Loughborough University

# FLUID MECHANICS 2

(22WSB802)

Semester 2 2023 In Person Examination

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

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Answer **ALL** questions.

Questions carry the marks shown.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

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**1.** A rocket is propelled through burning hydrogen and oxygen. The hot burned gas (specific heat capacity ratio $k = 1.22$, gas constant $R = 519.6\ \mathrm{J\ kg^{-1}K^{-1}}$) leaves the combustion chamber at $T_i = 3517\ \mathrm{K}$ and $p_i = 25\ \mathrm{bar}$ before being accelerated to reach supersonic speed through a convergent-divergent rocket nozzle, as shown in **Figure Q1**. The nozzle has a throat area of $A_{th} = 0.4\ \mathrm{m}^2$. Assume that the burned gas behaves as an ideal gas, and its velocity at the inlet of the rocket nozzle is negligibly small, and the flow process through the nozzle is isentropic. The nozzle geometry is optimized for a back pressure $P_b = 1.174 \times 10^{-2}\ \mathrm{bar}$ (i.e. correctly expanded flow at the exit such that the gas exit pressure matches the back pressure). Useful formulae are given at the end of the question.

a) What is the Mach number $M_e$ and velocity $u_e$ at the exit of the nozzle? [8 marks]

b) What is the mass flow rate $\dot{m}$ through the nozzle? [6 marks]

c) What should be the area of the exit $A_e$? [4 marks]

d) If the total mass of the rocket $m_r$ is 40 metric tons and the drag force is assumed to be negligible, what is the magnitude of acceleration of the rocket? Assume the rocket is oriented vertically. [5 marks]

e) What can be done to the geometry of the nozzle such that the propulsion force can be increased. [3 marks]

f) During the initial stage when the back pressure $p_b$ is close to 1 bar, will the exit flow still be supersonic? Draw a sketch to show the pressure and Mach number distributions along the nozzle. The back pressure which leads to a normal shock at the exit of the nozzle is 0.35 bar. [6 marks]

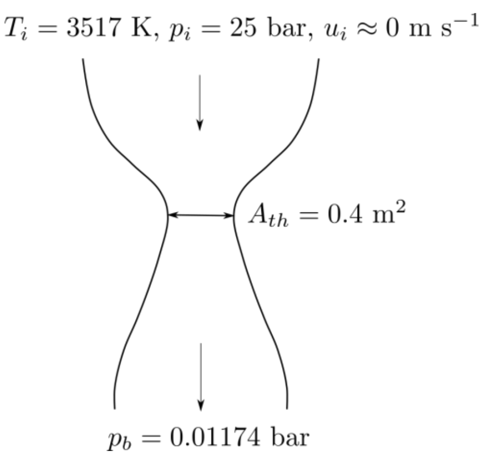


**Figure Q1.** Isentropic compressible flow inside a rocket nozzle

**Useful equations:**

Isentropic flow: $\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)\mathrm{M}^2$ ; $\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{M}^2\right]^{\frac{k}{k-1}}$;

speed of sound in an ideal gas: $c = \sqrt{kRT}$.

**2.** An asymmetric aerofoil (chord length $c$ and span $w$) is tested inside a wind tunnel with an angle of attack $\alpha = 0°$, as shown in **Figure Q2A**. The measured pressure difference between the upper surface and lower surface ($\delta P(x) = P_{lower}(x) - P_{up}(x)$) at different location is approximated by the following polynomial:

$$\frac{\delta P(x)}{\rho_\infty U_\infty^2/2} = c_1\left(\frac{x}{c}\right)^5 + c_2\left(\frac{x}{c}\right)^4 + c_3\left(\frac{x}{c}\right)^3 + c_4\left(\frac{x}{c}\right)^2 + c_5\left(\frac{x}{c}\right),$$

where $\rho_\infty$ and $U_\infty$ are the upstream density and velocity, respectively, and $c_1, c_2, c_3, c_4$ and $c_5$ are fitting coefficients (i.e., constants).

a) Derive the expression for the lift coefficient $C_L$ of the aerofoil as functions of the five fitting coefficients. [10 marks]

b) Explain what the stall condition is and what caused the stall condition. [4 marks]

Now consider an aeroplane equipped with two aerofoils (i.e. wings) where their angles of attack and configurations (i.e. clean, slotted and double-slotted shown in **Figure Q2B**) can be adjusted depending on the phase of the flight (e.g. taking off, cruising, or landing). The lift and drag characteristics of each of the aerofoil are shown in **Figure Q2B**. The total mass of the aeroplane $m_a$ is 70 metric tons and each of the aerofoil has a planform area $A_a = 75\ \mathrm{m}^2$. Assume the air behaves as an ideal gas with gas constant $R = 287\ \mathrm{J\ kg^{-1}K^{-1}}$.

c) Which configuration should be chosen for taking off, cruising and landing, respectively? Justify your choice. [6 marks]

d) What is the **minimum** speed $u_{min}$ required for take-off and landing respectively with and without the flaps? What is the implication for the infrastructure required in the airport to ensure safe take-off and landing. Use a safety factor $\beta = 1.2$ for the required lift force. The temperature and pressure on the ground $T_g = 300\ \mathrm{K}$ and $p_g = 1\ \mathrm{bar}$, respectively. [6 marks]

e) Sketch the flow surrounding the wings with and without the extended flaps during the take-off and explain why the extension of flaps can increase the lift coefficient? [3 marks]

f) What is the angle of attack $\alpha$ and the minimum thrust power $\dot{W}_t$ required to overcome the wing drag for cruising at speed $u_c = 558\ \mathrm{km\ h^{-1}}$ and at $12000\ \mathrm{m}$? The temperature at 12000 m is assumed to be $T_c = 216\ \mathrm{K}$ and the pressure $P_c = 0.194\ \mathrm{bar}$. [6 marks]

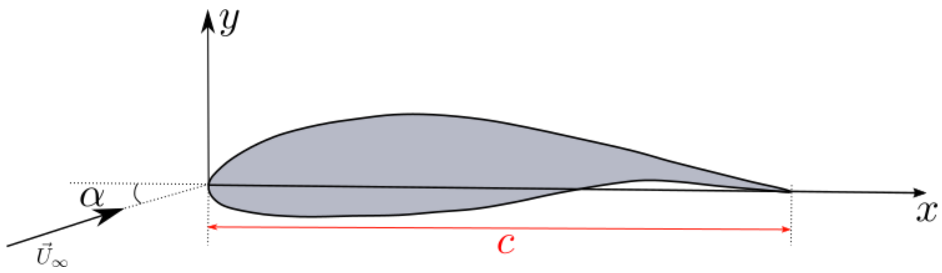


**Figure Q2A.** Flow across an aerofoil

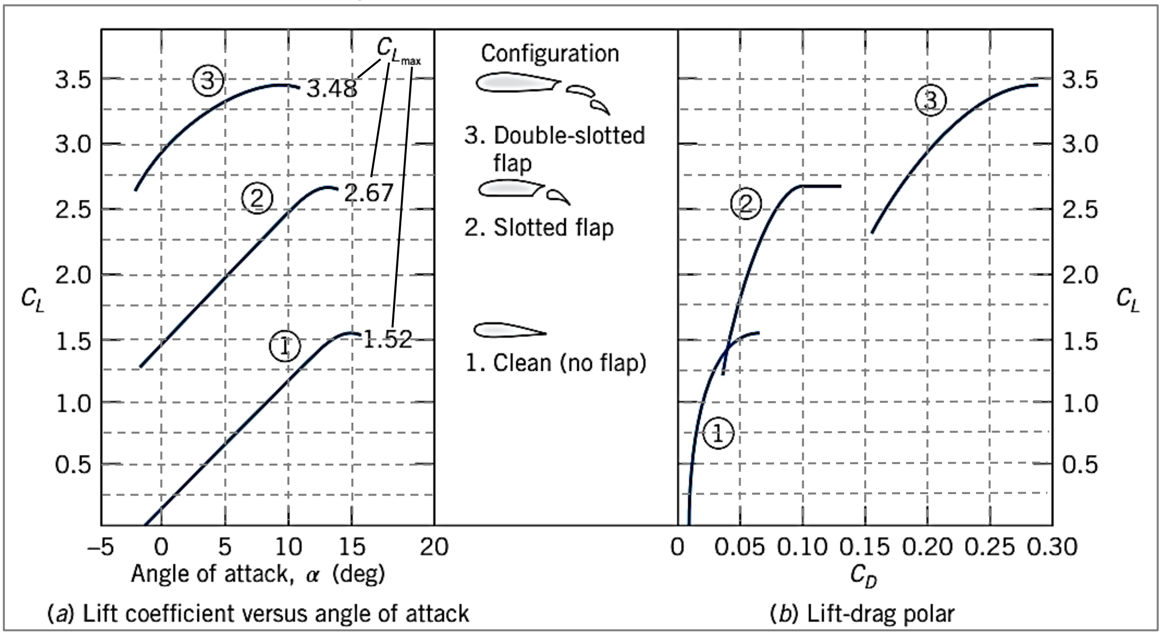


**Figure Q2B**. Lift and drag coefficients $C_L$, $C_D$ of each of the wings.

**3.** Fresh water (density $\rho_w = 998\ \mathrm{kg\ m^{-3}}$, dynamic viscosity $\mu_w = 1.002 \times 10^{-3}\ \mathrm{Pa\ s}$) is pumped from a nearby water reservoir to a slightly elevated trench (height difference $\Delta z = 1.5\ \mathrm{m}$), which is then used for irrigation.

The initial design (design A) is depicted in **Figure Q3A**. The pipe is constructed of PVC (roughness $\varepsilon = 0.002\ \mathrm{mm}$), has a total length $L = 200\ \mathrm{m}$, and has a constant diameter that is to be determined. An inline pump is positioned near the reservoir and receives a constant power input of $\dot{W}_{pump} = 200\ \mathrm{W}$. The characteristic pump supply curve is given in **Figure Q3C**.

Useful correlations are given at the end of the question.

a) Determine the pumping head $h_p$, and pump efficiency $\eta_p$, when supplying a flowrate $\dot{V} = 200\ \mathrm{litre\ min^{-1}}$. [3 marks]

b) Calculate the minimum pipe diameter $D$ required to deliver a flowrate of $200\ \mathrm{litre\ min^{-1}}$. Minor losses may be neglected due to the large pipe length. Where iteration is required, perform 2 cycles of iteration. You can consider using an initial guess value of $f = 0.015$, or use an alternative strategy. [15 marks]

Due to a lack of a power source near the reservoir, it is suggested that the pump could be brought closer towards the trench (Design B - **Figure Q3B**). For both designs, the pipe length from the reservoir to point **P** is $100\ \mathrm{m}$ and point **P** is at an elevation of $2\ \mathrm{m}$ relative to the reservoir. Point **P** coincides with a flange joining the pipe lengths together.

c) Determine the flowrate for this new design, if all other factors remain constant. [2 marks]

d) For **both** designs, calculate the gauge pressure in the pipe at point **P**, respectively. Use the pipe diameter calculated in part (b). [9 marks]

e) For **both** designs, comment on what will be the impact of a minor seal failure at point P (i.e. there is a leak path through the flange join). [4 marks]

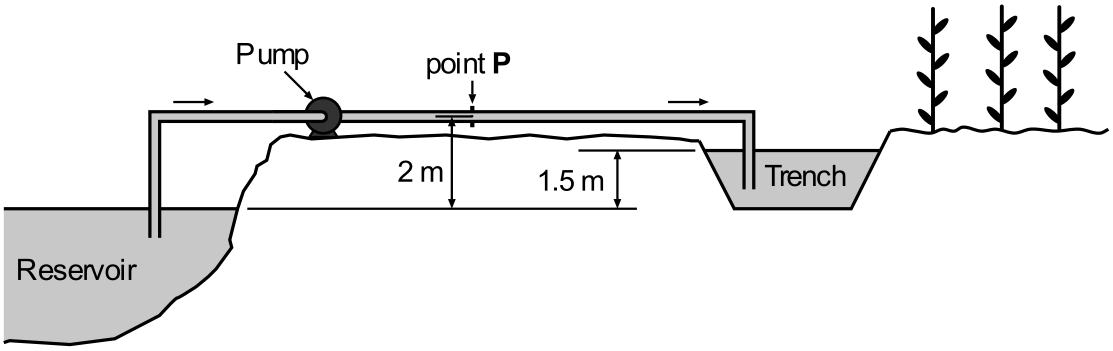


**Figure Q3A**: Sketch of design A (not to scale)

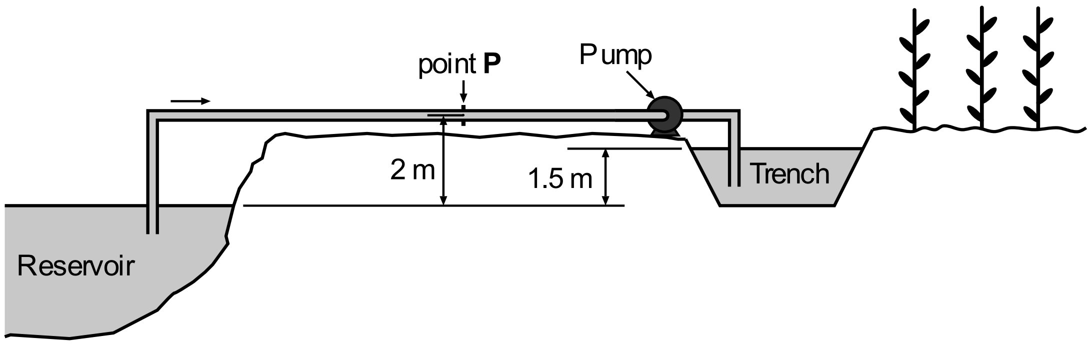

**Figure Q3B**: Sketch of design B (not to scale)

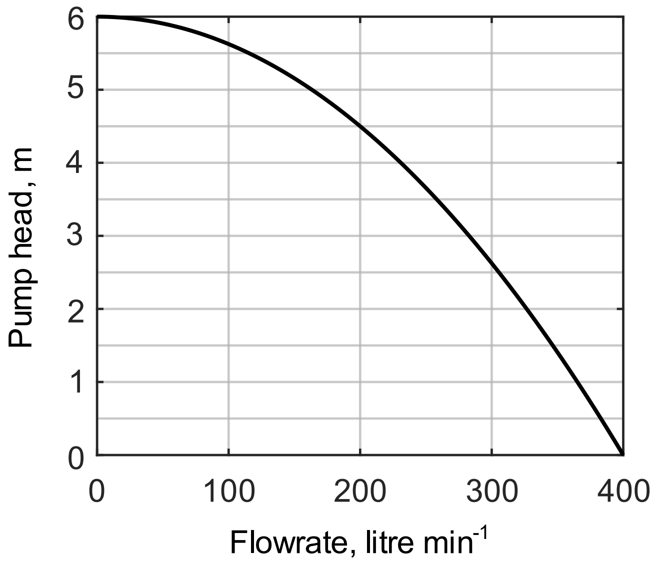


**Figure Q3C**: Pump supply characteristic curve

**<u>Useful correlations:</u>**

Friction factor ($f$):

i. $\frac{1}{\sqrt{f}} = -1.8 \log\left(\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$ for turbulent flow ($Re > 4000$);

ii. $f = \frac{64}{\text{Re}}$ for laminar flow ($Re < 2300$).

Kinetic energy correction factor:

i. $\alpha = 1.05$ for turbulent flow;

iii. $\alpha = 2$ for laminar flow.

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