain-Gauge Rosette

45° Rosette



$$\gamma_{xz} = \varepsilon_0 + \varepsilon_{90} - 2\varepsilon_{45}$$

60° Rosette



$$\begin{cases} \varepsilon_x = \dfrac{2}{3}\left(\varepsilon_{30} - \dfrac{\varepsilon_{90}}{2} + \varepsilon_{150}\right) \\ \gamma_{xz} = \dfrac{2}{\sqrt{3}}\left(\varepsilon_{150} - \varepsilon_{30}\right) \end{cases}$$