Loughborough University

Structural Dynamics and Earthquake Engineering

22CVP371

Semester 1 2022-23 | In-Person Exam Paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

ANSWER **TWO** questions from Section A – "Structural Dynamics"

AND **ONE** question from Section B – "Earthquake Engineering":

Each question in Section A is worth **25 marks**; each question in Section B is worth **50 marks**.

A formula sheet for the dynamics of SDoF (single degree of freedom) oscillators is attached.

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SECTION A – "Structural Dynamics"
(ANSWER **TWO** questions from Section A)

1. Figure Q1(a) shows plan and 3D view of a spatial RC (reinforced concrete) frame, consisting of a single thick flat slab supported by four columns at its corners. The total mass of the slab is *M*= 169 Mg (including any dead and imposed load), which can be assumed to be uniformly distributed. The Young's modulus of the concrete is *E*= 35 GPa, and all the dimensions of the frame are offered within Figure Q1(a).

   For the purposes of the dynamic analysis, it is assumed that: *i*) the slab is rigid in its own plan; *ii*) the slab is rigid in bending; *iii*) the torsional stiffness of the columns is negligible.

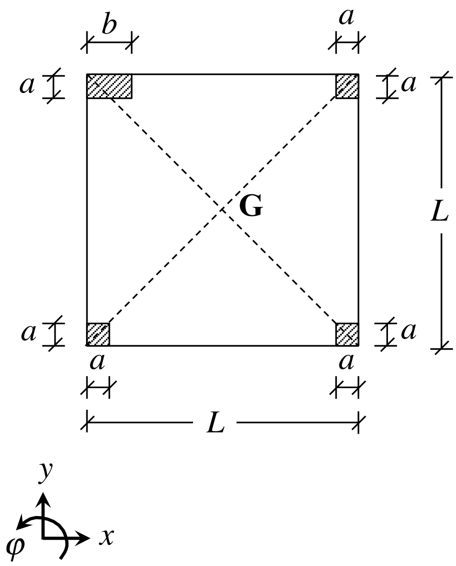
   (a) **Define** the array **<u>u</u>**(*t*) of the dynamically significant degrees of freedom and **calculate** the associated mass matrix **<u>M</u>**. [5 marks]

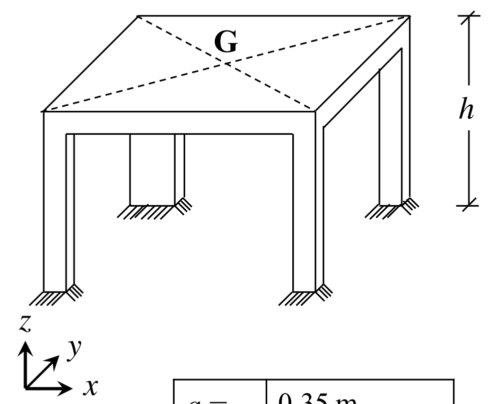
   (b) **Calculate** all the elements of the stiffness matrix **<u>K</u>**. [8 marks]

   (c) **Comment** on the relevance and implications of the assumptions *i*), *ii*) and *iii*) stated above. [**Note:** Two marks are allocated for each assumption correctly discussed, up to six marks in total.] [6 marks]

   (d) Figure Q1(b) shows the frequency response functions (FRFs) for a ground shaking in the *x* direction (—— solid line) and *y* direction (– – – dashed line). What interventions would you recommend to mitigate the effects of the seismic action on the structure? Briefly justify your answer. [**Note:** Three marks are allocated for each meaningful intervention correctly stated and briefly justified, up to six marks in total.] [6 marks]

Question 1 continues/…





a =	0.35 m
b =	0.75 m
L =	6.50 m
h =	3.70 m
E =	35 GPa
M =	169 Mg

Figure Q1(a) – Single-storey 3D reinforced concrete (RC) structure

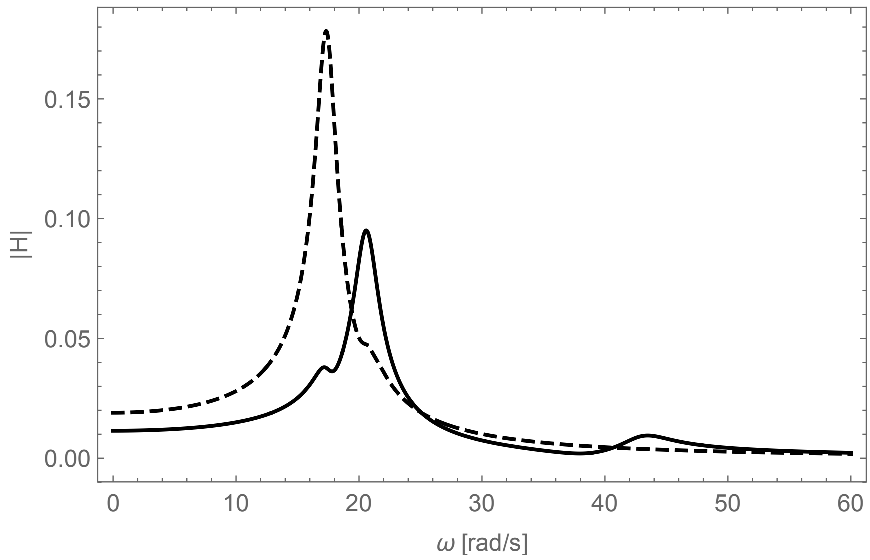


Figure Q1(b) – Moduli of the frequency response functions in the *x* and *y* directions

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2. Figure Q2 shows a SDoF portal frame, consisting of a rigid beam supported by two steel columns with fixed-fixed ends and further restrained by an inverted-V steel brace.

All the geometrical dimensions are offered in the same figure, including: area A_C and second moment of area I_C of the columns' cross section; cross-sectional area A_B of the pinned-pinned bracing rods.

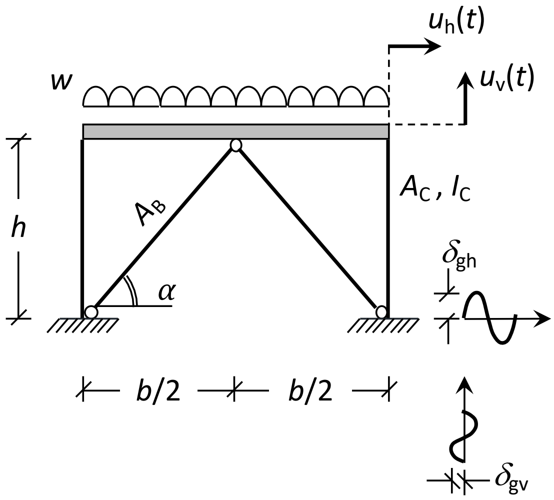
The figure also shows the applied gravitational load w, which is uniformly distributed over the beam and is the only source of mass to be considered in the analyses.

(a) Assuming that the Young's modulus of the steel is E= 210 GPa, **calculate** the natural periods of vibration in both the horizontal (T_{0h}) and vertical (T_{0v}) directions. [**Hint:** The columns and bracing system offer different contributions to the frame's stiffness in the horizontal and vertical directions.] [12 marks]

(b) Ground vibrations caused by a nearby excavation site are approximated as harmonic, with frequency ν_g= 15 Hz and amplitude of the horizontal ground shaking δ_{gh}= 1.5 mm. Assuming that the viscous damping ratio is ζ_0= 0.03, **calculate** the amplitude of the steady-state horizontal displacements experienced by the portal frame at the top level. [**Hint:** For damped systems, the contribution of the "homogeneous solution" $u_{hom}(t)$ to the dynamic response $u(t)$ becomes negligible in steady-state conditions.] [5 marks]

(c) The same excavation also results in a harmonic ground shaking in the vertical direction, characterised by the same frequency ν_g= 15 Hz and amplitude δ_{gv}= 0.5 mm. Assuming the same value of viscous damping ratio, ζ_0= 0.03, **calculate** the amplitude of the steady-state vertical accelerations of the portal frame at the top level. [**Hint:** The natural frequencies of vibration of the portal frame in the horizontal and vertical directions are different.] [8 marks]

Question 2 continues/…



h=	3.50 m	A_C=	96 cm^2
b=	9.50 m	I_C=	9,232 cm^4
w=	40.0 kN/m	A_B=	12.5 cm^2
E=	210 GPa	δ_{gh}=	1.5 mm
ζ_0=	0.03	δ_{gv}=	0.5 mm
		ν_g=	15 Hz

Figure Q2 – SDoF portal frame

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3. (a) Figures Q3(a), (b) and (c) show the force-displacement hysteretic loops for three different types of energy dissipation devices, namely viscoelastic, elasto-plastic and nonlinear viscous dampers. **Associate** the correct mechanism of energy dissipation to each of the three plots, and briefly **describe** how the different devices work. [**Note:** The answer to this question is not expected to exceed 250 words in total. A set of figures can be used to support your descriptions of the different devices.] [9 marks]

(b) A linear structure with n dynamically significant DoFs (degrees of freedom) is subjected to time-varying forces collected in the $(n\times1)$ vector $\underline{\mathbf{f}}(t)$. If $\underline{\mathbf{M}}$ is the mass matrix, $\underline{\mathbf{C}}$ is the damping matrix and $\underline{\mathbf{K}}$ is the stiffness matrix, **derive** the mathematical expression of the $(n\times n)$ matrix $\underline{\mathbf{H}}(\omega)$ that collects the frequency response functions (FRFs) of the dynamic system. [**Note:** You can start your derivation from the matrix equation of motion for a multi-DoF structure in the time domain.] [5 marks]

(c) For the same structural dynamics problem as in Question 3(b), **derive** the state-space equations of motion. [5 marks]

(d) For the same structural dynamics problem as in Question 3(b), and assuming a vector of external forces $\underline{\mathbf{f}}(t_i)$ discretised at the time instants $t = t_1, t_2, \ldots, t_N$, **derive** the mathematical expression of the Newmark's β-method [**Note:** For ease of derivation, you may neglect the contribution of the damping forces, i.e., you can assume that the damping matrix $\underline{\mathbf{C}}$ in the Newmark's β-method is a $(n\times n)$ zero matrix.] [6 marks]

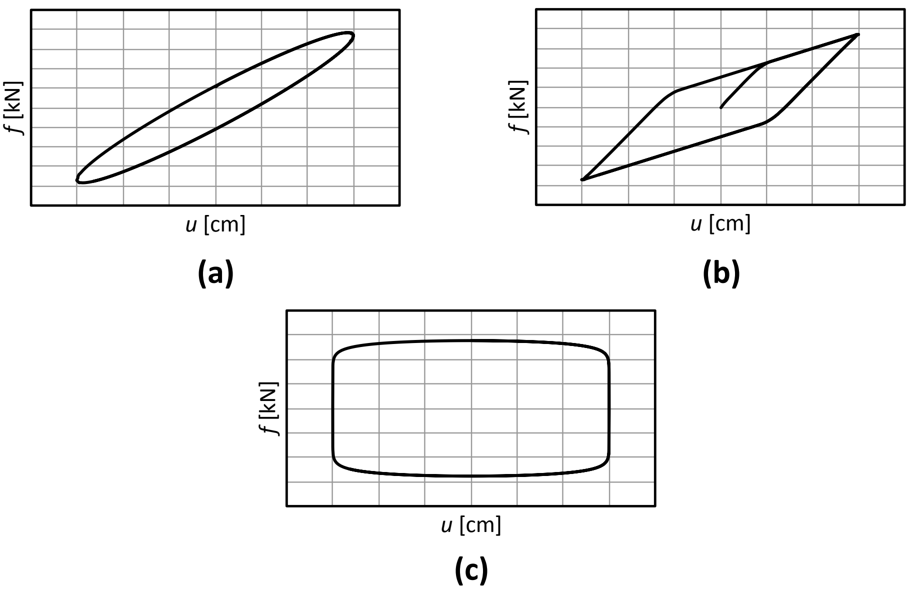

Figure Q3 – Force-displacement hysteretic loops recorded for different types of energy dissipation devices

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SECTION B - "Earthquake Engineering"
(ANSWER **ONE** question from Section B)

4. Figure Q4(a) shows the elevation of a 3-bay 4-storey steel moment resisting frame, which is 16.5 m wide and 13.1 m tall. The structure is regular in both plan and elevation. The geometrical dimensions h_i (i= 1,…,4) and b_j (j= 1,2,3) of the frame are offered in the same figure, along with the gravitational loads w_i, the Young's modulus E, and the second moments of area I_{min} and I_{max} of the columns.

Note: All the columns have the same cross section, but the outer columns are oriented such that the major axis bending occurs in the plane of the frame (strong orientation), while the inner columns experience the minor axis bending (weak orientation).

Note: The vertical dot-dashed centreline (CL) drawn in the figure is the frame's axis of symmetry.

The response of the frame to the design seismic actions needs to be quantified using the lateral force method, as formulated in the Eurocode 8 (EC8), considering both the damage limitation requirement (DLR, with return period T_{R}= 95 years) and the no-collapse requirement (NCR, with T_{R}= 475 years). Figure Q4(b) provides the mathematical expressions of the four-branch spectra for both levels of the seismic action, along with the values of the defining parameters a_{g}, S, T_{B}, T_{C}, T_{D} and β.

One can assume that:

- The beams are rigid, implying a shear-type behaviour for the frame.
- The fundamental period of vibration of the frame is T_1= 0.35 s.
- The equivalent viscous damping ratio for the DLR is ζ= 0.02.
- The behaviour factor for the NCR is q= 3.5.

(a) For the DLR, **calculate** and **draw** the seismic-induced deformed shape of the frame, clearly indicating the maximum displacement and the maximum inter-storey drift.
[14 marks]

(b) For the NCR, **calculate** and **draw** the seismic-induced bending moment and shear force diagrams for the leftmost column, denoted as A-C-D-E in Figure Q4(a).
[16 marks]

(c) For the NCR, **calculate** the seismic-induced bending moment at the leftmost first-floor and fourth-floor beam ends in Figure Q4(a), denoted as node E in the beam E-F and node A in the A-B beam, respectively. [8 marks]

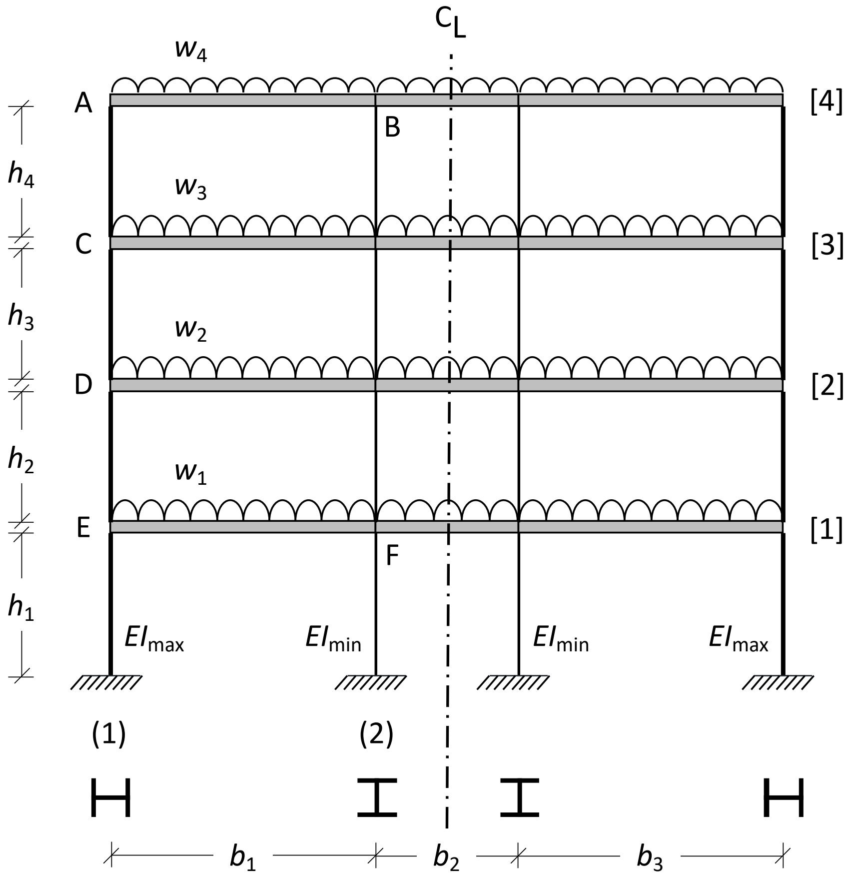
(d) **Discuss** how realistic is the assumption of shear-type behaviour for the frame.
[**Note:** The answer to this question is not expected to exceed 75 words.] [6 marks]

Question 4 continues/…

…/question 4 continued

(e) **Explain** why the seismic design of framed structures typically entails "weak beams" and "strong columns". [**Note:** The answer to this question is not expected to exceed 75 words. A figure can be used to support the description of the phenomenon.]

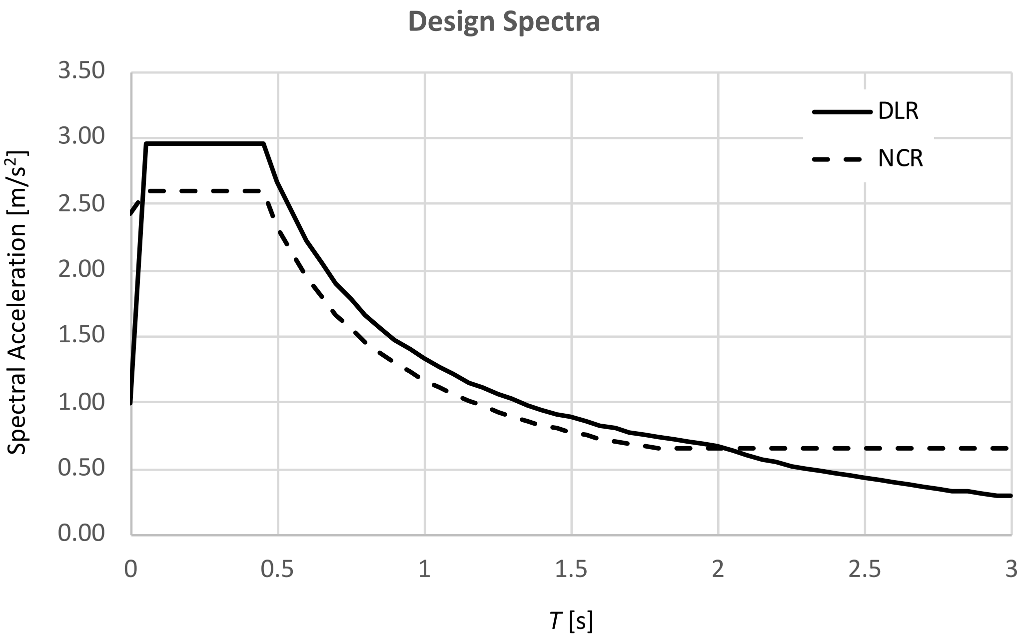
[6 marks]



$b_1 = b_3 =$	6.50 m	$E =$	205 GPa
$b_2 =$	3.50 m	$I_{max} =$	125,000 cm^4
$h_1 =$	3.50 m	$I_{min} =$	55,000 cm^4
$h_2 = h_3 = h_4 =$	3.20 m	$w_1 = w_2 = w_3 =$	42.0 kN/m
		$w_4 =$	35.0 kN/m

Figure Q4(a) – Moment-resisting steel frame

Question 4 continues/…



Elastic design spectrum for the DLR
(T_R= 95 years)

$$S_e(T)=\begin{cases} a_g \cdot S \cdot \left[1+\frac{T}{T_B}\cdot(\eta\cdot 2.5-1)\right] & , \quad 0\le T<T_B \\ a_g\cdot S\cdot\eta\cdot 2.5 \quad , & T_B\le T\le T_C \\ a_g\cdot S\cdot\eta\cdot 2.5\cdot\left[\frac{T_C}{T}\right] \quad , & T_C<T\le T_D \\ a_g\cdot S\cdot\eta\cdot 2.5\cdot\left[\frac{T_C\cdot T_D}{T^2}\right] \quad , & T_D<T\le 4\text{ s}\end{cases}$$

where a_g= 0.9 m/s^2 , and:

$$\eta=\sqrt{\frac{10}{5+100\,\zeta}} \quad \ge \quad 0.55$$

Elasto-plastic design spectrum for the NCR
(T_R= 475 years)

$$S_d(T)=\begin{cases} a_g\cdot S\cdot\left[\frac{2}{3}+\frac{T}{T_B}\cdot\left(\frac{2.5}{q}-\frac{2}{3}\right)\right] \quad , & 0\le T<T_B \\ a_g\cdot S\cdot\frac{2.5}{q} \quad , & T_B\le T\le T_C \\ a_g\cdot S\cdot\frac{2.5}{q}\cdot\left[\frac{T_C}{T}\right] \ \ge\ \beta\cdot a_g \quad , & T_C<T\le T_D \\ a_g\cdot S\cdot\frac{2.5}{q}\cdot\left[\frac{T_C\cdot T_D}{T^2}\right] \ \ge\ \beta\cdot a_g \quad , & T>T_D\end{cases}$$

where a_g= 3.3 m/s^2 , and:

$$\beta=0.2$$

Other design parameters which are in common between the two spectra are:

- S= 1.1
- T_B= 0.05 s
- T_C= 0.45 s
- T_D= 2.00 s

Figure Q4(b) – Definition of the seismic spectra

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5. (a) **Describe** the phenomenon of stress build-up at fault locations, leading to a seismic event. [**Note:** The answer to this question is not expected to exceed 75 words. A figure can be used to support the description of the phenomenon.] [5 marks]

(b) **Provide** the basic definition of a seismic wave. Briefly discuss the four types of seismic waves that are generated by an earthquake. [**Note:** The answer to this question is not expected to exceed 150 words. Figures can be used to illustrate the different types of seismic waves.] [8 marks]

(c) **Explain** the meaning and purpose of the magnitude and intensity scales. [**Note:** The answer to this question is not expected to exceed 100 words.] [6 marks]

(d) As a structural consultant, among the methods of seismic analysis allowed by Eurocode 8, which one would you **recommend** for the following cases? **Explain** why.

- New two-storey masonry building, without any irregularity in plan and in elevation;
- New tall building, in which dissipative elasto-plastic braces are installed;
- Existing multi-storey reinforced-concrete frame, built before new seismic provisions were enforced in the area;
- Steel chimney for a new industrial power plant.

[**Note:** Two marks are allocated for each bullet point above. At least one justifying argument is expected for each case.] [8 marks]

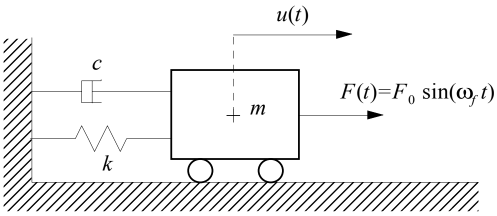
(e) **Provide** two examples of irregularity in plan and two examples of irregularity in elevation for a multi-storey building, and explain why they might negatively affect the seismic response of the structure. [**Note:** Three marks are allocated for each example, up to a maximum of 12 marks. At least one justifying argument is expected for each example.] [12 marks]

(f) **Explain** how the use of base isolation systems allows reducing the design seismic forces in a building. [**Hint:** You can refer, for instance, to considerations about the earthquake response spectra and/or the energy content of seismic records in the frequency domain.] [**Note:** The answer to this question is not expected to exceed 100 words. A figure can be used to support your explanation.] [5 marks]

(g) **Explain** how the presence of soft soil layers (e.g., soft clay) beneath a structure may affect its seismic response. Also **explain** why in these circumstances the use of base isolation systems may be inappropriate. [**Note:** The answer to this question is not expected to exceed 150 words.**]** [6 marks]

A Palmeri

<u>Formula Sheet</u>: Dynamics of SDoF Oscillators



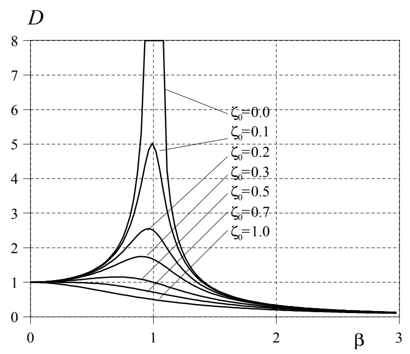
$$m\,\ddot{u}(t)+c\,\dot{u}(t)+k\,u(t)=F_0\,\sin\left(\omega_{\mathrm{f}}\,t\right)$$

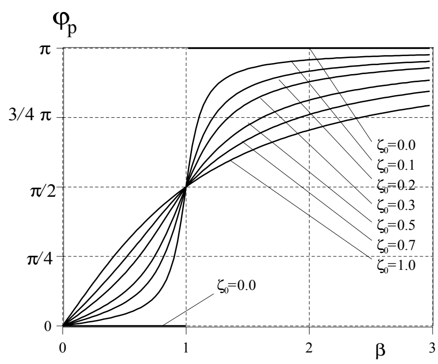
$$u(t)=u_{\mathrm{h}}(t)+u_{\mathrm{p}}(t)$$

$$u_{\mathrm{h}}(t)=\exp\left(-\zeta_0\,\omega_0\,t\right)\left[\bar{C}_1\,\cos\left(\bar{\omega}_0\,t\right)+\bar{C}_2\,\sin\left(\bar{\omega}_0\,t\right)\right]$$

$$u_{\mathrm{p}}(t)=u_{\mathrm{stat}}\,D(\beta)\,\sin\left(\omega_{\mathrm{f}}\,t-\varphi_{\mathrm{p}}\right)$$

$$\beta=\frac{\omega_{\mathrm{f}}}{\omega_0}\qquad D(\beta)=\frac{1}{\sqrt{\left(1-\beta^2\right)^2+\left(2\,\zeta_0\,\beta\right)^2}}\qquad \tan(\varphi_{\mathrm{p}})=\frac{2\zeta_0\,\beta}{1-\beta^2}\qquad \left\{0\le\varphi_{\mathrm{p}}<\pi\right\}$$





Complete quadratic combination (CQC) coefficients

$$\rho_{ij}=\frac{8\,\zeta^2\left(1+\beta_{ij}\right)\beta_{ij}^{1.5}}{\left(1-\beta_{ij}^2\right)^2+4\,\zeta^2\,\beta_{ij}\left(1+\beta_{ij}\right)^2}\;,\quad \text{with}\quad \beta_{ij}=\min\left\{\frac{T_i}{T_j},\frac{T_j}{T_i}\right\}\quad\left(0<\beta_{ij}\le 1\right)$$