Loughborough University

# ROBOTICS AND CONTROL

22WSC104

Semester 1 2022 In-Person Exam paper

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This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

# ROBOTICS AND CONTROL

(22WSC104)

January 2023 | 2 Hours

Answer **ALL** questions.

Questions carry the marks shown.

Any University-approved calculator is permitted.

*A range of formulae and tables likely to be of benefit in the solution of these questions are provided at the rear of the paper.*

**1.**

a) Sketch approximate Bode plots for the following transfer functions, marking relevant frequencies, noting the presence or absence of any resonant peaks, and giving the gradients of any asymptotes.

i. $$G_1(s) = \frac{s+1}{100s+1}$$ [6 marks]

ii. $$G_2(s) = \frac{100}{(s^2+0.08s+1)}$$ [6 marks]

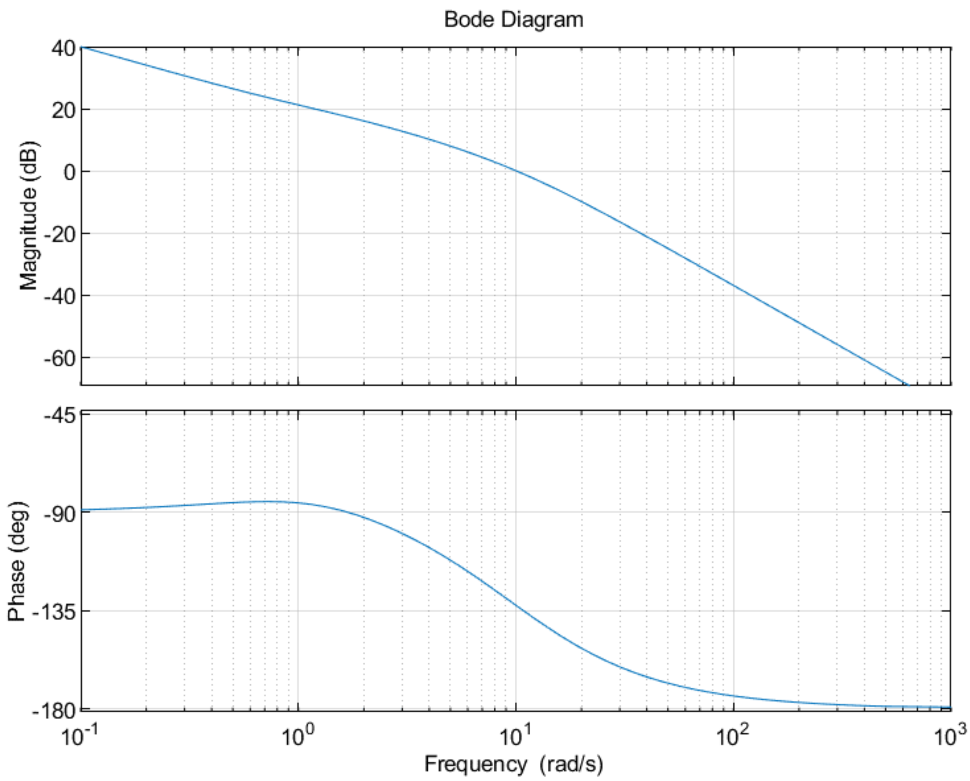


**Figure Q1**

Figure Q1 shows the Bode plot of an open-loop stable system (a robotic servomechanism).

b) Briefly explain how the Nyquist stability theorem implies that the servomechanism is stable under unity feedback. [4 marks]

c) Estimate the gain and phase margins under unity feedback. [4 marks]

**2.** The heating element of an electric shower has a time constant $T_1 = 5s$ and heats the water by $A = 2$ degrees C for each change in flow of 1 litre/min. The water temperature is then measured by a thermocouple of time constant $T_2 = 0.5s$ and fed back to a controller which implements a transfer function $C(s)$.

A block diagram is shown in Figure Q2, showing the output temperature $Y(s)$ sensed by the thermocouple, and the temperature reference $R(s)$.

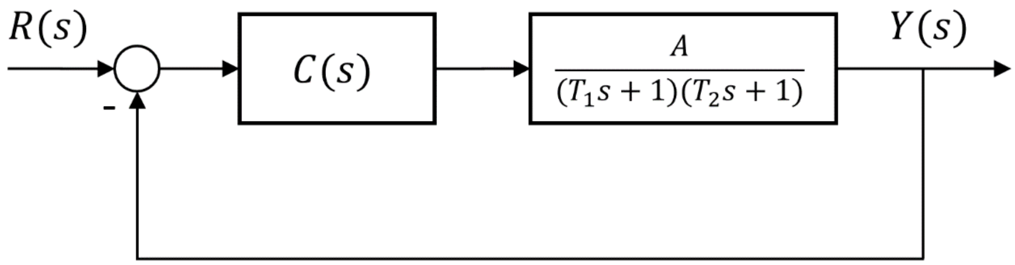


**Figure Q2**

a) Show that at an angular frequency $\omega$ the gain $|G(j\omega)|$ and phase $\angle G(j\omega)$ of the shower's transfer function

$$G(s) = \frac{A}{(T_1 s + 1)(T_2 s + 1)}$$

is given by:

$$|G(j\omega)| = \frac{A}{\left(\sqrt{T_1^2\omega^2 + 1}\right)\left(\sqrt{T_2^2\omega^2 + 1}\right)}$$

$$\angle G(j\omega) = -\tan^{-1}(T_1\omega) - \tan^{-1}(T_2\omega)$$

[5 marks]

b) Design a PI controller for $C(s)$ to give a target 0dB crossover frequency of 0.5 rad/s with a phase margin of 60 degrees. [5 marks]

c) For this controller design, sketch the expected response of the closed-loop system to a unit step in $R(s)$, stating approximate values of steady-state error, overshoot and rise time. [5 marks]

d) Briefly discuss possible advantages and disadvantages of adding an additional derivative term to $C(s)$ to implement PID control. Can the disadvantages be mitigated in practice? [5 marks]

**3.** The rotation speed $\omega$ of a horizontal-axis wind turbine is regulated to a rated speed $\omega_r$ by pitching the turbine blades to an angle $\theta$. Blade pitch is controlled by a PI controller in a feedback loop as shown in Figure Q3(a).

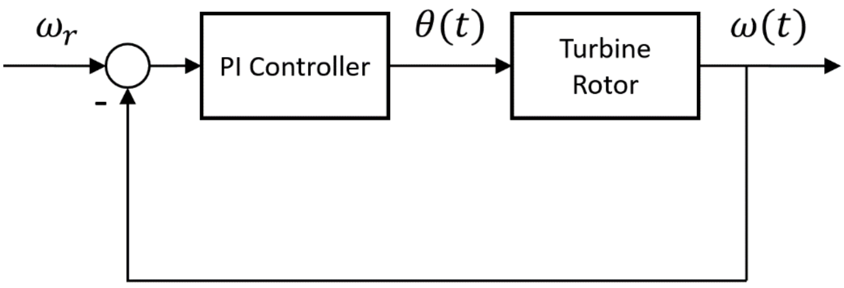


**Figure Q3(a)**

The dynamic behaviour of a wind turbine varies with wind speed. An approximate Bode plot of a 5MW turbine at a wind speed of 15m/s is shown in Figure Q3(b).

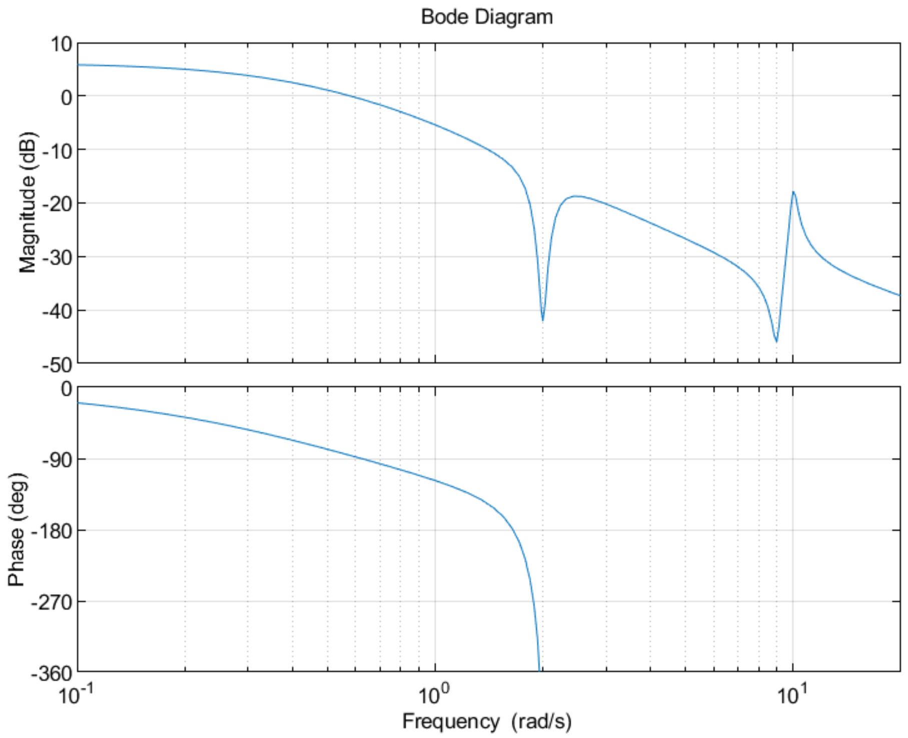


**Figure Q3(b)**

The control objectives are to achieve low steady state error so that $\omega(t) \approx \omega_r$ and to ensure that the closed-loop response is well damped (e.g. $\zeta > 0.6$) for a range of wind speeds (12 m/s to 25 m/s).

a) Discuss the suitability of a PI controller for the wind turbine and explain how the controller parameters $K$ and $T_i$ could be designed, noting any difficulties and limitations, and possible methods to account for them in the design (there is no need to calculate suitable parameters).

Some things you may wish to consider in your answer:

- Can a PI controller be used to meet these control objectives?
- How can we choose a target 0dB crossover frequency (bandwidth) and phase margin for controller design?
- Is there a limit to the rise and/or settling times that are achievable in closed-loop operation?
- How can we ensure robustness of the controller to changes in turbine behaviour at different wind speeds?
- Is there a way to make the controller account for these changes to turbine behaviour, and hence the Bode plot, at different wind speeds?

You may assume that the control engineer carrying out the design has access to Bode plots of the turbine at various wind speeds between 12 m/s and 25 m/s.

[10 marks]

**4.** **Figure Q4** shows three reference frames, {A}, {B} and {C}. Referring to the figure:

- Derive the homogeneous transformation ${}^{A}_{B}T$ [3 marks]
- Derive the homogeneous transformation ${}^{A}_{C}T$ [3 marks]
- Derive the homogeneous transformation ${}^{B}_{C}T$ [3 marks]

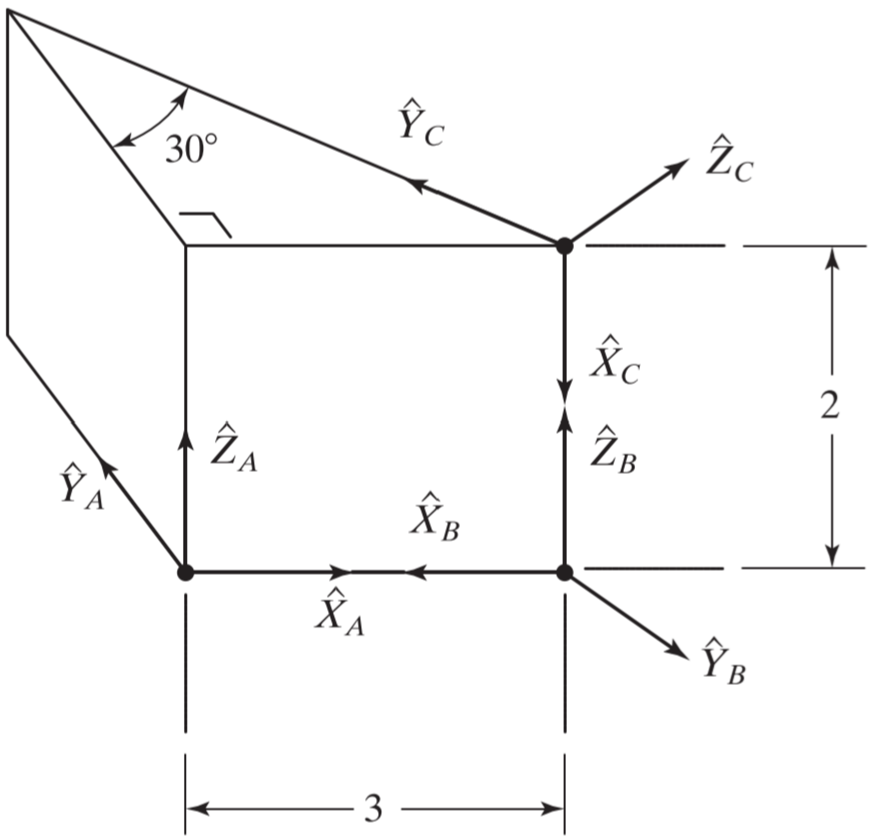


**Figure Q4**

**5.** The arm with three degrees of freedom shown in Figure Q5 has joints 1 and 2 perpendicular, and joints 2 and 3 parallel. As pictured, all joints are at their zero location. Note that the positive sense of the joint angles is indicated.

a) Using the modified Denavit-Hartenberg convention used in class, assign the link frames {0} to {4} showing the attachment of the frames on the attached copy of **Figure Q5 on Page 18** which must be handed in with your answer book. [4 marks]

b) Copy the table below in your answer book and complete the table of Denavit-Hartenberg parameters that describe the manipulator of **Figure Q5**. [4 marks]

| $i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |

c) Derive the transformation matrices ${}^0_1T$, ${}^1_2T$, ${}^2_3T$ and ${}^3_4T$. [4 marks]

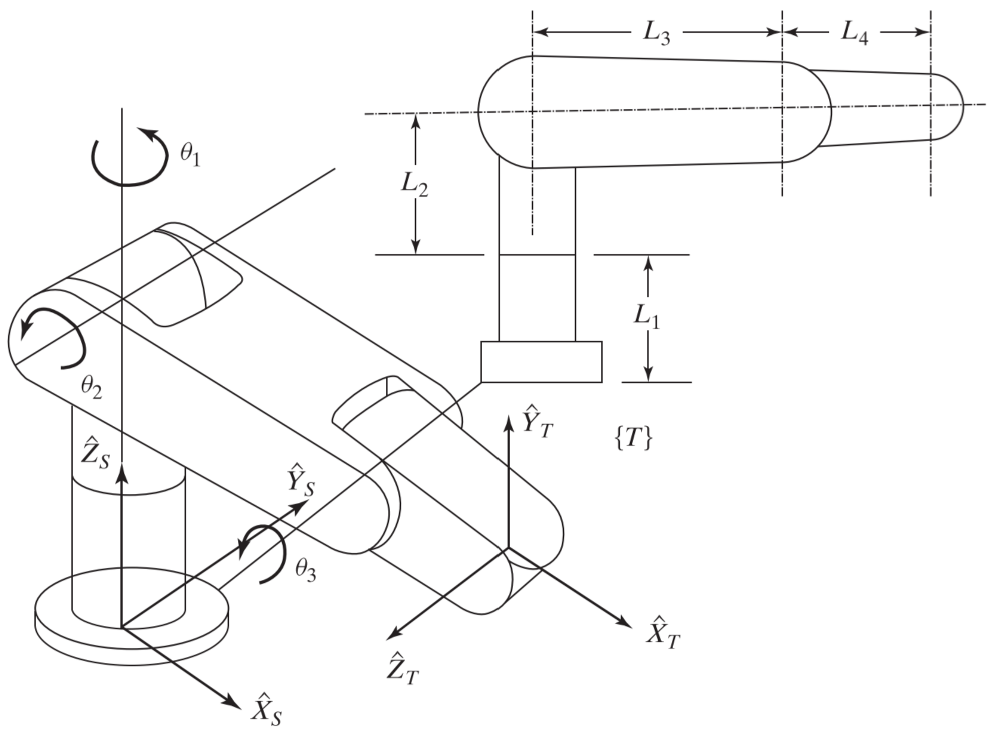


**Figure Q5** Two views of a 3R manipulator

**6.** In class we have seen the general formulation of the link transformation ${}^{i-1}_{i}T$.based on the Denavit-Hartenberg parameters, which refers to the frame assignment shown in **Figure Q6**

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

where c is cosine, s is sine.

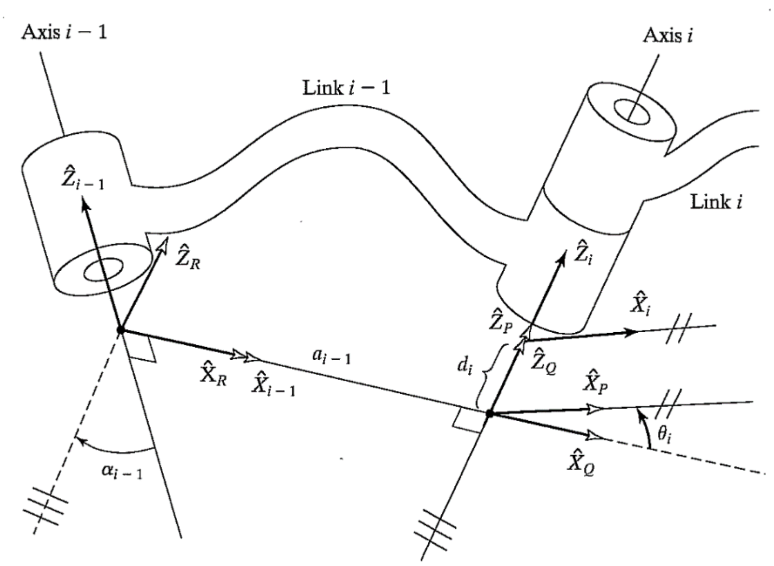


**Figure Q6**

Explain why, referring to degrees of freedom or other physical arguments, some rigid transformation cannot be achieved. [3 marks]

**7.** You have a 1 DOF planar robot arm that needs to be programmed to move according to the following requests about its angle θ:

Start point: $\theta_1$ = 90.0 deg

Via point: $\theta_2$ = -120.0 deg

Goal point: $\theta_3$ = 60.0 deg

with:

$t_{d12}$ (time from start to via point) = 4.0 sec

$t_{d23}$ (time from via point to end) = 4.0 sec

Max acceleration during blends: 60 deg/$s^2$

a) Calculate $\theta_{12}$, $\theta_{23}$, $t_1$, $t_2$, and $t_3$ for a two-segment linear spline with parabolic blends. Use the formulas at the end of the paper. [3 marks]

b) Sketch plots of position, velocity, and acceleration of θ. [3 marks]

c) Calculate the minimum acceleration needed to execute the required movements in the allowed time. [2 marks]

d) Explain why the trajectory might not necessarily go through the via point and propose a possible solution to make it sure it does. [2 marks]

**8.** The two-link planar manipulator shown in **Figure Q8** has two parallel rotary joints. It is similar to the example we saw in class, except now the joint range limits in degrees are $0 < \theta_1 < 90$ and $-90 < \theta_2 < 180$.

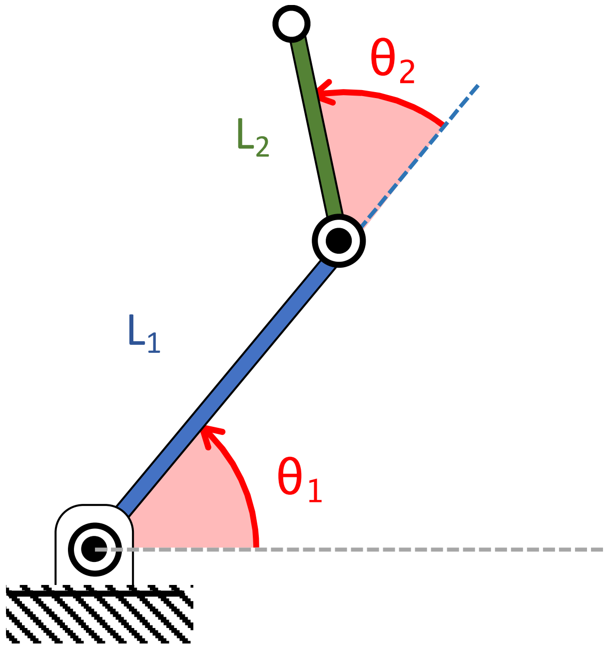


**Figure Q8**

a) Knowing that for this arm the first link is twice as long as the second (i.e. $L_1 = 2L_2$) sketch the approximate reachable workspace (an area) of the tip of link 2. You could use $L_1 = 40$, $L_2 = 20$ for a scaled drawing, or any equivalent lengths. [6 marks]

b) For this workspace, highlight the dexterous workspace, if any. [1 mark]

**9.** You are asked to design a robotic cell and select or design a ‘pick-and place’ robotic manipulator for picking cubic components, one at a time, from a conveyor belt, and placing one component at a time into a nearby container.

Explain how you would design the system in order to use a robotic manipulator with a minimum number of degrees-of-freedom. [9 marks]

Your answer should include:

- Your design specifications for the robotic cell and any assumptions you have made.
- The minimum number of degrees of freedom necessary to perform the task.
- A description or a sketch of two manipulators with different kinematic configurations which are most appropriate to perform the task.

**J. M. Fleming**
**M. Zecca**

## Useful information (control systems)

### *Selected Laplace transforms*

| *t-domain (t>0)* | *s-domain* |
|---|---|
| Unit step | $\frac{1}{s}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ |
| $e^{-at}\ \sin \omega t$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| $e^{-at}\ \cos \omega t$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |

### *Some results about Laplace Transforms*

| $f(t)$ | $F(s)$ |
|---|---|
| $f(t-T)$ | $e^{-Ts}\,F(s)$ |
| $e^{-at}\,f(t)$ | $F(s+a)$ |
| $f'(t)$ | $sF(s)-f(0)$ |
| $\int_0^t f(\tau)\,d\tau$ | $\frac{F(s)}{s}$ |



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***Step response of second-order systems*** - *For systems well approximated by* $\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$

Percentage overshoot:

$$O_{\%} = 100e^{\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Peak time:

$$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

Settling time to p% of steady-state:

$$T_s = \frac{\ln(100/p)}{\zeta\omega_0}$$

Rise time:

$$T_r \approx \frac{1.8}{\omega_0}$$

With phase margin $\phi_{PM}$ in degrees, $\zeta \approx \phi_{PM}/100$ for $\phi_{PM} < 60^o$, and $\phi_{PM} = 65^o$ gives $\zeta \approx 0.7$

***Common SISO compensator types (PID and related controllers)***

PID controller:

$$C_{pid}(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right)$$

PI controller:

$$C_{pi}(s) = K\left(1 + \frac{1}{T_i s}\right)$$

Lag compensator:

$$C_{lag}(s) = K\frac{s + 1/T_i}{s + p_{lag}}$$

PD controller:

$$C_{pd}(s) = K(1 + T_d s)$$

Lead compensator:

$$C_{lead}(s) = K\frac{T_d s + 1}{\frac{s}{p_{lead}} + 1}$$

***Frequency-domain design formulae for PID-type controllers***

where $G_C$, $\phi_C$ are required controller gain and phase at a chosen design frequency $\omega_d$

PI controller:

$$T_i = \frac{1}{\omega_d \tan(-\phi_C)}$$

PID controller:

$$\tan\phi_C = T_d\omega_d - \frac{1}{T_i\omega_d}$$

PD controller:

$$T_d = \frac{\tan\phi_C}{\omega_d}$$

Controller gain (all cases):

$$K = \frac{G_c}{\sqrt{1 + \tan^2\phi_C}}$$

**Useful information**

## Cross Product (or Vector Product)

**A x B = V**

Vector **V** is perpendicular to the vectors **A** and **B**

If $A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

then $V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ with: $v_1 = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} = a_2 b_3 - a_3 b_2$

$$v_2 = -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} = a_3 b_1 - a_1 b_3$$

$$v_3 = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

## Dot Product (or Scalar Product)

$$A \bullet B = \|A\| \, \|B\| \, \cos\theta$$

where $\|A\|$ and $\|B\|$ denote the length (magnitude) of **A** and **B** and $\theta$ is the angle between them, or

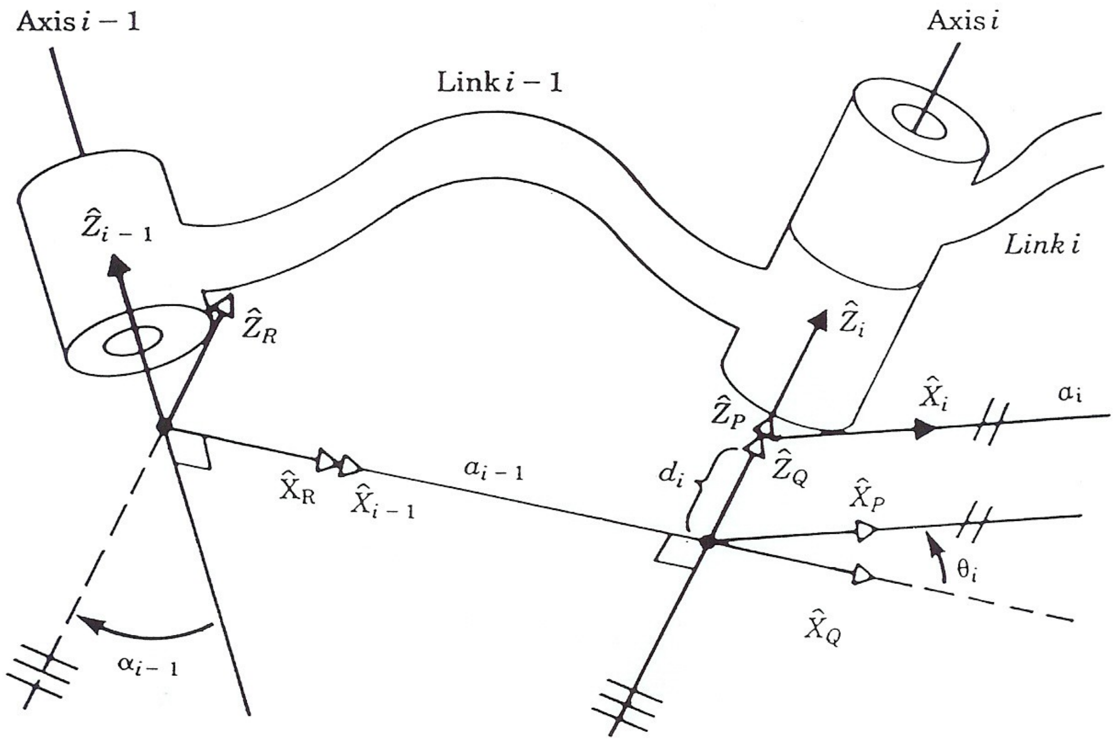
If $A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ then,

$$A \bullet B = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\|A\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

## LINK PARAMETERS ACCORDING TO THE CONVENTION OF CRAIG

**(Three intermediate frames {P}, {Q} and {R} between frames {*i*} and {*i*-1} are also shown in this figure)**

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**ROTATION MATRICES AND GENERAL HOMOGENEOUS TRANSFORMS**

Rotation matrices in 3x3 and 4x4 form

$$\textbf{Rot (X, θ)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**3 x 3** **4 x 4**

$$\textbf{Rot (Y, θ)} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

**Similarly**

$$\textbf{Rot (Z, θ)} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Similarly**

## General homogeneous transform

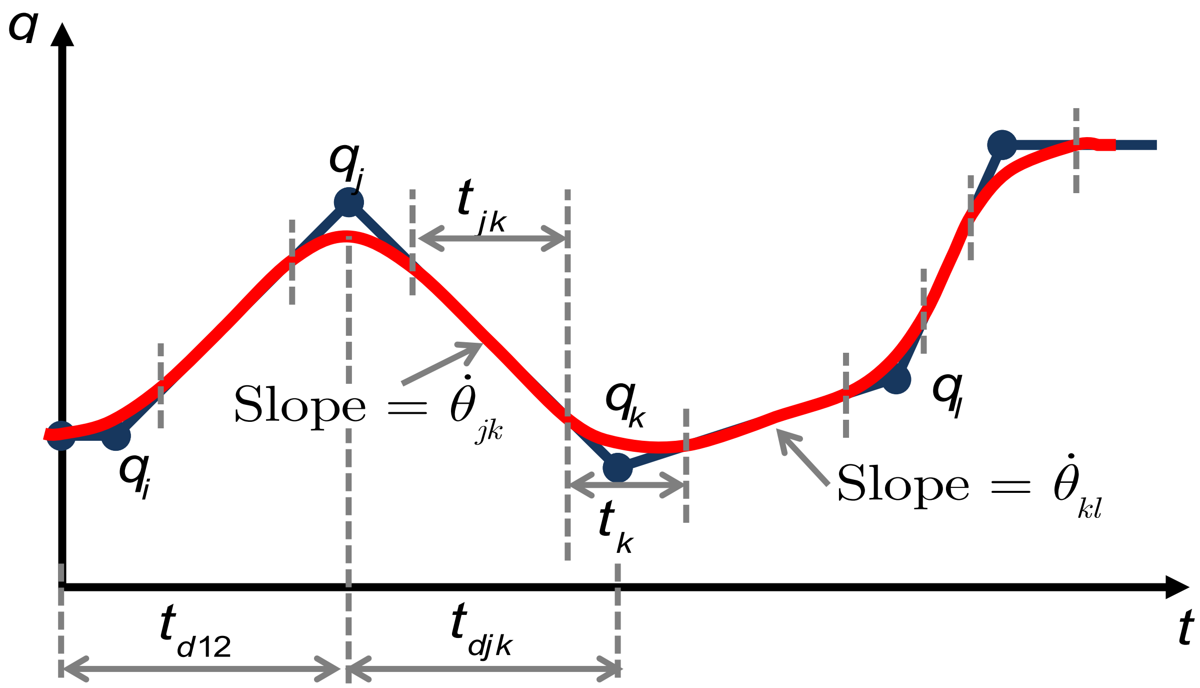
**The general transformation matrix from frame {B} to {A} in terms of the 3x3 rotation matrix $^{A}_{B}R$ and the vector $^{A}P_{\text{BORIG}}$ of the origin of frame {B} w.r.t. frame {A} is:**

$$^{A}_{B}T = \left[\begin{array}{ccc:c} & ^{A}_{B}R & & ^{A}P_{\text{BORIG}} \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{array}\right]$$

**The inverse transformation matrix, $^{B}_{A}T$, of $^{A}_{B}T$ can also be written in terms of $^{A}_{B}R$ and $^{A}P_{\text{BORIG}}$ as follows:**

$$^{B}_{A}T = \left[\begin{array}{ccc:c} & ^{A}_{B}R^{T} & & -^{A}_{B}R^{T} \cdot {}^{A}P_{\text{BORIG}} \\ \hdashline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{array}\right]$$

## Linear function with parabolic blends for a path with via points




| For the first segment | For the last segment | For the interior path points |
|---|---|---|
| $\ddot\theta_1 = \text{sign}\left(\dot\theta_2 - \dot\theta_1\right)\left\|\ddot\theta_1\right\|$<br>$t_1 = t_{d12} - \sqrt{t_{d12}^2 - 2\frac{\theta_2 - \theta_1}{\ddot\theta_1}}$<br>$\dot\theta_{12} = \frac{\theta_2 - \theta_1}{t_{d12} - \frac{1}{2}t_1}$<br>$t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$ | $\ddot\theta_n = sign\left(\dot\theta_{n-1} - \dot\theta_n\right)\left\|\ddot\theta_n\right\|$<br>$t_n = t_{d(n-1)n} - \sqrt{t_{d(n-1)n}^2 - 2\frac{\theta_n - \theta_{n-1}}{\ddot\theta_n}}$<br>$\dot\theta_{(n-1)n} = \frac{\theta_n - \theta_{n-1}}{t_{d(n-1)n} - \frac{1}{2}t_n}$<br>$t_{(n-1)n} = t_{d(n-1)n} - t_n - \frac{1}{2}t_{n-1}$ | $\dot\theta_{jk} = \frac{\theta_k - \theta_j}{t_{djk}}$<br>$\ddot\theta_k = \text{sign}\left(\dot\theta_{kl} - \dot\theta_{jk}\right)\left\|\ddot\theta_k\right\|$<br>$t_k = \frac{\dot\theta_{kl} - \dot\theta_{jk}}{\ddot\theta_k}$<br>$t_{jk} = t_{djk} - \frac{1}{2}t_j - \frac{1}{2}t_k$ |



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ROBOTICS & CONTROL (22WSC104)

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Student Registration Number: _________.

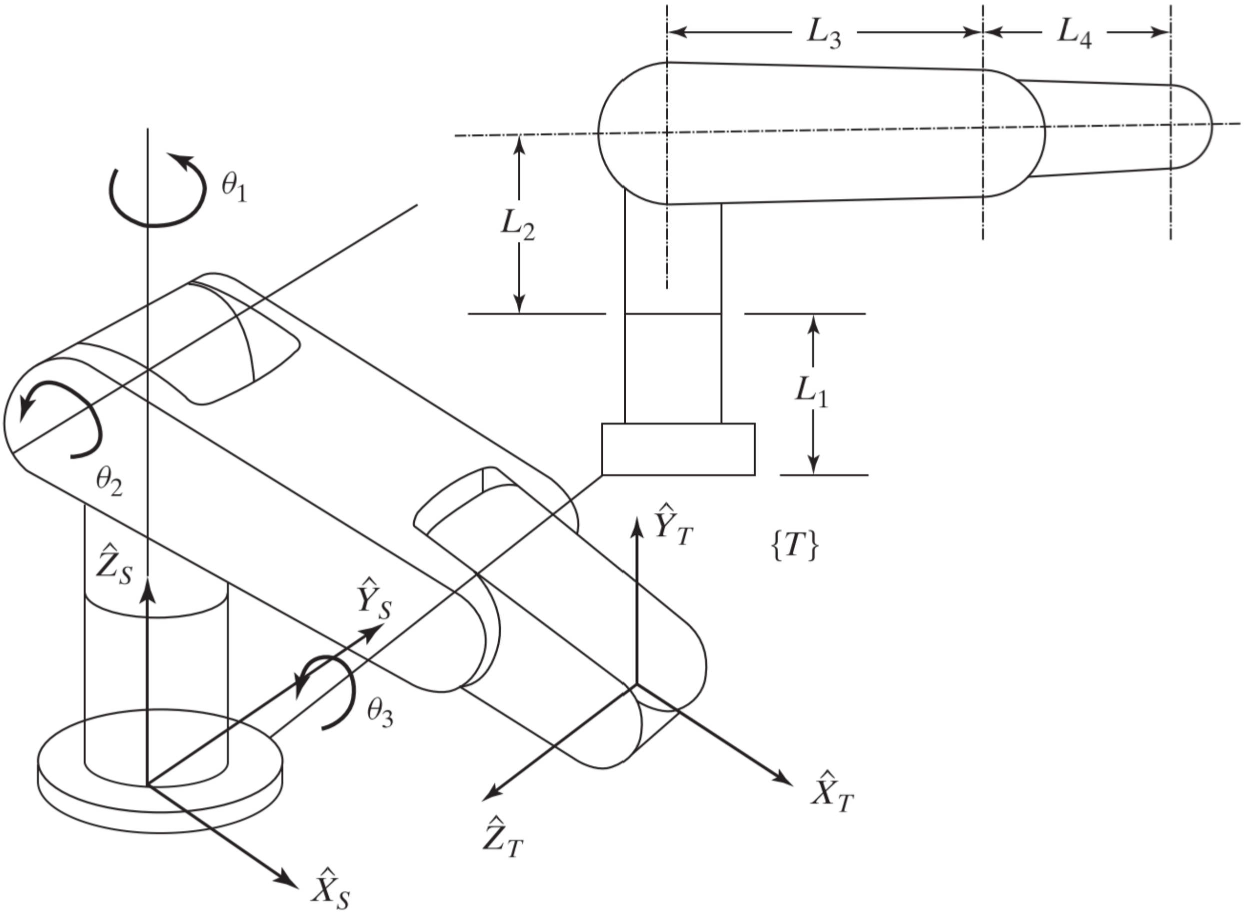


**Figure Q5**

Table for the DH parameters of Exercise 5

| $i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 0 | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |