Loughborough University

NONLINEAR DYNAMICS

22WSD102

Semester 1 2022 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

NONLINEAR DYNAMICS
(22WSD102)

January 2023 2 Hours

Answer **ALL** questions.

Questions carry the marks shown.

Any approved University calculator is permitted.

1.

a) How is the stress-strain curve of a material affected by the increase of strain rate and temperature? Explain using graphs. [4 marks]

b) Euler's method is one of the simplest numerical procedures for the solution of ordinary differential equations. Provide the graphical representation of the method and explain how it works. How can the accuracy of this method be improved? [4 marks]

c) A standard linear solid model is composed of a spring and a Maxwell model connected in parallel, as shown in **Figure Q.1**. Provide and explain the initial condition(s) for integration under creep and relaxation conditions. [4 marks]

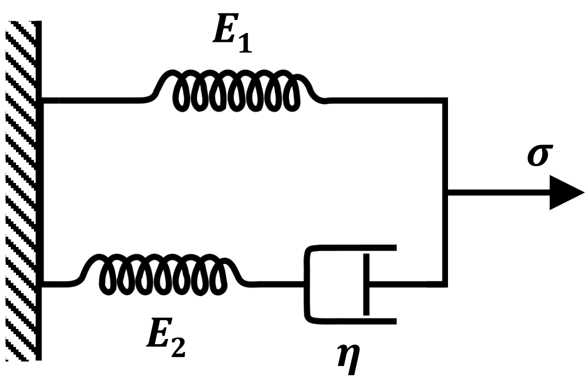

Figure Q.1

d) The spring E_1 was disconnected from the above apparatus and a stress relaxation experiment was performed using the Maxwell device.
A 10 cm long bar was stretched instantly until the stress reached $\sigma_0 = 200$ MPa. Two minutes after the initial loading, the stress measured in the bar decreased to 160 MPa.

Assuming that the material response can be modelled by a Maxwell system:

i. Calculate the relaxation time of the material defined as $\tau_r = \eta/E$. [2 marks]

ii. Calculate the viscosity of the dashpot in the model given that the initial stress was reached by extending the bar by 16 mm. [4 marks]

iii. Find the Young's modulus of the material 1.5 hours after the initial loading. Comment on your result. [4 marks]

You are reminded that the relaxation response of a material using a Maxwell model is given by the following expression:

$$\sigma(t) = \sigma_0 \exp\left(-\frac{E}{\eta}t\right)$$

where σ_0 is the initial stress applied to the material, E is the Young's modulus of the spring, η is the viscosity of the dashpot, and t is time.

2. Describe the following sources of nonlinear effects and briefly discuss one example for each source:

- Kinematics
- Material nonlinearities
- Boundary conditions
- Nonlinear body forces
- Geometric nonlinearities [15 marks]

3. Structures containing different nonlinear elements are excited with a constant amplitude sinusoidal force. Describe the Nyquist and FRF plot distortions for the following nonlinear elements when excited with low and high level amplitudes of the sinusoidal force. Sketch plots to support your answer.

- Hardening cubic stiffness
- Quadratic damping
- Coulomb friction
- Clearance [12 marks]

4. The equation of a mechanical system with nonlinear damping is provided below:

$$m\ddot{x} + \varepsilon\,(a\dot{x}^2 - c)\dot{x} + kx \;= 0$$

where m is the mass, x is the motion of the oscillator, ε is a small constant, a is the coefficient of nonlinear damping, c is the linear damping, and k is the linear stiffness.

a) <u>Normalise</u> the above equation of motion and <u>take dimensions off</u> by using the characteristic “length” x_c and characteristic frequency $\omega_0 = \sqrt{\frac{k}{m}}$. [8 marks]

b) The above system exhibits a <u>limit cycle</u>. Briefly describe the dynamics of the limit cycle and the corresponding trajectories. Draw a sketch to support your answer. [5 marks]

5. The equation describing the nonlinear oscillations of an undamped pendulum is provided below:

$$\ddot{\theta} + sin\theta = 0$$

a) Find the singular points. [2 marks]

b) Write an expression for the potential energy and sketch a qualitative plot of the potential energy with respect to the pendulum angle. [4 marks]

c) Classify the singular points using the potential energy. Sketch the trajectories to display the local dynamics (near the singular points), as well as the global dynamics. [5 marks]

d) What changes to the dynamics (and to the trajectories) when positive viscous damping is added to the system (provide a sketch of the trajectories to support your answer)? [5 marks]

6. The equation of motion of the forced Rayleigh oscillator is given below:

$$\ddot{u} - \varepsilon\left(1 - \frac{1}{3}\dot{u}^2\right)\dot{u} + \omega_0^2\, u = \varepsilon\, f_0 \cos\omega t$$

Using the method of multiple time scales show that the zero and first order of ε equations of motion are given by:

$$D_0^2\, u_0 + \omega_0^2\, u_0 = 0$$

$$D_0^2\, u_1 + \omega_0^2\, u_1 = -2D_0D_1u_0 + D_0u_0 - \frac{1}{3}(D_0u_0)^3 + f_0\cos(\omega_0\tau_0 + \sigma\tau_1)$$

[14 marks]

It is given that the fast time is $\tau_0 = t$ and the slow time is $\tau_1 = \varepsilon\, t$.

The form of the response is $u(t) = u_0(\tau_0, \tau_1) + \varepsilon\, u_1(\tau_0, \tau_1) + \ldots$

$D_0 = \frac{\partial}{\partial\tau_0}$ and $D_1 = \frac{\partial}{\partial\tau_1}$.

7. The fatigue behaviour of a high-strength alloy used in dynamic applications under alternating stress conditions with zero mean stress is given by the following expression:

$$(\sigma_a)^n \cdot N_f = 0.362\,K$$

where σ_a is the stress amplitude, N_f the number of cycles to failure, and n, K are material constants.

The number of cycles to failure for this alloy at different stress levels is given in the following table:

σ_a (MPa)	120	80
N_f (cycles)	10^5	10^7

Also, the endurance limit for this material was found equal to 50 MPa.

Estimate the life of a component made of this material when subjected to an amplitude of i) 60 MPa, ii) 40 MPa. [8 marks]

S Theodossiades
K Baxevanakis