Loughborough University

# Mechanical and Mathematical Principles of Fluid Mechanics
## 22CVA105

Semester 2 2023 | In-Person Exam Paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

This examination consists of two sections.
Answer **ONE QUESTION in Section A.**
Answer **THREE QUESTIONS in Section B.**

All questions carry equal marks.

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## SECTION A
(Answer **ONE QUESTION in Section A as an essay)**

1. With reference to the work of Betterle and Botter (2021), discuss the contention that catchment nestedness has a significant impact upon similarity of hydrological response.
[25 marks]

2. With reference to the work of Zhang et al. (2022), discuss the effectiveness of traditional extreme value modelling versus that which tries to incorporate non-stationarities.
[25 marks]

3. In contrast to studies that highlight the importance of Reynolds numbers of 2300 and 2900 for defining laminar and turbulent flow, Avila et al. (2010) emphasise a Reynolds number of 1720. Explain what arises at this Reynolds number and evaluate its importance.
[25 marks]

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## SECTION B
(Answer **THREE QUESTIONS in Section B)**

4. a) Explain briefly how a pitot tube works. [3 marks]

   b) Describe how such pitot tube instruments have been used at the Vallée de la Sionne field site in Switzerland to gain insights into snow avalanche dynamics. [3 marks]

   c) In order to derive the Bernoulli equation it is necessary to assume that the flow is isentropic. Briefly explain what this means, why the assumption is required and what the primary mechanism is for invalidating this assumption in a real flow of a fluid at constant bulk temperature. [6 marks]

   d) An orifice plate is a constriction placed within a pipe such that there is a sudden transition in the cross-sectional area. If a typical discharge coefficient for a venturimeter is $C_d \sim 0.97$, suggest a value for the discharge coefficient of the orifice plate and explain your reasoning. [2 marks]

   e) A horizontal pipe has an initial diameter of 8 cm, which decreases linearly and smoothly such that it is 4 cm at the end of the section under consideration here. Halfway along this length, the pipe bends through an angle of 20°. Assuming that this bend and the change in diameter cause negligible energy loss, determine the force, *F*, acting upon the bend that a fitting needs to support and the direction in which this force acts. Illustrate this with an arrow indicating the approximate direction with respect to the direction of the inlet velocity. You should assume that the flowing fluid is water at room temperature and that the bulk velocity at the start of the working section is 0.6 $ms^{-1}$. The flow at the inlet is driven by 3.8 m of pressure head. The pipe system is sketched in Fig. 4e. [11 marks]

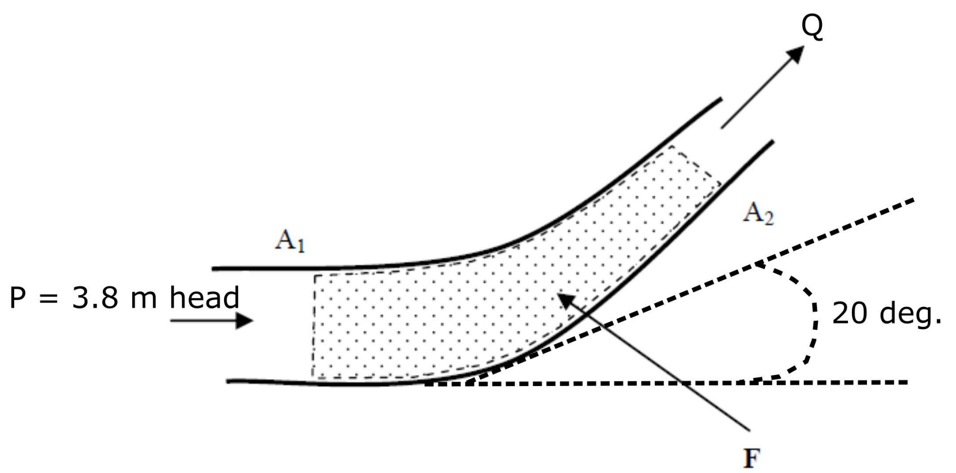


**Figure 4e**

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5. a) The velocity profile in a particular pipe flow is given by:

$$u(r) = -\frac{(R^2 - r^2)}{4\mu}\frac{d}{dx}(P + \rho g z)$$

where $R$ is the pipe radius, $r$ is the distance from the centre of the pipe, $x$ is the downstream coordinate, $P + \rho g z$ is the piezometric pressure and $\mu$ is the dynamic viscosity.

Show by integration that the discharge in the pipe can be expressed as:

$$Q = -\frac{\pi}{8\mu}R^4\frac{d}{dx}(P + \rho g z)$$

[8 marks]

b) What is the difference between a hydraulically rough and a hydraulically smooth pipe flow? [2 marks]

c) Calculate the pipe diameter required to carry a discharge of 0.6 $m^3$/s along a pipe ($f$ = 0.005) with a head loss of 1 m for every 30 m of pipe. [4 marks]

d) Two reservoirs have water surface elevations of 221 m and 183 m respectively. They are joined by a pipe which is 1000 m long in total and contains two sections in series. The first section has a diameter of 0.1 m, and the second has a diameter of 0.2 m. The friction factor for both is 0.006 and minor losses are negligible. If the discharge is 0.02 $m^3$/s, how long is the first section of pipe? [8 marks]

e) Figure Q5e shows typical performance curves for a pump. Explain what each of the curves A, B and C shows. [3 marks]

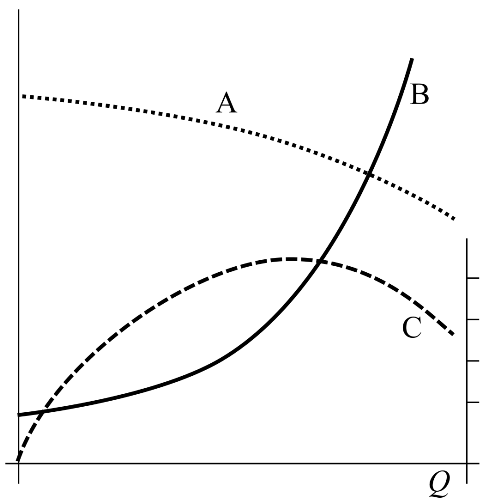


**Figure Q5e**

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6. a) You have been tasked with modelling air motions along a mountain valley. In your report you state that you have ignored the effect of density variation where it might contribute to the dynamic terms, but have included it when it contributes to hydrostatic body forces. What is this assumption better known as? [2 marks]

b) Provide a brief fluid mechanical explanation of dimictic mixing, and explain why this is important for the ecological productivity of a lake. [5 marks]

c) The heat transfer rate per unit area, $q$, in one spatial dimension is defined according to:

$$q = -k\frac{dT}{dx}$$

where $T$ is temperature and $k$ is the thermal conductivity. Using this equation state the dimensions of the thermal conductivity. Show your working. Recall that one Joule represents the product of force and distance. [5 marks]

d) An empirical expression for estimating the Nusselt number is given by:

$$Nu = 0.33Pr^{1/3}Re^{1/2}\left(\frac{T_s}{T_a}\right)^{0.117}$$

where the notation follows that used in class. Explain how it is possible to determine if the Nusselt number applies to free or forced convection from this expression and state which is the case. [2 marks]

e) Head loss of 5 m arises in a 15 m long pipe with 12 cm diameter where the flowing air is at a temperature of 350˚K and has a bulk velocity of 4 ms$^{-1}$. Given that the heat transfer rate per unit area is known to be 65.7 W m$^{-2}$, what is the temperature difference, $\Delta T$, between the walls of the pipe and the air? Show your working.

The Nusselt number for such a pipe flow is given by:

$$Nu = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

Question 6 continues/…

where *f* is the Darcy-Weisbach friction factor. Air at 350K has a density of 0.9950 kg $m^{-3}$ a specific heat capacity of 1.009 kJ $kg^{-1}K^{-1}$, a dynamic viscosity of $2.082 \times 10^{-5}$ N s $m^{-2}$, and a thermal conductivity of 0.0300 $Wm^{-1}K^{-1}$. The following equations will be of use:

$$h = \frac{q}{\Delta T}$$

$$Nu = \frac{hD}{k}$$

$$Pr = \frac{C_p \mu}{k}$$

[11 marks]

C Keylock
T Marjoribanks