

Mathematical Modelling of Structures

22CVA104

Semester 2 2023

Online Short-window Exam paper

This is an online short-window examination, meaning you have a total of **2 hours plus an additional 30 minutes** to complete and submit this paper. The additional 30 minutes are for downloading the paper and uploading your answers when you have finished. If you have extra time or rest breaks as part of a Reasonable Adjustment, you will have further additional time as indicated on your exam timetable.

It is your responsibility to submit your work by the deadline for this examination. You must make sure you leave yourself enough time to do so.

It is also your responsibility to check that you have submitted the correct file.

Exam Help

If you are experiencing difficulties in accessing or uploading files during the exam period, you should contact the Exam Helpline. For urgent queries please call **01509 222900**.

For other queries email examhelp@lboro.ac.uk

You may handwrite and/or word process your answers, as you see fit.

Answer **THREE** questions in total:

- **TWO** questions from Section A – “Mathematics”.
- **THE** question from Section B – “Analysis of Beams and Frames”.

All questions in the same section carry equal marks.

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SECTION A – MATHEMATICS
(Answer **TWO** questions in this section)

1. **Solve** the following 1st order ODEs:

(a) $\frac{dy}{dx} = e^y \sin(x)$, separating variables, subject to the boundary condition:
 $y(\pi/2) = -1$

[7 marks]

(b) $\frac{dy}{dx} + \frac{1}{x} y = \ln(x)$, assuming $x > 0$

[8 marks]

(c) $\frac{dy}{dx} = \frac{y^2}{x^2} + \frac{y}{x} + 2$ using the substitution $z = y \cdot x$ and the formula

$$\int \frac{1}{z^2+2} dz = \frac{1}{\sqrt{2}} \tan^{-1}(z/\sqrt{2})$$

[10 marks]

2. **Solve** the following 2nd order ODEs:

(a) $\frac{d^2y}{dx^2} + 3 \frac{dy}{dx} + 2y = 0$,

[4 marks]

(b) $2 \frac{d^2y}{dx^2} + 2y = e^{-x}$, subject to the boundary conditions: $y(0) = 0$, $y'(0) = 0$

[15 marks]

(b) $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} - 4y = 0$

[6 marks]

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3. Given the mass-spring oscillator shown in Figure Q3, with mass $m = 20 \text{ kg}$, spring constant $k = 20 \text{ N/m}$ and external forcing $f(t) = 400 \sin(20t)$, undertake the following tasks:
- (a) **Determine** the equation of motion of the oscillator, assuming there is no friction between the mass and the floor. [6 marks]
 - (b) **Solve** the equation of motion of the oscillator, subject to the conditions $x(0) = 0$ and $x\left(\frac{\pi}{2} \sqrt{\frac{m}{k}}\right) = 0$. [15 marks]
 - (c) Is the oscillator close to resonance? Why? [4 mark]

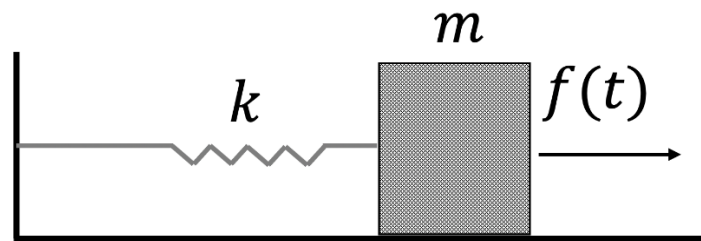


Figure Q3

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SECTION B – ANALYSIS OF BEAMS AND FRAMES
(Answer **Question 4** in this section)

4. (a) The beam shown in Figure Q4(a) has a frictionless hinge at point **A** and a roller at point **C**.
- i) **Calculate** all support reactions. [4 marks]
 - ii) **Sketch** the bending moment diagram and the shear force diagram, indicating all principal values. [10 marks]
 - iii) **Calculate** the axial force at point B at the middle of the beam. [2 marks]

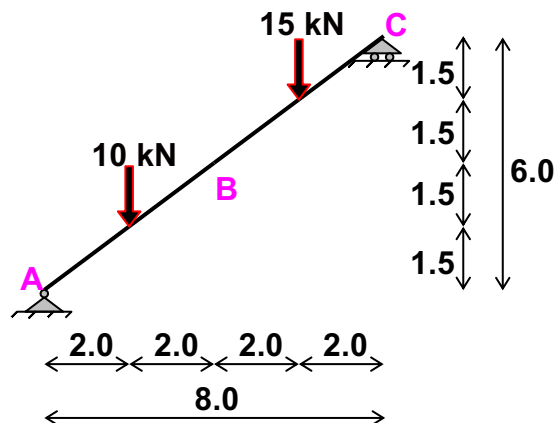


Figure Q4(a) - Beam under point loads [all geometrical dimensions are in m]

- (b) For the frame shown in Figure Q4(b):
- i) **Calculate** all support reactions. [12 marks]
 - ii) **Draw** the axial force diagram and the bending moment diagram for the whole structure, indicating all principal values. [16 marks]
 - iii) **Sketch** the deflected shape. [6 marks]

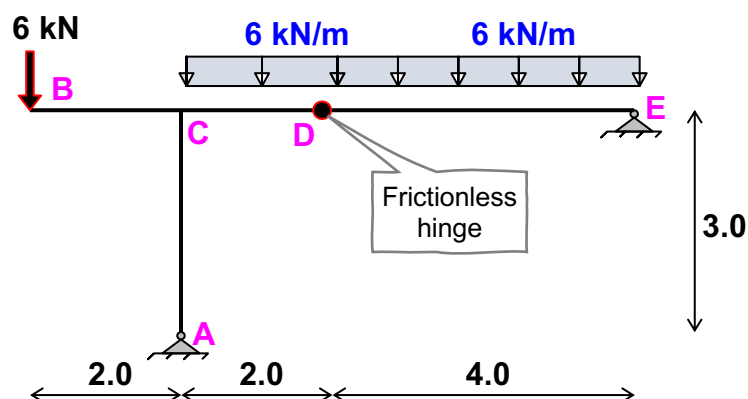


Figure Q4(b) - Frame under point and distributed loads
[all geometrical dimensions are in m]

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