Loughborough University

DESIGN OF MACHINE ELEMENTS

22WSB403

Semester 2 2023 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

DESIGN OF MACHINE ELEMENTS

(22WSB403)

Semester 2 2023 | 2 Hours

Answer **ALL** questions.

Questions carry the marks shown.

Any University-approved calculator is permitted.

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the rear of the paper.

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1. Answer the following short questions.

a) With the aid of a sketch, describe the face width of a gear. [2 marks]

b) When would you use a solid lubricant? Name TWO reasons. [2 marks]

c) Why should the keyway be made of softer material than the hub? [2 marks]

d) What factor can lead to uneven load distribution across the thickness of a spur gear tooth? [2 marks]

e) Why is a preload needed for some types of belts? [2 marks]

2. A single stage enclosed reduction gearbox is to be used in a hoist to transmit 1.2 kW from an electric motor running at 1500 rpm. The gearbox reduction is 4:1 using simple straight cut, full tooth involute spur gears with a pressure angle of 20°, a module of 3 mm and a facewidth of 15 mm. The number of teeth on the steel input pinion is 12 and the gear wheel is to be made of cast iron, the hoist can be considered to be subject to moderate shock. For this gearbox determine the following:

a) Pitch circle diameter of both gears and the centre distance between the two shafts. [2 marks]

b) Contact ratio [2 marks]

c) Pitch line velocity. [2 marks]

d) Velocity factor. [2 marks]

e) Transmitted load. [2 marks]

f) Tooth bending stress [2 marks]

g) Contact stress in the gear teeth. [4 marks]

h) If the pinion is mounted mid-way on a 15 mm diameter shaft of 100 mm length (i.e. supported by a rolling element bearing on each end), calculate a suitable dynamic load to be used for selection of the bearings used on the pinion shaft with an L10 life of 30,000 hours and select a suitable bearing from the table. [4 marks]

i) The gearbox operation is expected to increase the working temperature of the shafts up to 80°C. Determine the thermal expansion along the length if the shaft is made of carbon steel. What accommodation would you make for this in your bearing selection and bearing mounting? Justify your decisions. [4 marks]

j) If the gearbox required enhanced reliability of 99%, would you change the bearing specification and if so, what specification would you choose from the table? [2 marks]

k) What would be the most cost-effective lubrication system for the bearings in this case [2 marks]

l) From a manufacturing and maintenance viewpoint which would be the best way to mount the shafts in the gearbox, how would the casing be split? [2 marks]

3. A motor is mounted on a pivoted table which is supported by a frictionless pivot joint at "A" and by a vertical spring in compression, which acts at 600 mm from "A". See **Figure Q3**.

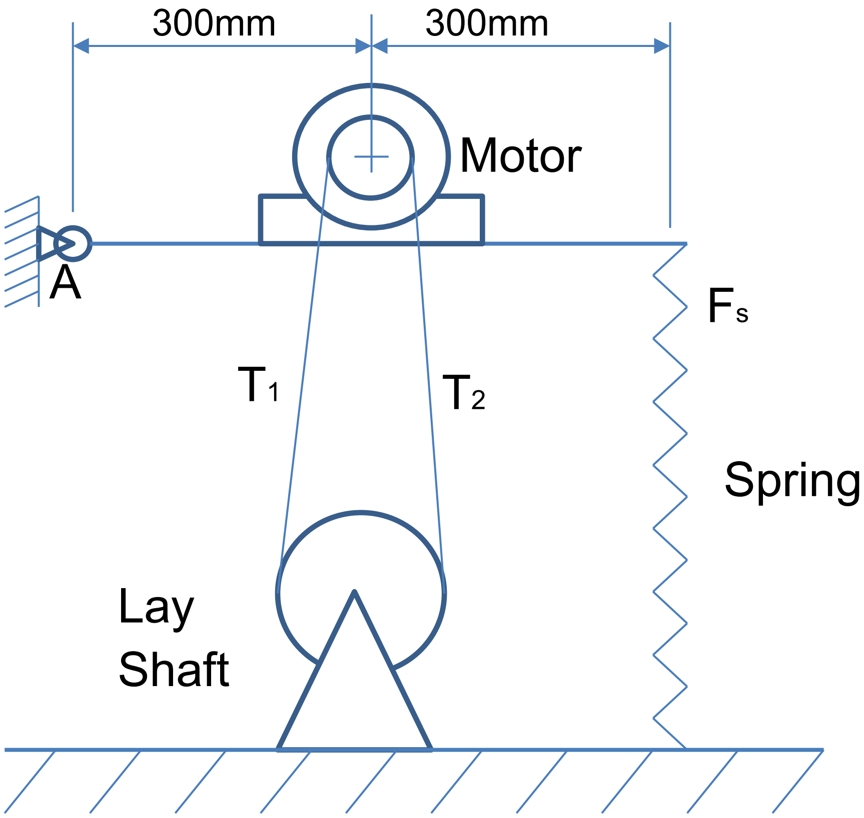

Figure Q3. Motor and Lay Shaft Configuration

The motor drives a lay shaft via a friction belt

The table mass may be ignored.

The motor weighs 150 kg, resulting in a mass force centred 300 mm from "A".

a) Draw a free body diagram of the motor drive pulley showing transmitted shaft torque and the forces transmitted by the belt. [4 marks]

b) If $\mathbf{T_1 > T_2}$ what is the direction of rotation of the motor pulley when observed from the pulley end (ref: **Figure Q3**.) [2 marks]

c) The typical operating speed of the motor is 1800 RPM and the pulleys are sized such that the lay shaft runs at 600 RPM. Calculate the required lay shaft pulley diameter if the motor pulley diameter is 150 mm. [4 marks]

/continued

d) Referring to your free body diagram, if the belt system is considered as 100% efficient, derive an expression for the motor pulley torque in terms of T_1, T_2 and the diameter of the motor pulley if T_1 and T_2 act vertically. [5 marks]

Under normal running conditions, the motor produces 100 Nm of torque.

The spring force has been adjusted such that the belt will be on the point of slipping when the torque reaches 100 Nm.

The slipping point is given by the following equation:

$$\boldsymbol{T_1 = T_2 \times e^{\mu\beta}}$$

Where μ= Coeff. of Static Friction = 0.3

Use β = Angle of belt to pulley contact = π rads

e) Calculate the values of T_1 and T_2 at 100 Nm torque at the point of slipping by equating your expression derived in part (d) and the slip equation. [6 marks]

f) Draw a free body diagram (FBD) of the motor and table arrangement and calculate the required value of F_s provided by the spring assuming $\beta = \pi$ rads. Note that your FBD does not include the motor pulley but does contain the forces imparted by the pulley. [9 marks]

4. A compression spring has squared ends (one inactive coil each end) and is made of carbon steel wire (G = 79 GPa). The mean coil diameter is 20 mm, and the wire diameter, d = 3.20 mm. the coil outer diameter is 23.20 mm. The spring has a free length of 155 mm, and 19 total coils.

a) Calculate the pitch of the spring at the free length. [4 marks]

b) Calculate the solid height for the spring. [2 marks]

c) Calculate the deflection to compress the spring to its solid height. [2 marks]

d) Calculate the spring constant for the spring. [4 marks]

e) Determine the force to compress the spring to its solid height. [2 marks]

f) Determine whether the spring will buckle before it reaches its solid height if the spring is unrestrained. [4 marks]

g) Select a suitable spring material if the force on the spring fluctuates from 0 to 44.5 N. Allow a safety factor of 5. [6 marks]

h) **Figure Q4** shows a coiled compression spring that has been loaded against a support by means of a bolt and nut. After the nut has been tightened to the position shown, an external force F is applied to the bolt as indicated, and the deflection of the spring is measured as F is increased. A plot of the resulting force-deflection curve is shown. Briefly but clearly state the reasons the curve changes at points A, B, and C. [6 marks]

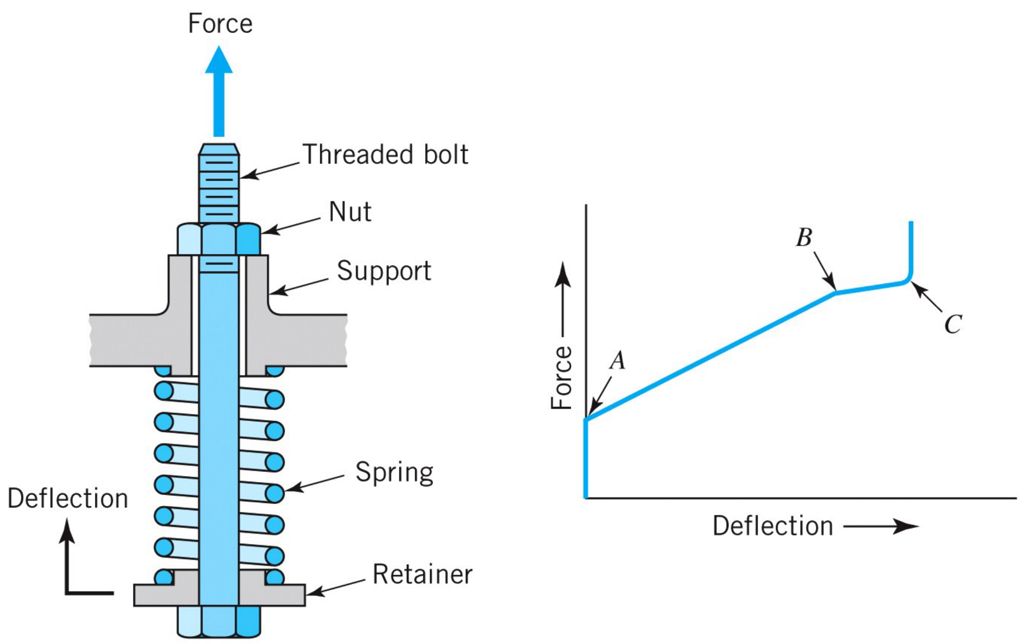

Figure P12.16

Figure Q4. Bolt and spring assembly

Y.M. Goh
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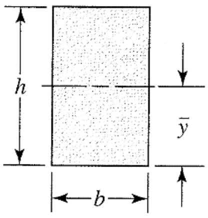
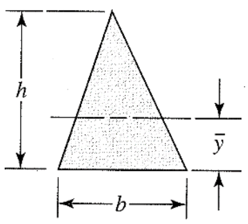
Formula Sheet



Rectangle

$$A = bh \qquad I = \frac{bh^3}{12} \qquad Z = \frac{bh^2}{6}$$

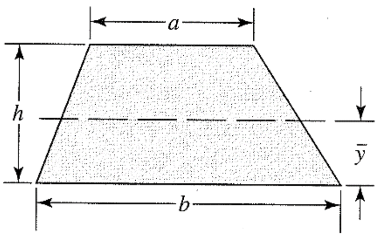
$$\rho = 0.289h \qquad \bar{y} = \frac{h}{2}$$



General triangle

$$A = \frac{bh}{2} \qquad I = \frac{bh^3}{36} \qquad Z = \frac{bh^2}{24}$$

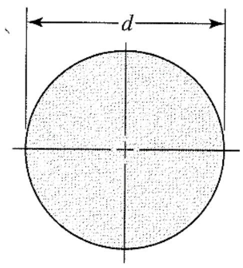
$$\rho = 0.236h \qquad \bar{y} = \frac{h}{3}$$



General trapezoid

$$A = \frac{h}{2}(a + b) \qquad I = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \qquad Z = \frac{h^2}{12}\,\frac{(a^2 + 4ab + b^2)}{(a + 2b)}$$

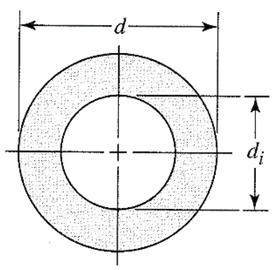
$$\rho = \frac{h}{6}\sqrt{2 + \frac{4ab}{(a + b)^2}} \qquad \bar{y} = \frac{h}{3}\,\frac{(2a + b)}{(a + b)}$$



Circle

$$A = \frac{\pi d^2}{4} \qquad I = \frac{\pi d^4}{64} \qquad Z = \frac{\pi d^3}{32}$$

$$J = \frac{\pi d^4}{32} \qquad \rho = \frac{d}{4}$$



Hollow circle

$$A = \frac{\pi}{4}(d^2 - d_i^2) \qquad I = \frac{\pi}{64}(d^4 - d_i^4) \qquad Z = \frac{\pi}{32d}(d^4 - d_i^4)$$

$$J = \frac{\pi}{32}(d^4 - d_i^4) \qquad \rho = \sqrt{\frac{d^2 + d_i^2}{16}}$$

/continued

Shafts Design

$S_e = C_{surf} * C_{size} * C_{load} * C_{temp} * C_{rel} * S_e'$

The surface finish factor, C_{surf} is given by (S_{ut} in MPa):

$$C_{surf} = aS_{ut}^{b}$$

Surface finish	a	b
Ground	**1.58**	**-0.085**
Machined or cold drawn	**4.51**	**-0.265**
Hot rolled	**57.7**	**-0.718**
Forged	**272.0**	**-0.995**

The size factor, C_{size} is given by:

$$C_{size} = \begin{cases} \left(\dfrac{d}{7.62}\right)^{-0.1133} & \text{for d} \le 50\,\text{mm} \\ 1.85d^{-0.19} & \text{for d} > 50\,\text{mm} \end{cases}$$

The load factor, C_{load} is given as:

$$C_{load} = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

For temperatures between -57°C and 204°C, the temperature factor, C_{temp} can be taken as 1.

The reliability factor, C_{rel}:

Nominal reliability	C_{rel}
0.5	**1.0**
0.9	**0.897**

/continued

0.99	**0.814**
0.999	**0.753**

For a rotating shaft with steady torque and fully reversed bending, the ASME design equation is:

$$d = \left[\frac{32N}{\pi} \sqrt{\left(\frac{K_t M}{S_e}\right)^2 + \frac{3}{4}\left(\frac{T}{S_y}\right)^2} \right]^{\frac{1}{3}}$$

where N = safety factor, T = steady torque and M = fully reversed bending moment.

$\Delta L = L\,\alpha\,\Delta T$

Flexible Transmission

$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D-d}{2 \cdot C}\right)$$

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D-d}{2 \cdot C}\right)$$

$$F_r = \sqrt{F_1{}^2 + F_2{}^2 - 2F_1 F_2 \cos\theta}$$

$$\text{Power} = (F_1 - F_2) \cdot V$$

$F_c = \rho \cdot V^2 \cdot A$ **where Fc = centrifugal force**

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu \cdot \theta_d}$$

Chain length in number of pitches, L is given as:

$$L = \frac{N_1 + N_2}{2} + \frac{2C}{p} + \left(\frac{N_2 - N_1}{2\pi}\right)^2 \frac{p}{C}$$

L = number of pitches,
N_1 = number of teeth in the driving sprocket,
N_2 = number of teeth in the driven sprocket,
C = centre distance (m),
p = chain pitch (m).

The exact centre distance, C is given as:

$$C = \frac{p}{8}\left[2L - N_2 - N_1 + \sqrt{(2L - N_2 - N_1)^2 - \frac{\pi}{3.88}(N_2 - N_1)^2}\right]$$

$f_2 = 19/N_1$

Bolted Joints

Pitch diameter, d_p = d - 0.649519p

Minor diameter, d_r = d - 1.226869p

$$A_t = \frac{\pi}{16}(d_p + d_r)^2$$

$$F_i := f_p \cdot S_p \cdot A_t$$

$T = KF_i d$

$$k = \frac{AE}{l}$$

$$C = \frac{k_b}{k_b + k_m}$$

$P_b = CP$

$P_m = (1-C)P$

$$f = 2\pi^2 \frac{\mu n}{P} \frac{R}{c}$$

Lubrication, Seals and Bearings

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu \cdot N_s}{P}$$

$h_o = c - e$ $\quad\quad$ $\boldsymbol{\varepsilon} = e/c$ $\quad\quad$ $h_o/c = 1 - \boldsymbol{\varepsilon}$

$$\Delta T = \frac{8.3 \times 10^{-6} P}{1 - \frac{1}{2}(Q_s/Q)} \times \left(\frac{(r/c)f}{Q/rcN_sL}\right)$$

/continued

$$\varphi = \sqrt{\frac{1-\left(\frac{p_n}{p_o}\right)}{n+\ln\left(\frac{p_o}{p_n}\right)}}$$

where p_n is downstream pressure following the n^{th} labyrinth (Pa); and n is the number of fins.

$$\dot{m} = A\alpha\gamma\varphi\sqrt{\rho_0 p_0}$$

Leakage flow through an axial bush seal $Q = \dfrac{\pi\phi c^3\left(p_0^2 - p_a^2\right)}{24\mu L p_a}$

Leakage flow through a radial bush seal $Q = \dfrac{\pi c^3\left(p_0^2 - p_a^2\right)}{12\mu p_a}$

$P = X F_r + F_a$

$F_r' = f_w \times F_r$

$C = P(L/10^6)^{1/k}$

Gears design

$$d_p = mN$$

$$C_d = \frac{1}{2}\left(d_p + d_g\right)$$

$$F_t = \frac{P}{v}$$

$$\sigma = \frac{K_v F_t}{WmY}$$

For cut or milled gears $K_v = \dfrac{6,1 + V}{6,1}$

For cast iron , cast gears $K_v = \dfrac{3,05 + V}{3,05}$

For hobbed or shaped gears $K_v = \dfrac{3,56 + \sqrt{V}}{3,56}$

For shaved or ground gears $K_v = \sqrt{\dfrac{5,56 + \sqrt{V}}{5,56}}$

$$r_1 = \frac{d_P \sin\varphi}{2}$$

$$r_2 = \frac{d_G \sin\varphi}{2}$$

$$\sigma_c = -Z_E \left[\frac{K_v^{'} F_t}{W \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{0.5}$$

$$Contact\ Ratio = \frac{\left(R_{go}^2 - R_{gb}^2\right)^{1/2} + \left(R_{po}^2 - R_{pb}^2\right)^{1/2} - a\ sin\emptyset}{\pi\ m\ cos\emptyset}$$

R_{go} = Radius of Outside Dia of Gear
R_{gb} = Radius of Base Dia of Gear
R_{po} = Radius of Outside Dia of Pinion
R_{pb} = Radius of Base Dia of Pinion

Springs

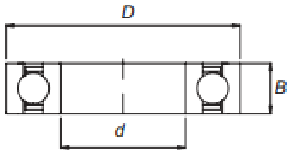
$$C = \frac{D}{d}$$

$$k = \frac{dG}{8C^3 N_a}$$

$$\tau_{\max} = \frac{K_s 8FD}{\pi d^3} \text{ where the direct shear factor } K_s = 1 + \frac{1}{2C}$$

$$\tau_{\max} = \frac{K_w 8FD}{\pi d^3} \text{ where the Wahl factor } K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Table 4.7 Selected example single row deep groove ball bearing ratings



d (mm)	D (mm)	B (mm)	Basic dynamic load rating C (N)	Basic static load rating C_o (N)	Speed limit for grease lubrication (rpm)	Speed limit for oil lubrication (rpm)	Code
15	24	5	1570	800	28 000	34 000	61802
	32	8	5600	2850	22 000	28 000	16002
	32	9	5600	2850	22 000	28 000	6002
	35	11	7850	3750	19 000	24 000	6202
	42	13	11 500	5400	17 000	20 000	6302
17	26	5	1690	930	24 000	30 000	61803
	35	8	6060	3250	19 000	24 000	16003
	35	10	6060	3250	19 000	24 000	6003
	40	12	9550	4750	17 000	20 000	6203
	47	14	13 600	6550	16 000	19 000	6303
	62	17	23 000	10 800	12 000	15 000	6403
20	32	7	2750	1500	19 000	24 000	61804
	42	8	6900	4050	17 000	20 000	16004
	42	12	9400	5000	17 000	20 000	6004
	47	14	12 800	6550	15 000	18 000	6204
	52	15	16 000	7800	13 000	16 000	6304
	72	19	30 800	15 000	10 000	13 000	6404
25	37	7	4400	2600	17 000	20 000	61805
	47	8	7600	4750	14 000	17 000	16005
	47	12	11 300	6550	15 000	18 000	6005
	52	15	14 050	7800	12 000	15 000	6205
	62	17	22 600	11 600	11 000	14 000	6305
	80	21	36 000	19 300	9000	11 000	6405
30	42	7	4500	2900	15 000	18 000	61806
	55	9	11 300	7350	12 000	15 000	16006
	55	13	13 400	8300	12 000	15 000	6006
	62	16	19 600	11 200	10 000	13 000	6206
	72	19	28 200	16 000	9000	11 000	6306
	90	23	43 700	23 600	8500	10 000	6406

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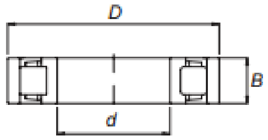
Table 4.9 Selected example cylindrical roller bearing ratings



d (mm)	D (mm)	B (mm)	Basic dynamic load rating C (N)	Basic static load rating C_o (N)	Speed limit for grease lubrication (rpm)	Speed limit for oil lubrication (rpm)	Code
15	35	11	12 600	10 200	18 000	22 000	NU202E
	42	13	19 500	15 300	16 000	19 000	NU302E
25	52	15	28 700	27 000	11 000	14 000	NU205E
	62	17	40 300	36 500	9500	12 000	NU305E
30	62	16	38 100	36 500	9500	12 000	NU206E
	72	19	51 300	48 000	9000	11 000	NU306E
50	90	20	64 500	69 500	6300	7500	NU210E
	110	27	111 000	112 000	5000	6000	NU310E
	130	31	131 000	127 000	5000	6000	NU410
100	180	34	252 000	305 000	3200	3800	NU220E
	250	58	430 000	475 000	2400	3000	NU420
200	360	58	766 000	1 060 000	1500	1800	NU240E
	420	80	990 000	1 320 000	1300	1600	NU340
600	870	118	2 750 000	510 000	600	700	NU10/600

/continued

Lewis form factor table

N Number of teeth	Y $\phi = 20^{o}$ $a = 0.8m$ $b = m$	Y $\phi = 20^{o}$ $a = m$ $b = 1.25m$
12	0.33512	0.22960
13	0.34827	0.24317
14	0.35985	0.25530
15	0.37013	0.26622
16	0.37931	0.27610
17	0.38757	0.28508
18	0.39502	0.29327
19	0.40179	0.30078
20	0.40797	0.30769

Elastic coefficient Z_E ($(MPa)^{0.5}$)

		GEAR MATERIAL					
PINION MATERIAL	E_{pinion} (GPa)	Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminium Bronze	Tin Bronze
Steel	200	191	181	179	174	162	158
Malleable Iron	170	181	174	172	168	158	154
Nodular Iron	170	179	172	170	166	156	152
Cast Iron	150	174	168	166	163	154	149
Aluminium Bronze	120	162	158	156	154	145	141
Tin Bronze	110	158	154	152	149	141	137

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Estimate for overload factors

	DRIVEN MACHINE			
DRIVING MACHINE	Uniform. For example: continuous duty generator sets.	Light Shock. For example: fans, low speed centrifugal pumps, variable duty generators, uniformly loaded conveyors, positive displacement pumps.	Moderate Shock. For example: high speed centrifugal pumps, reciprocating pumps and compressors, heavy duty conveyors, machine tools drives and saws.	Heavy Shock. For example: punch presses and crushers.
Uniform. For example: electric motors or constant speed gas turbines.	1	1.25	1.5	1.75
Light Shock. For example: variable speed drives.	1.2	1.4	1.75	2.25
Moderate Shock. For example: multi cylinder engines.	1.3	1.7	2	2.75

Table: Linear Co-efficient of Thermal Expansion

Material (at 20 ^{o}C)	α units $10^{-6}/^{o}C$
Grey Cast Iron	10.8
Pure Cast iron	12.0
Forged Iron	11.3
Austenitic Stainless Steel	16.0
Ferritic Stainless Steel	9.9
Carbon Steel	11.9
Brass	19.0
Aluminium	23.0

/continued

Reliability factors for bearings

% RELIABILITY	90	95	96	97	98	99
	L_{10}	L_5	L_4	L_3	L_2	L_1
a_1	1.0	0.62	0.53	0.44	0.33	0.21

Table Spring materials

Spring Material	Minimum UTS (MPa)	Torsional shear stress (% of Tensile strength)	Relative cost
Hard Drawn ASTM A227	1015	50	0.9
Oil tempered	1150	45	0.95
Music Wire	1530	55	1.0
Berylium copper	1050	50	1.1
Stainless steel A313	1620	50	1.2

Buckling conditions for helical compression springs