Loughborough University

23CGC051
Transfer Processes

Semester 1 2023/24 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Answer **THREE** questions. Each question carries 25 marks.

Candidates should show full working for calculations and derivations.

1. Air at 20°C flows over a heated plate at a velocity of 10 m s^{-1}. The plate is 0.7 m long, 0.5 m wide and at a constant temperature of 110°C (Figure Q1).

(a) Using relevant equations from Table Q1, show that the average heat transfer coefficient across the plate (between $x = 0$ and $x = L$) is tw ice the local heat transfer coefficient at the trailing edge ($x = L$), where x is the distance from the leading edge of the plate and L is the plate length. [4 marks]

(b) Calculate the average heat transfer coefficient across the plate and the total amount of heat transferred per unit time. [8 marks]

(c) Calculate the heat transfer coefficient, h, thickness of the hydrodynamic boundary layer, δ, and thickness of the thermal boundary layer (BL), δ_t, at the trailing edge of the plate. [6 marks]

(d) What is the temperature at 0.2 m downstream of the leading edge of the plate and 0.5 mm above the plate surface (clue: x = 0.2 m, y = 0.5 mm)? [7 marks]

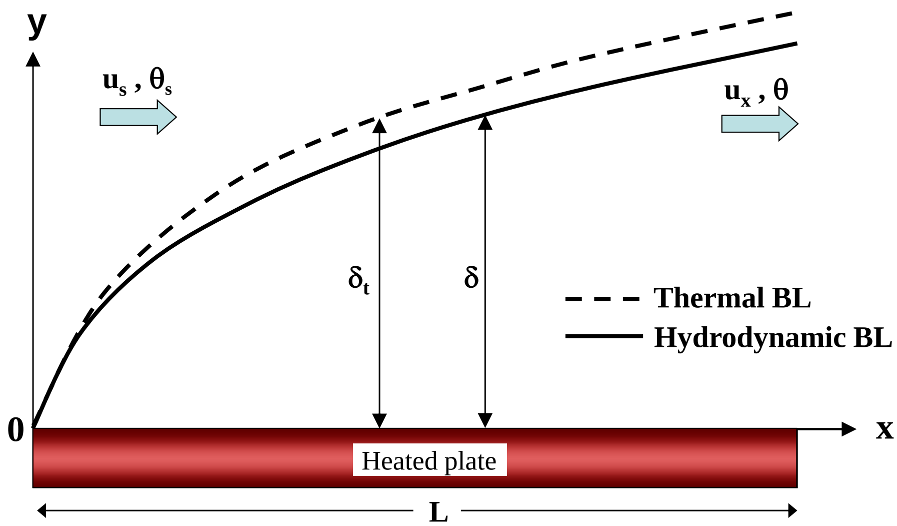

Figure Q1. Development of boundary layers when fluid flows above a flat plate.

<u>Relevant Data</u>

Critical Reynolds number, Re_c = 4×10^5

Air properties at 65°C (average temperature in the boundary layer)

Prandtl number, Pr = 0.71

Kinematic viscosity, ν = 1.94×10^{-5} m^2 s^{-1}

Thermal conductivity, k = 0.0289 W m^{-1}K^{-1}

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Q1 Continued/…

Table Q1. Characteristics of boundary layers for laminar flow over a flat plate

Thickness of hydrodynamic boundary layer at distance x from the leading edge	$\delta = 4.64x\ Re_x^{-1/2} = 4.64x\left(\frac{u_s \rho x}{\mu}\right)^{-1/2}$
Thickness of thermal boundary layer	$\delta_t = \delta\left(\frac{C_p \mu}{k}\right)^{-1/3} = \delta Pr^{-1/3}$
Average Nusselt number from $x = 0$ to $x = L$ (h_m is the average heat transfer coefficient)	$(Nu_L)_m = \frac{h_m L}{k} = 0.65\, Pr^{1/3}\, Re_L^{1/2}$
Local Nusselt number at distance x from the leading edge (h is the local heat transfer coefficient)	$Nu_x = \frac{hx}{k} = 0.323\, Pr^{1/3}\, Re_x^{1/2}$
Local temperature in the thermal boundary layer (y is the distance from the plate surface and θ_0 is the plate temperature)	$\theta = \theta_0 + (\theta_s - \theta_0)\left[\frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3\right]$

Other symbols have their usual meaning.

2. A horticultural company plans to produce pesticide in the form of a spherical pellet which would be suspended in a greenhouse. The pesticide sublimes slowly to release the active ingredient into the atmosphere making a very dilute mixture. Treat the pellet as a homogeneous material (A) which diffuses into stagnant atmosphere of pure air (B) under isothermal conditions at 20°C.

(a) By stating the assumptions, explain why the flux of pesticide (A) in the atmosphere at a radius *r* from the centre of the sphere is given by:

$$N_A = -D_{AB} C \frac{dy_A}{dr} = \frac{Q_A}{4\pi r^2} \qquad (2.1)$$

where Q_A (kmol/s) is the rate of pesticide sublimation at the surface of the spherical pellet. [5 marks]

(b) By integrating Equation (2.1) and using appropriate boundary conditions, show that the mass transfer coefficient for a spherical pesticide particle of diameter *d* is given by:

$$\frac{k_g d}{D_{AB}} = 2 \qquad (2.2)$$

[5 marks]

(c) If the pellet mass reduces to 1/8 of its initial value after 240 days of usage, determine the relationship between the initial diameter of the pellet (d_0) and its final diameter (d_f).

[4 marks]

(d) Writing material balance on the shrinking pesticide particle, derive an expression for the initial diameter of the pellet (d_0), if the pellet mass is reduced to 1/8 of the initial pellet mass in 240 days (t_f). [8 marks]

(e) Using the data below, calculate the initial diameter of the pellet (d_0) for the case mentioned in part (c) above. [3 marks]

<u>Relevant Data (at 20°C)</u>

Vapour pressure of A at the surface of the pellet is 3.5 Pa.

Diffusion coefficient, D_{AB} = 5.2 × 10^{-6} m^2/s

The pesticide A has a solid density of 1300 kg/m^3 and a relative molecular mass (RMM) of 135.

1 atm = 1.013 x10^5 Pa

R = 8314 J $kmol^{-1}$ K^{-1}

3. Air is passed at 3.5 m s^{-1} through a smooth 25 mm diameter pipe, that is 6 m long. The initial temperature of the air is 290 K and the temperature of the tube wall is constant at 350 K.

(a) Applying differential heat balance and heat transfer equations, show that the outlet air temperature, θ_o, can be calculated as shown in Equation (3.1):

$$\theta_o = \theta_w - (\theta_w - \theta_i)\left[\left(\exp\left(-\frac{4hL}{\rho u d C_p}\right)\right)\right] \qquad (3.1)$$

where θ_i is the initial temperature of air, θ_w the temperature of the tube wall, L the tube length, h the heat transfer coefficient, ρ the density of air, u the air velocity, d the diameter of the pipe, and C_p the specific heat capacity of air. [4 marks]

(b) Using Equation (3.1) and the universal velocity profile method calculate the:

(i) heat transfer coefficient and outlet temperature of the air, θ_o; [9 marks]

(ii) temperature difference across the laminar sub-layer at the tube outlet; [3 marks]

(iii) temperature differences across the buffer layer and the turbulent core at the tube outlet. [4 marks]

(c) Which of the three flow regions (laminar sub-layer, buffer layer and turbulent zone) offers the smallest thermal resistance to heat flow and how do you know this? [2 marks]

(d) Calculate the pressure drop along the pipe. [3 marks]

<u>Relevant physical properties of air</u>

Density, ρ = 1.09 kg m^{-3}

Specific heat capacity, C_p = 1.01 kJ kg^{-1} K^{-1}

Dynamic viscosity, μ = 1.95 × 10^{-5} N s m^{-2}

Thermal conductivity, k = 2.77 × 10^{-2} W m^{-1} K^{-1}

Relevant equations are provided on the next page.

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Universal velocity profile equations for turbulent flow in a smooth cylindrical tube at 2,500 < Re < 100,000 (symbols have their usual meanings):

Heat transfer coefficient	$h = \dfrac{0.032 C_p \rho u Re^{-1/4}}{1 + 0.82 Re^{-1/4}\left[(Pr - 1) + \ln\left(\frac{5}{6}Pr + \frac{1}{6}\right)\right]}$
Friction factor	$\dfrac{\tau}{\rho u^2} = 0.0396 Re^{-1/4}$
Difference in temperature over the laminar sub-layer ($y^+ < 5$)	$\theta_5 = \theta_w - \dfrac{5 q \mu}{k u^* \rho}$
Difference in temperature over the buffer layer (5 < y^+ < 30)	$\theta_5 - \theta_{30} = \dfrac{5q}{C_p \rho u^*} \ln(5Pr + 1)$

Prandtl number: $Pr = \frac{C_p \mu}{k}$

Reynolds number: $Re = \frac{u \rho d}{\mu}$

Dimensionless velocity: $y^+ = \frac{y u^* \rho}{\mu}$

Shearing velocity: $u^* = \sqrt{\frac{\tau}{\rho}}$

Heat transfer equation: $dQ = h \pi d (\theta_w - \theta) dL$

Heat balance equation: $dQ = \rho u \frac{\pi d^2}{4} C_p d\theta$

Pressure drop along pipe: $\Delta p = \frac{4 \phi \rho u^2 L}{d}$

Assume fully developed flow in the pipe.

4. A bubble of gas rises through pure water at 20°C that is contained in a vertical tube with an inner diameter of 18 mm and a water height of 0.65 m, kept at 1 bara. The gas contains a mixture of 15% v/v Cl_2 (RMM = 71) in nitrogen. Assume that the bubble is spherical with a diameter of 7.5 mm and that its rise velocity is given approximately by $V_b = 0.35\sqrt{gd}$, where g is the gravitational acceleration and d is the tube diameter. The equilibrium relationship for chlorine in nitrogen / water at 1 bara and 20°C is given by $y^* = 367x$ (where y^* and x denote mole fractions). Ignore the effects of hydrostatic pressure changes and assume that the bubble diameter remains approximately constant during its rise.

(a) Using Higbie's penetration theory, calculate the liquid side mass transfer coefficient. [5 marks]

(b) Find the molar composition of the bubble leaving the tube. [10 marks]

(c) Calculate the amount of chlorine absorbed by the water. [5 marks]

(d) Confirm that the bubble size remains approximately constant. [5 marks]

Relevant Data

Gravitational acceleration = 9.8 m s^{-2}

Diffusion coefficient of chlorine in water (20°C) = 1.4×10^{-9} m^2 s^{-1}

Density of water at 20°C = 1000 kg m^{-3}

RMM of water = 18

R = 8314 J $kmol^{-1}$ K^{-1}

END OF PAPER

Dr G Vladisavljevic, Dr HCH Bandulasena