Loughborough University

HEAT TRANSFER

22WSB801

Semester 2 2023 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

HEAT TRANSFER
(22WSB801)

Summer 2023 | 2 Hours

Answer **ALL** questions.

Questions carry the marks shown.

Any approved University calculator is permitted.

A range of graphs and formulae likely to be of benefit in the solution of these questions are provided at the rear of the paper.

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1. Use the lumped system approach in solving the following problems.

a) A hot wire anemometer probe consists of an iron wire which is a 5 mm long cylinder with a diameter of 0.1 mm. Air flows across the cylinder at 10 $m.s^{-1}$.
If the kinematic viscosity of air is $1.02 \cdot 10^{-5}$ $m^2.s^{-1}$, select a suitable correlation from the data sheet and evaluate the Nusselt number and hence the convective heat transfer coefficient between the cylinder and the air. Take the thermal conductivity of air as $2.577 \cdot 10^{-2}$ $W.m^{-1}.K^{-1}$. [10 marks]

b) The air temperature is 20.0°C and electrical current through the cylinder maintains its temperature at 22.0°C. If electrical current to the wire is suddenly switched off how long will it take for the wire to reach a temperature of 20.1°C? Take the density, specific heat and thermal conductivity of iron as 7870 $kg.m^{-3}$, 447 $J.kg^{-1}.K^{-1}$ and 80.2 $W.m^{-1}.K^{-1}$ respectively. [10 marks]

2. A heat exchanger with two shells and four tube passes is used to heat 3 $kg.s^{-1}$ of pressurised water from 35°C to 110°C with 1.5 $kg.s^{-1}$ water entering at 250°C.

a) Use the NTU method to find the required heat transfer area. Assume that the specific heat capacity for water is 4180 $J.kg^{-1}.K^{-1}$ and the overall heat transfer coefficient is equal to1500 $W.m^{-2}.K^{-1}$. [11 marks]

b) Calculate outlet temperature of the water which is used for heating. [4 marks]

3. Evaporation of droplets in gases occurs in a wide range of applications, such as evaporation of fuel sprays in combustion devices or the evaporation of respiratory droplets which facilitates virus transmission. In this question, the evaporation process of a spherical water droplet in gaseous nitrogen is considered, as in **Figure Q3**. Initially, the gaseous nitrogen contains no water moisture, and its temperature is uniform at 800 °C. As the evaporation process starts, a temperature gradient starts to develop in the nitrogen while the temperature sufficiently far away from the droplet remains at 800 °C. The total pressure in the gas domain is maintained at 1 bar throughout the evaporation process. The gravity is ignored such that the evaporation process can be considered to be spherical symmetric (i.e., all variables are only functions of the radial distance from the centre of the droplet r and time t). The following thermophysical properties of the gas domain and water can be considered as constants throughout the evaporation process: thermal conductivities ($k_{gas} = 0.07\ \mathrm{W\,m^{-1}K^{-1}}$), latent heat of evaporation of water $h_{fg} = 2257.4\ \mathrm{kJ\,kg^{-1}}$ and densities ($\rho_{water} = 1000\ \mathrm{kg\,m^{-3}}$). The Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}}K^{-4}$.

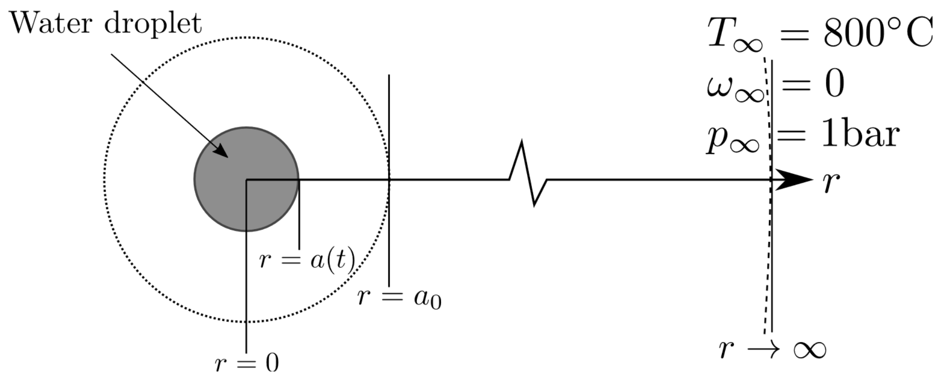

Figure Q3. Evaporation of a water droplet in nitrogen

a) If the initial temperature of the water droplet is at 10 °C, is there any evaporation? Why? [4 marks]

Next, consider a water droplet with an initial radius $a_0 = 3\ \mathrm{mm}$ and its temperature is maintained at 100 °C throughout the evaporation process.

b) What is the net amount of thermal radiation it receives from the dry nitrogen initially? Assume the droplet behaves as a black body. [3 marks]

c) What is the initial conductive heat transfer rate between the dry nitrogen and the droplet? Assume a 1 micrometre thick thermal boundary layer with a linear temperature profile. [3 marks]

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d) What is the initial evaporation rate $\dot{m}_{ev,0}$ $(\mathrm{kg\,s^{-1}})$ of the droplet? [3 marks]

e) Show that the initial rate of change of droplet radius $\dot{a}(t=0) = -\dot{m}_{ev,0}/(4\pi\rho_{water}a_0^2)$, and calculate its numerical value? [hint: think about the conservation of mass inside the droplet.] [4 marks]

After a short initial period, the evaporation process can be considered as a quasi-steady process and the evaporation rate is limited by the mass diffusion rate of water vapour into gaseous nitrogen. It can be shown analytically that the mass fraction of water vapour in gaseous nitrogen at different radial distance, i.e., $\omega(r)$, during this quasi-steady process is:

$$\frac{\omega(r)-\omega_\infty}{\omega_s-\omega_\infty} = \frac{B_M+1}{B_M}\left[1-\left(\frac{1}{B_M+1}\right)^{a/r}\right].$$

where ω_∞ and ω_s are the water vapour mass fraction at infinity and on the droplet surface respectively, assumed to be constant during the evaporation process, B_M is the Spalding number of mass diffusion that remains constant during the evaporation, r is the radial distance from the centre of the droplet, and a is the droplet radius.

f) Based on the Fick's law of diffusion: $\dot{m}_d = -\rho_{air}DA\frac{d\omega}{dr}$, where D is the mass diffusivity of water vapour in nitrogen $(\mathrm{m^2 s^{-1}})$ and A is the evaporation area, show that the mass diffusion rate of water vapour at the droplet surface is:

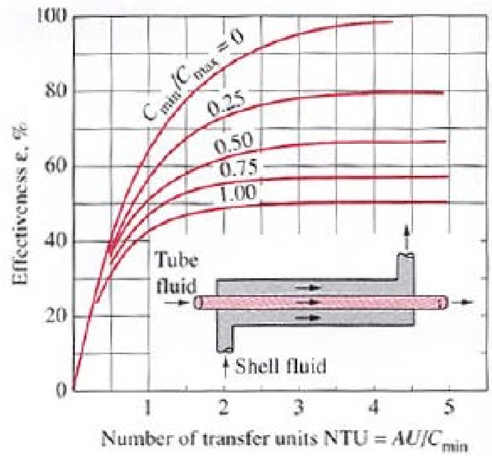
$$\dot{m}_d = 4\pi\rho_{air}Da\frac{(\omega_s-\omega_\infty)\ln(B_M+1)}{B_M}.$$

Please note that $d(a^{cx}) = ca^{cx}\ln a$ and $\frac{d\left(f(g(x))\right)}{dx} = \frac{df}{dg}\frac{dg}{dx}$. [6 marks]

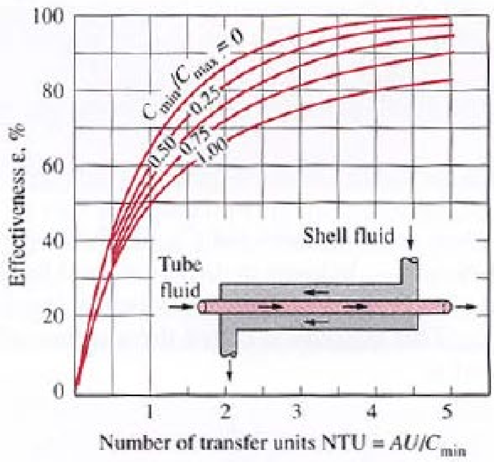
g) Prove the following established correlation in this quasi-steady diffusion-controlled regime (i.e. the evaporation rate $\dot{m}_{ev}$ equals to the mass diffusion rate $\dot{m}_d$): $a^2 = a_0^2 - ct$, where a is the instantaneous droplet radius and will shrink as time goes on, a_0 is the initial droplet radius, i.e. $t=0$, and c is a constant. [5 marks]

J Szmelter
H Zhao

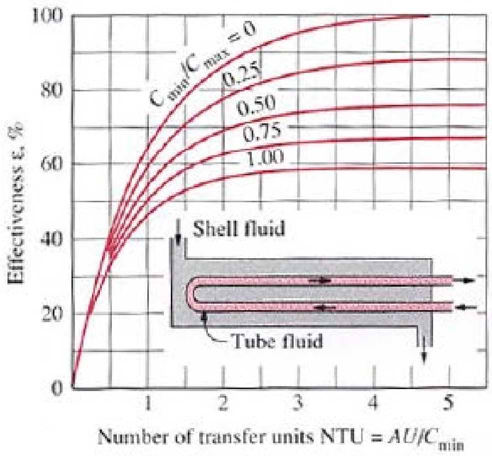
Graphs that may be of use:



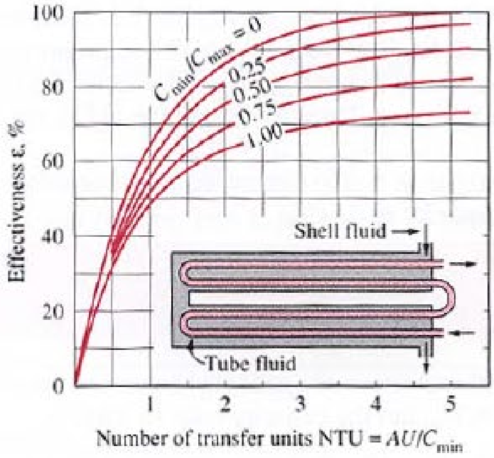
(a) Parallel-flow



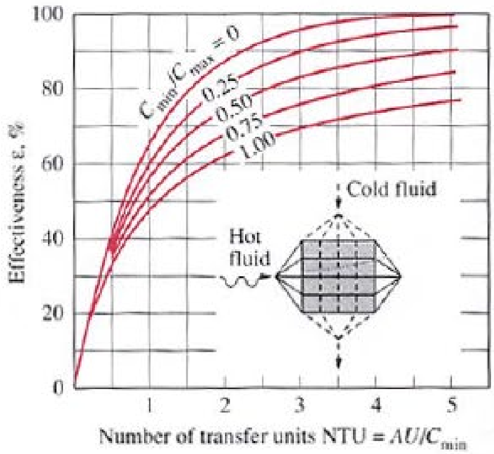
(b) Counter-flow



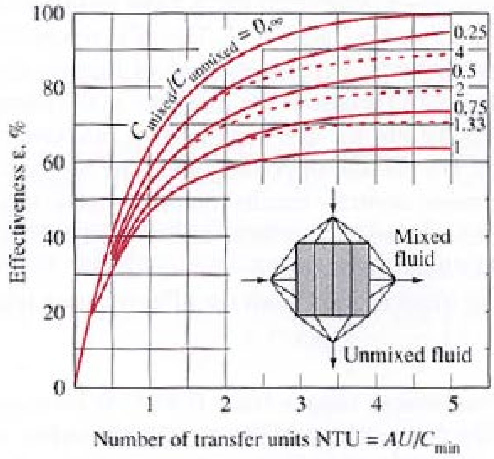
(c) One-shell pass and 2, 4, 6, tube passes



(d) Two-shell passes and 4, 8, 12, tube passes



(e) Cross-flow with both fluids unmixed



(f) Cross-flow with one fluid mixed and the other unmixed

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Equations that may be of use:

General conduction equation	One-dimensional	
	Rectangular coordinates	$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+q_g=\rho c\frac{\partial T}{\partial t}$
	Cylindrical coordinates	$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+q_g=\rho c\frac{\partial T}{\partial t}$
Steady state 1-D conduction equation		
	Rectangular coordinates	$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+q_g=0$
	Cylindrical coordinates	$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+q_g=0$
Resistances in electrical analogy method $\dot{Q}=\frac{\Delta T}{R_{equivalent}}$ **Serial** $R_{equivalent}=\sum_i R_i$	Plane slab	Resistance $\frac{L}{kA}$
	Cylindrical thickness	Resistance $\frac{\ln(b/a)}{2\pi k.H}$
	Convection at a boundary	Resistance $\frac{1}{hA}$
Heat transfer from fins	General solution	$\theta=C_1\sinh(mx)+C_2\cosh(mx)$ $m^2=\frac{hP}{kA_c}$ and $\theta=(T-T_{ref})$ Efficiency: $\eta_{fin}=\frac{\dot{Q}_{fin,actual}}{\dot{Q}_{fin,ideal}}$; $\dot{Q}_{fin,ideal}=hA_{fin}\theta_b$; Effectiveness: $\varepsilon_{fin}=\frac{\dot{Q}_{total,fin}}{\dot{Q}_{total,nofin}}$;
	Adiabatic tip	Solution: $\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh mL}$ Heat Transfer: $Q=\sqrt{hPkA_c}.\theta_b.\tanh(mL)$ Efficiency: $\eta_{fin}=\frac{tanh(mL)}{mL}$ Heat loss form finned surface: $\dot{Q}=h\left(A_{unfin}+\eta_{fin}A_{fin}\right)\theta_b$
Lumped system analysis	For cooling objects under Convection	$\frac{dT}{dt}+\frac{hA}{\rho Vc}(T-T_a)=0$ or $\frac{d\theta}{dt}+m\theta=0$ where $\theta=T-T_a$ and $m=\frac{hA}{\rho Vc}$
	Solution to governing equation	$\theta=\theta_0e^{-mt}$
	Biot number	$Bi=\frac{h}{k/L}$
	For lumped capacity method	$Bi<0.1$

Equations that may be of use (continued):

Convective heat transfer non-dimensional parameters		
	[L] is an appropriate dimension	
	Reynolds number	$\mathrm{Re} = \frac{\rho U[L]}{\mu}$ or $\mathrm{Re} = \frac{U[L]}{\nu}$
	Nusselt number	$Nu = \frac{h[L]}{k}$
	Prandtl number	$\mathrm{Pr} = \frac{C_p \mu}{k}$
	Grashof number	$Gr = \frac{g\beta(T - T_\infty)[L^3]}{\nu^2}$
	Rayleigh number	$Ra = Gr.\mathrm{Pr}$

Forced Convection Correlations - Forced Convection $Nu = C.\mathrm{Re}^m\ \mathrm{Pr}^n\ .K$

	C	M	N	K	Conditions
Circular tube Internal Flow	1.86	1/3	1.3	$(d/l)^{1/3}(\mu/\mu_w)^{0.14}$	Laminar flow short tube Re < 2000
	3.66	0	0	1	Laminar flow long tube Re < 2000
	0.023	0.8	0.4	1	Turbulent flow Re > 2000
Circular Tube Air in cross flow	0.538	0.47	0	1 for air Other fluids multiply Nu by 1.1 $\mathrm{Pr}^{1/3}$	Laminar flow Re < 500
	0.240	0.6	0		Turbulent flow 500 < Re < 50000
Flat plates	0.332 0.644	½ ½	1/3 1/3	(local) 1 (mean)	Laminar flow Re < 5 x 10^5
	0.029 0.037	4/5 4/5	1/3 1/3	(local) 1 (mean)	Turbulent flow Re > 5 x 10^5

Free Convection correlations $Nu = C.(Gr\,\mathrm{Pr})^n.K$

	C	N	K	Conditions
Horizontal cylinder	0.47	¼	1	Laminar flow Gr_D.Pr < 10^9
	0.10	1/3	1	Turbulent Gr_D.Pr > 10^9
Vertical cylinder D << L	1.37	0.16	$(D/L)^{0.16}$	For Gr_D.Pr < 10^4
	0.60	¼	$(D/L)^{1/4}$	For Gr_D.Pr > 10^4
Vertical cylinder D >> L	0.59	¼	1 Also applies to Vertical plates	Laminar Gr_L.Pr < 10^9
	0.13	1/3		Turbulent Gr_L.Pr > 10^9
Horizontal Plate (note Le = Plate area/Perimeter)	0.54	¼	1 Hot-side up	Laminar Gr_{Le}.Pr < 10^9
	0.14	1/3	1 Hot-side up	Turbulent Gr_{Le}.Pr > 10^9
	0.25	¼	1 Hot-side down	Laminar Gr_{Le}.Pr < 10^9

Heat Exchangers	
LMTD $\Delta T_m = (\Delta T_1 - \Delta T_2)/\ln(\Delta T_1/\Delta T_2)$	Effectiveness $\varepsilon = \dot{Q}/\dot{Q}^*$ $\dot{Q}^* = [(mC_P)_{\min}(\Delta T)]$
$NTU = AU_m/C_{\min}$	Stream capacity ratio $C = C_{\min}/C_{\max}$