Loughborough University

23CGB018

Plant Engineering

Semester 2 2023/24 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Attempt **THREE** questions in total. Each question carries 25 marks.

Candidates should show full working for all calculations and derivations.

1. You have been asked by your manager to specify the material and cross-sectional shape of the beams that are to be used as part of a frame to support a storage tank. One such beam is horizontal before loading and your options are as follows: (a) material: aluminium or steel; (b) beam cross-section: rectangular or I-beam. Figure Q1 shows the configuration of the beam, which has simple supports at both ends.

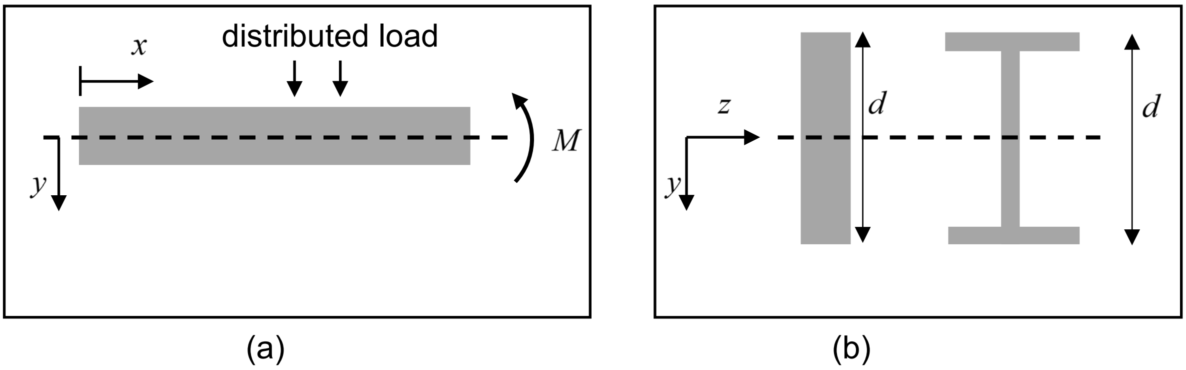

Figure Q1 The configuration of the beam; (a) a section of the beam indicating the long axis of the beam (x), the deflection direction (y) and the direction of loading producing a bending moment M; (b) the possible cross-sections of the beam (in the $y-z$ plane). The dashed line indicates the neutral axis; the beam is symmetric about the neutral axis.

When loaded, the beam is subject to a bending moment, M, given by

$$M = -\frac{q_0}{6L}x^3 + \frac{q_0 L}{6}x$$

with q_0 =90 kN m^{-1}, and the length of the beam $L = 8$ m. The rectangular and I-beams have the same cross-sectional area. Relevant data are provided at the end of the question.

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Q1 Continued/…

(a) Determine the distance along the beam, $x = x_{\max M}$, where the bending moment is greatest (in magnitude), and calculate the maximum bending moment. Provide numerical answers with units. [5 marks]

(b) Using the flexure formula, write an expression for the local normal stress σ_x at the location of maximum bending moment, $x = x_{\max M}$, at a position y relative to the neutral axis of the beam (Figure Q1b) in terms of the second moment of area, I.

[2 marks]

(c) Calculate the maximum normal stress due to bending (i.e. the maximum in magnitude) for each of the four possible beams under consideration (two materials each with two possible beam shapes). Sketch the whole beam, indicating the location(s) of the maximum (magnitude) normal stress and label whether the stress is compressive or tensile. [8 marks]

(d) State what your recommendation would be for the choice of material and beam cross-sectional shape under the specified loading conditions. Provide reasons for your answer, considering strength (yield stress) and beam shape. [5 marks]

(e) Following successive integration of the bending moment expression, the deflection curve for the beam can be written as:

$$v = \frac{1}{EI}\left[\frac{q_0}{120L}x^5 - \frac{q_0 L}{36}x^3 + C_1 x + C_2\right]$$

By applying appropriate boundary conditions, which should be clearly stated, determine the integration constants C_1 and C_2, in terms of q_0 and L. [5 marks]

Data

Depth $d = 610$ mm

Second moment of area for the rectangular beam $I = 5.43 \times 10^{-4}\ m^4$

Second moment of area for the I-beam $I = 8.75 \times 10^{-4}\ m^4$

Yield stress of stainless steel = 201 MPa

Yield stress of aluminium = 24 MPa

Design factor of safety = 2.5 (i.e. *margin* of safety = 1.5)

2. A simply supported beam, which forms part of a walkway, is loaded as shown in Figure Q2.

   (a) Draw a free body diagram for the whole beam and calculate the magnitude of the reaction forces at the supports. [7 marks]

   (b) With the aid of appropriate free body diagrams, derive expressions, in terms of the distance *x* from the left-hand end of the beam, for the shear force (*V*) and bending moment (*M*) acting over each section of the walkway. [18 marks]

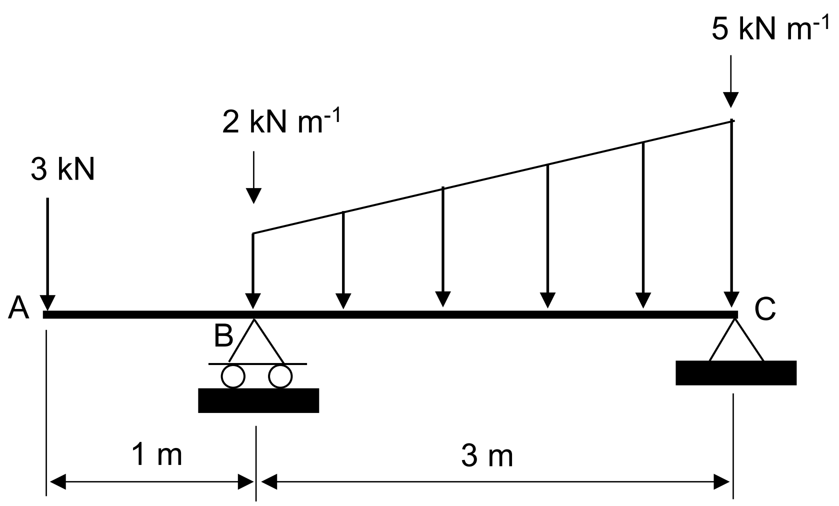

Figure Q2: Simply supported beam and loadings.

3. You have been asked to conduct mechanical design calculations for a cylindrical pressure vessel of inner diameter 1.6 m which is to be used for gas storage at 1.5 MPa.

   (a) Calculate the minimum permitted thickness of the vessel wall under the British Standards Institute (BSI) design code PD5500 for two alternative steels according to their allowable design stress provided below. Comment on what additional factors should be considered when selecting which steel to use. [7 marks]

   Design stress (for normal stress) for low alloy steel= 240 MPa

   Design stress (for normal stress) for stainless steel = 165 MPa

   (b) Having decided on the wall thickness, satisfying the constraint imposed in part (a), you now need to determine the principal stress (and principal planes) in the vessel wall. The longitudinal and circumferential tensile stresses are estimated to be as follows.

   Longitudinal stress σ_x = 27 MPa

   Circumferential stress σ_y = 54 MPa

   Construct and label the Mohr's circle for this case. State the values of the principal stresses and the angles of the principal planes. You may assume that the in-plane shear stresses are zero i.e. $\tau_{xy} = \tau_{yx} = 0$. [8 marks]

   (c) Use the Mohr's circle to determine the maximum (in-plane) shear stress and the angle at which the maximum shear stress occurs. [4 marks]

   (d) It is proposed to construct the vessel by bending a broad strip of steel into a helical form, and then joining by welding to form the cylindrical section of the vessel (see Figure Q3). The seam (joint) is designed to be at an angle of 30° to the x-axis. Determine the normal and (in-plane) shear stress on the joint plane. Considering the yield stresses below, comment on whether the design has a higher safety factor for shear or for tension on the welded joint.

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Q3 Continued/…

Estimated yield stress at the joint (in tension) =150 MPa

Estimated yield stress at the joint (in shear) = 87 MPa

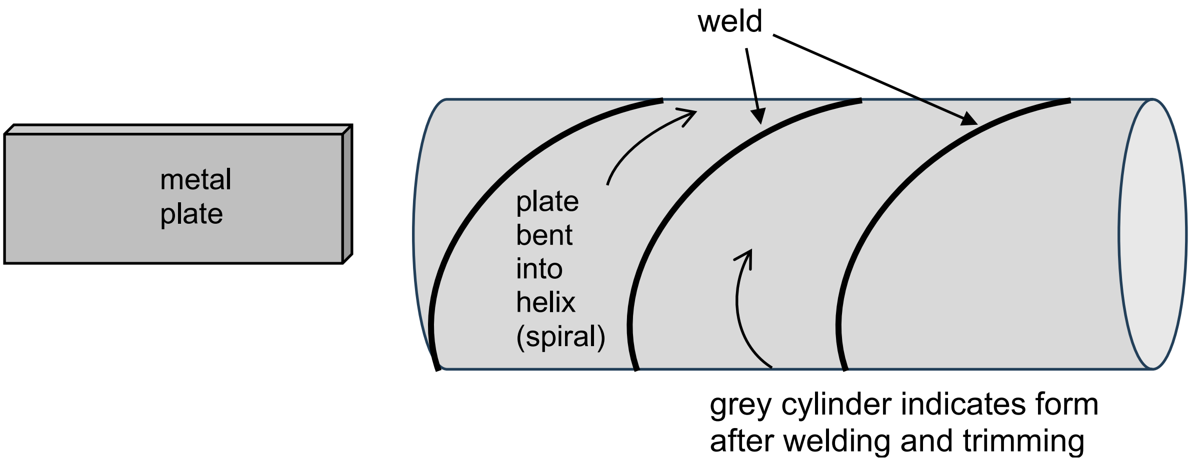

Figure Q3 Construction of cylindrical pressure vessel from metal plate bent into a helix (spiral) and welded on a plane 30° to the axis of the cylinder.

[6 marks]

4. A bar has the cross-section shown in Figure Q4 and is loaded by a force which acts in the downward direction. At a position along the bar the bending moment (*M*) acting is 50 kN m. The second moment of area for common shapes given in the List of Equations.

Calculate:

(a) The position of the neutral axis. [8 marks]

(b) The second moment of area for the cross-section. [10 marks]

(c) The maximum tensile and compressive bending (normal) stresses. [5 marks]

(d) The factor of safety against failure due to bending if the design stress is 45 MPa. [2 marks]

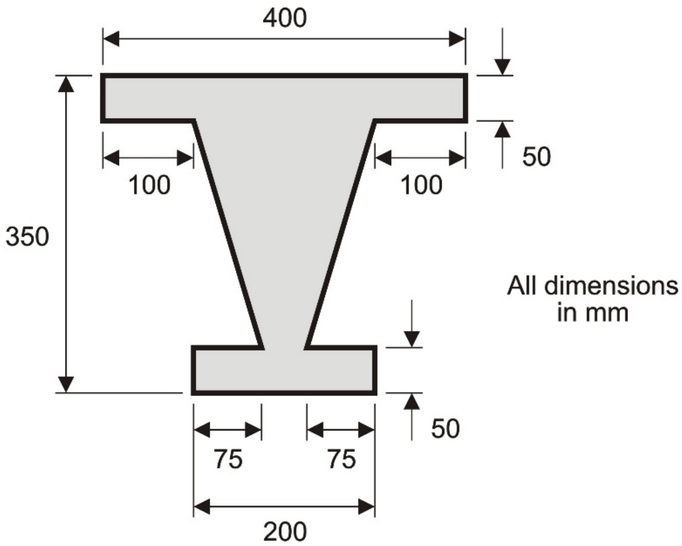

Figure Q4: Bar cross-section.

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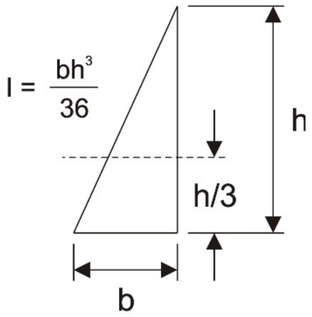
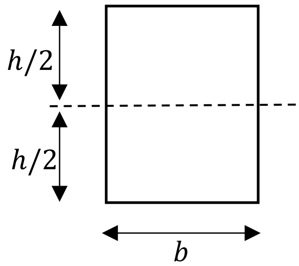
Dr HCH Bandulasena, Prof VJ Pinfield

List of Equations

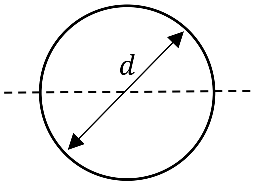
Frame rigidity evaluation, $m + r - 2j$

Second moment of area for common shapes. The dashed line indicates the centroidal axis.





$$I = \frac{bh^3}{12}$$



$$I = \frac{\pi d^4}{64}$$

Parallel axis theorem $\quad I_n = \sum \left(I_c + Ad^2 \right)$

Flexure Formula $\quad \sigma_x = \frac{M}{I} y$

Strain due to bending $\quad \varepsilon_x = -yv'' = -\frac{y}{\rho}$

Shear Formula $\quad \tau = \frac{VQ}{I_x b}$

Beam loading and deflection:

$$\frac{dV}{dx} = -q$$

$$\frac{dM}{dx} = V$$

$$M = -EI_x v''$$

$$V = -EI_x v'''$$

$$q = EI_x v''''$$

$$\frac{1}{\rho} = v''$$

Plane stress at an angle

$$\sigma_{x1} = \frac{\left(\sigma_x + \sigma_y\right)}{2} + \frac{\left(\sigma_x - \sigma_y\right)}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

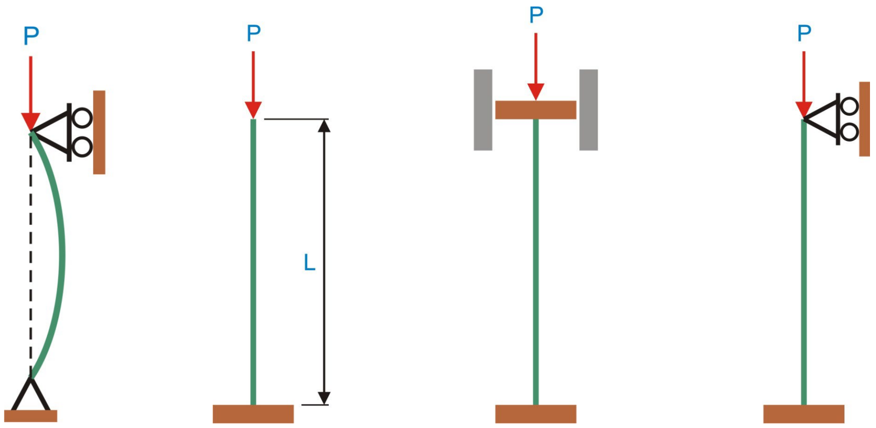
$$\tau_{x1y1} = -\frac{\left(\sigma_x - \sigma_y\right)}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Tensile load $\sigma = \frac{P}{A}$

Tensile strain $\varepsilon = \frac{\delta}{L}$

Stress concentration $K = \frac{\sigma_{\max}}{\sigma_{\text{nom}}}$

Buckling failure:
(for various support combinations)



Critical stress:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad P_{cr} = \frac{4\pi^2 EI}{L^2} \qquad P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

Storage tanks:

Shell plate thickness $t = \frac{D}{20S}\left[98w(H-0.3)+p\right]+c$

H : height from bottom of course to top of shell (m)

D : tank diameter (m)

w : max. specific gravity of tank contents (≥1.0)

S : design stress (MPa)

p : design pressure (mbar)

c : corrosion allowance (mm)

Pressure vessels:

Shell thickness $e \geq= \frac{pD_i}{2\sigma_d - p}$

Flat end thickness $e \geq CD\sqrt{p/\sigma_d}$

σ_d : design stress

p : design internal pressure

C : factor dependent on type