Structural Forms and Stress Analysis
23CVA103

Semester 2 2024 | In-Person Exam Paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

This examination consists of three sections.
Section A, question 1 is compulsory – you MUST answer this question.
Answer **ONE** question in Section B.
Answer **TWO** questions in Section C.

All questions carry equal marks.

A two-page Aide-Mémoire formula sheet is attached.

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SECTION A
(Answer **THIS** question)

1. (a) A beam is a structural member subjected to bending, and it is probably the most common structural element that designers must cope with. Figures Q1(a,b) show two beams with different support conditions.

i) Under the assumption that the material used is concrete, explain why steel reinforcement is needed.

[4 marks]

ii) The beams in Figures Q1(a,b) are subjected to the same load. Sketch qualitatively how each beam deforms under the considered loading condition.

[6 marks]

iii) For each beam in Figures Q1(a,b), identify the tension and compression sides and sketch where the steel reinforcement is strictly needed.

[6 marks]

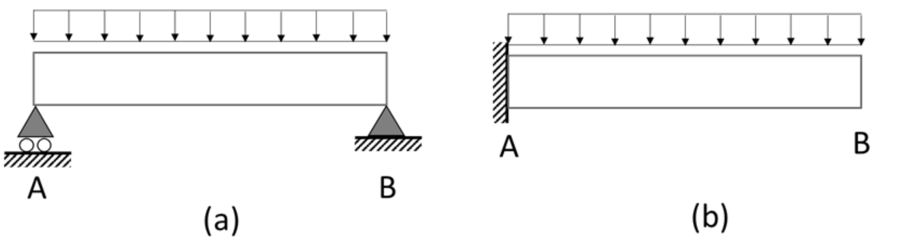

Figure Q1 – (a) Simply supported beam; (b) Cantilever beam.

(b) In structural design, ensuring lateral stability is crucial. Various fundamental strategies are employed to achieve this goal when buildings are subjected to lateral forces. Taking a typical industrial building composed of parallel portal frames (as illustrated in Figure Q1(c) as a case study). Describe the different strategies used to resist lateral loads in plane x-y and in plane y-z.

[9 marks]

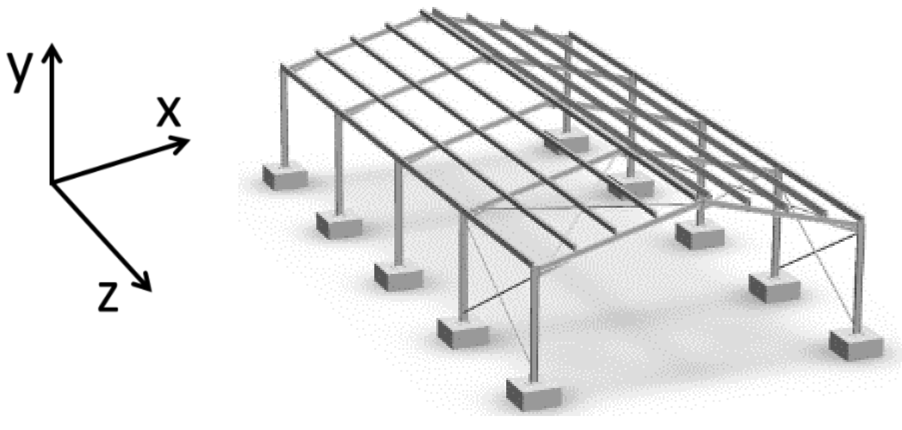

Figure Q1(c)– A typical industrial steel building.

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SECTION B
(Answer **ONE** question)

2. Figures Q2(a) and Q2(b) depict two cross-sections that could be used for the design of a horizontal steel beam. Both cross-sections have a width of 225 mm and a height of 300 mm. The cross-section in Figure Q2(a) has a uniform thickness of 15 mm, while the cross-section in Figure Q2(b) has 15 mm thickness for the flanges and 30 mm thickness for the web. The beam has a total length L = 3.50 m.

   (a) Determine the change in length experienced by both beams after applying a tensile axial load P with a magnitude of 200 kN at the centroid. Assume the Young modulus E for the steel to be equal to 210 GPa.

   [9 marks]

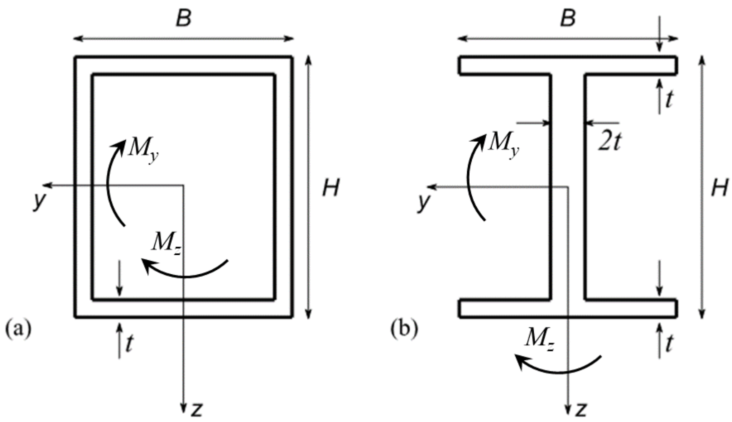
   (b) Consider now the beam to be subjected to the same tensile axial load P plus two bending moments M_y and M_z as shown in Figures Q2(a) and Q2(b).

   i) Indicate for both cross-sections the points with maximum compressive and tensile stress.

   [2 marks]

   ii) Then, assuming the admissible normal stress to be σ_{adm} = 250 N/mm^2 and the moment M_y to have a magnitude of 80 kNm, determine the maximum elastic bending moment M_z that could be applied to both beams.

   [14 marks]



$B = 225\ mm$
$H = 300\ mm$
$t = 15\ mm$
$L = 3.5\ m$
$E = 210\ GPa$
$\sigma_{adm} = 250\ N/mm^2$

Figure Q2 – (a) Rectangular hollow section and (b) I-section for steel beam

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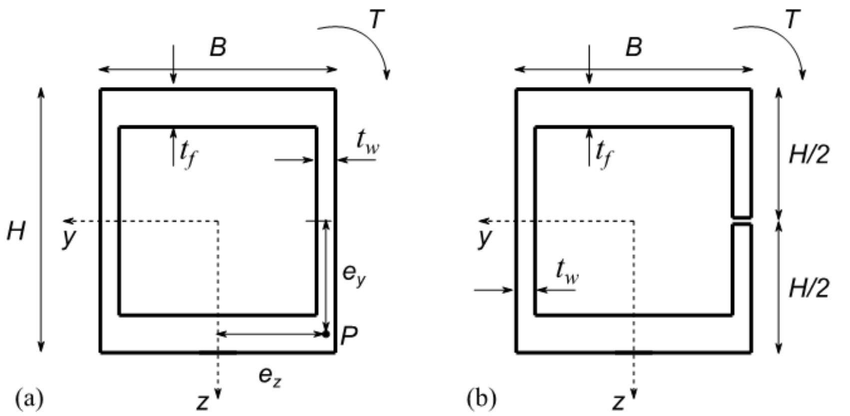
3. Figure Q3(a) shows a 400 mm wide and 500 mm deep rectangular hollow section, with web thickness t_w = 10 mm and flange thickness t_f = 20 mm. The cross section is subjected to a compressive load P acting with eccentricity e_y = 240 mm with respect to the horizontal axis of symmetry and eccentricity e_z = 195 mm with respect to the vertical axis of symmetry as shown in the figure.

   (a) Assuming the maximum admissible elastic normal stress to be σ_{adm} = 200 N/mm^2 (both in compression and in tension), determine the magnitude of the maximum elastic load P that can be safely applied to the cross-section shown in Figure Q3(a).
   [8 marks]

   (b) Assuming an external torque T = 25 kNm is further applied to the cross section shown in Figure Q3(a), determine the magnitude of the maximum shear stress, clearly indicating where this is found in the cross section.
   [9 marks]

   (c) Figure Q3(b) shows a similar cross section, in which the right web has been cut in the middle (neglect the size of the cut). Determine the maximum shear stress in the section of Figure Q3(b) for effect of the external torque T = 25 kNm. Clearly indicate where the maximum shear stress is found in this case.
   [8 marks]



$B = 400\ mm$
$H = 500\ mm$
$t_w = 10\ mm$
$t_f = 20\ mm$
$e_y = 240\ mm$
$e_z = 195\ mm$
$T = 25\ kN.m$
$\sigma_{adm} = 200\ N/mm^2$

Figure Q3 – (a) Rectangular hollow section; (b) Open section due to cut on right vertical side (cut size considered negligible).

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SECTION C
(Answer **TWO** questions)

4. Figure Q4 shows a steel plate (Young modulus E= 200 GPa; Poisson ratio υ= 0.3) subjected to planar state of stress with σ_x= 25 MPa, σ_z= -10 MPa, and τ_{xz}= 25 MPa.

   (a) Draw the Mohr circle of complex stresses and find the magnitude of the principal stresses and the orientation of the corresponding principal directions with respect to the *xz* reference system.
   [10 marks]

   (b) Draw the Mohr circle of complex strains and calculate the maximum compressive strain, tensile strain, and shear strain.
   [10 marks]

   (c) Calculate the axial strain α_m measured by the strain gauge along m-m direction, as shown in Figure Q4, knowing that the angle it forms with the horizontal direction *x* is α_{xm}= 60°.
   [5 marks]

σ_x =	25 MPa
σ_z =	-10 MPa
τ_{xz} =	25 MPa
E =	200 GPa
υ =	0.30
α_{xm} =	60°

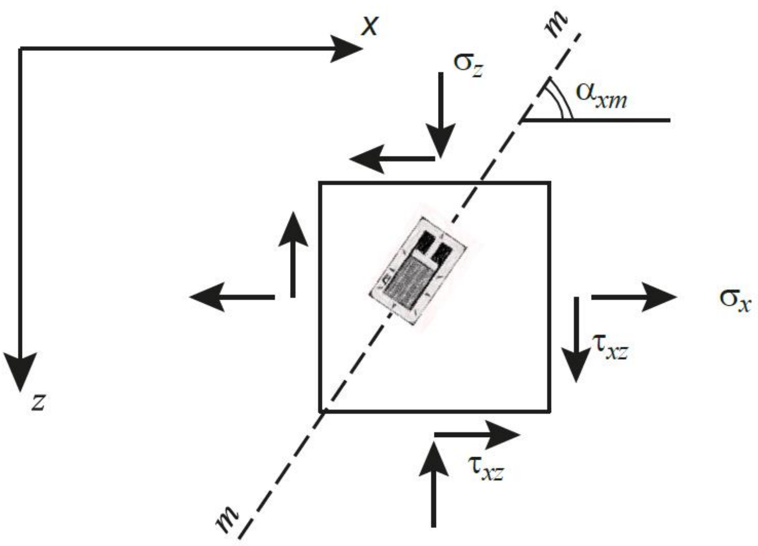

Figure Q4 –Steel plate under planar stress conditions equipped with a strain gauge that measures the axial strain along the *m-m* direction.

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5. Figure Q5(a) shows an inverted T cross-section with outer dimensions and thickness *a*= 180 mm and *t*= 20 mm, respectively. The centroid **G** is located along the axis of symmetry, *z*, at a distance z_G= 52.4 mm from the outer edge of the flange. The material is steel, with Young's modulus *E*= 200 GPa and yield stress σ_{yld}= 355 MPa.

The cross-section is used to fabricate a column of unknown height *h*. The column is simply supported in the *x*-*y* plane and cantilevered in the *x*-*z* plane, as illustrated with the qualitative buckling modes in Figure Q5(b).

(a) Calculate the second moments of area I_{yy} and I_{zz} of the cross section with respect to the principal axes *y* and *z.*

[7 marks]

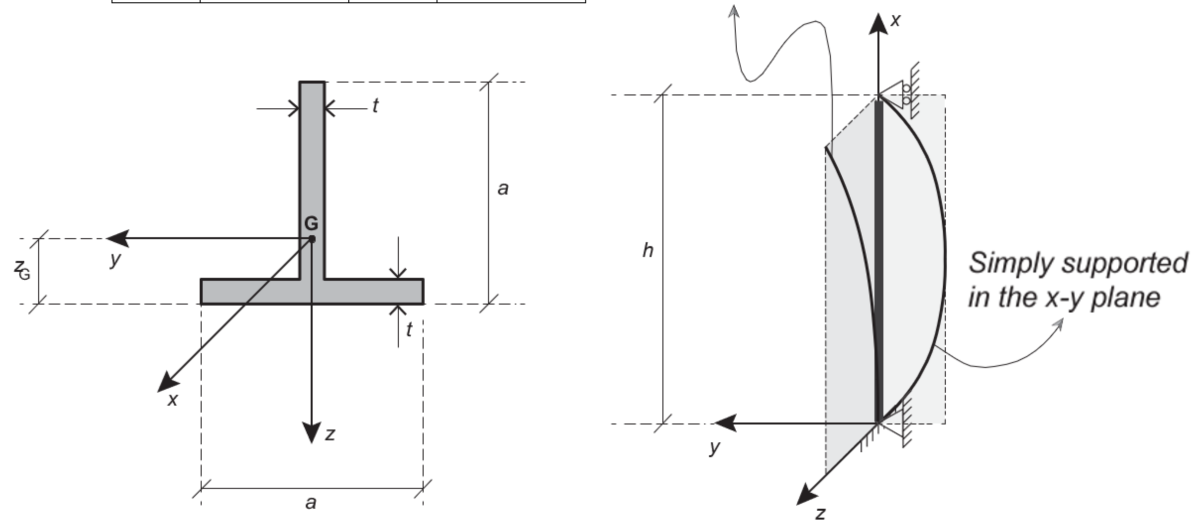
(b) Knowing that the Euler's buckling load in the *x*-*z* plane is $P_{E(yy)}$= 1,246 kN, calculate the height of the column. <u>Hint</u>: the buckling load $P_{E(yy)}$ results in the column bending about the *y* axis; therefore, $P_{E(yy)}$ is proportional to the flexural stiffness $E\ I_{yy}$.

[9 marks]

(c) Calculate the Rankine's failure load in the *x*-*y* plane, $P_{R(zz)}$.

[9 marks]

a =	180 mm	*E* =	200 GPa
t =	20 mm	σ_{yld} =	355 MPa
z_G =	52.4 mm	$P_{E(yy)}$ =	1,246 kN



(a) (b)

Figure Q5 – (a) Cross section and (b) Column with different restrains in the *x*-*y* and *z*-*x* planes.

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6. A timber cross-section (E_t= 32 GPa), b= 40 cm wide and d= 50 cm deep, is reinforced by a single steel plate (E_s= 200 GPa), t= 5 cm thick, firmly bonded to the lower side of the timber (see Figure Q6(a)).

The resulting steel-timber composite section is used to fabricate a simply supported beam, L= 6.00 m long. The beam is loaded with a pair of point loads F= 60.0 kN, applied at a distance $L/4$ to the beam's ends (see Figure Q6(b)).

Calculate:

(a) The position of the centroid of the composite cross-section of Figure Q6(a).

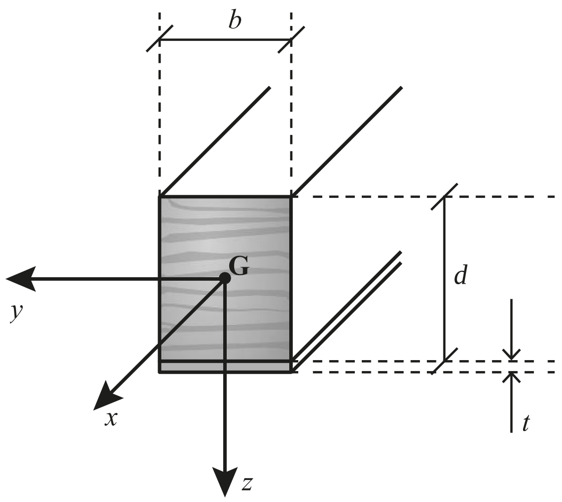
[10 marks]

(b) The flexural stiffness of the composite cross-section, $(E\,I)_{comp}$.

[7 marks]

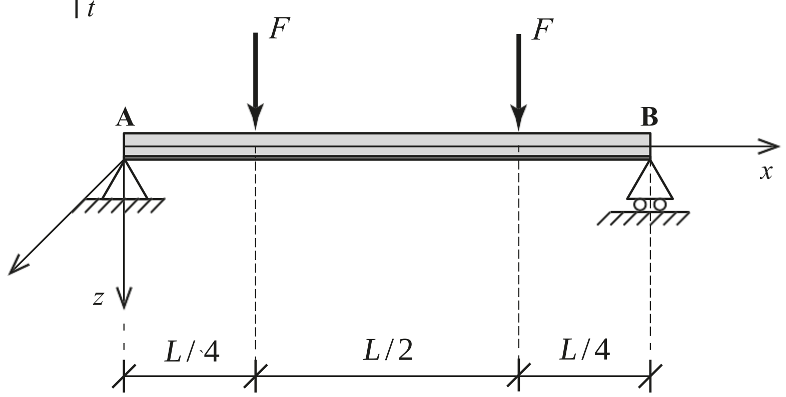
(c) The maximum values of the direct stress σ in the timber and steel, clearly indicating whether they occur in tension or compression for the load conditions specified in Figure Q6(b).

[8 marks]



(a)

E_t =	32 GPa
E_s =	200 GPa
b =	40 cm
d =	50 cm
t =	5 cm
F =	60.0 kN/m
L =	6.00 m



(b)

Figure Q6 – (a) Steel-timber composite cross section and (b) Simply supported beam.

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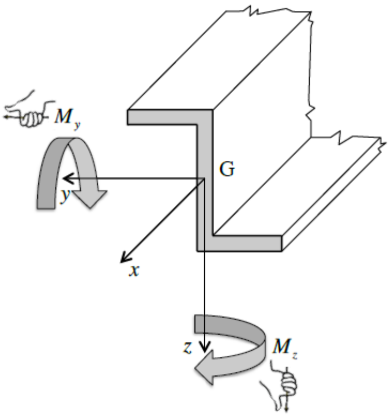
Aide-Mémoire Formula Sheet (Page 1 of 2)

Buckling Load

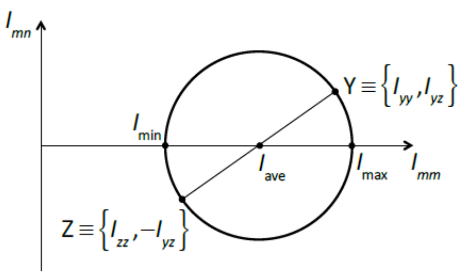
$$P_E = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 E}{\lambda^2} \quad ; \quad \lambda = \frac{L_e}{\rho}$$

$$P_R = \frac{P_E P_c}{P_E + P_c}$$

Unsymmetrical Bending



$$\sigma_x = \beta y + \gamma z \quad ; \quad \begin{cases} \beta = -\dfrac{M_z I_{yy} + M_y I_{yz}}{I_{yy} I_{zz} - I_{yz}^2} \\ \gamma = \dfrac{M_y I_{zz} + M_z I_{yz}}{I_{yy} I_{zz} - I_{yz}^2} \end{cases}$$



Elastic Constitutive Law

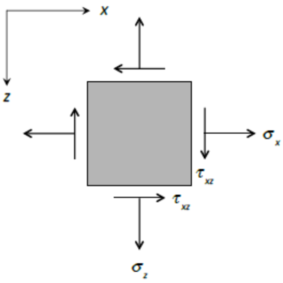
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y - \nu\sigma_z\right) \quad ;$$

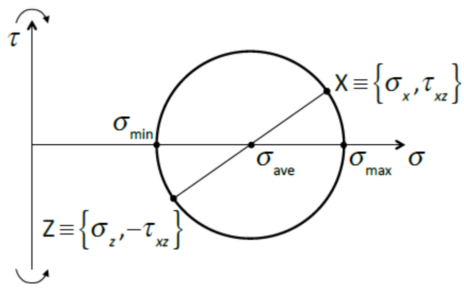
$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y - \nu\varepsilon_z\right]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \quad ; \quad \tau_{xy} = G\gamma_{xy}$$

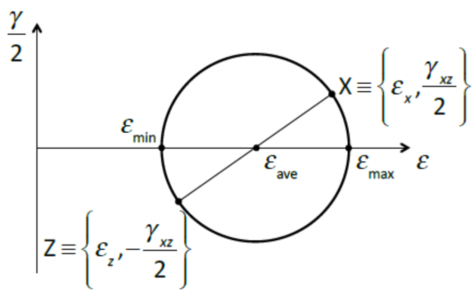
$$G = \frac{E}{2(1+\nu)}$$

Mohr's Circle for Stress State





Mohr's Circle for Strain State



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Aide-Mémoire Formula Sheet (Page 2 of 2)

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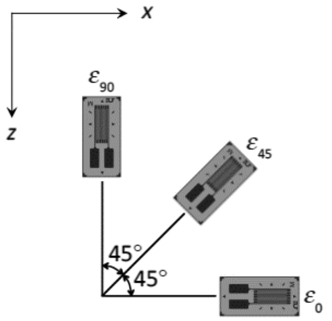
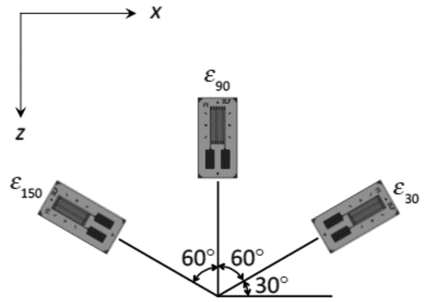
Strain-Gauge Rosette

45° Rosette



$$\gamma_{xz} = \varepsilon_0 + \varepsilon_{90} - 2\varepsilon_{45}$$

60° Rosette



$$\begin{cases} \varepsilon_x = \dfrac{2}{3}\left(\varepsilon_{30} - \dfrac{\varepsilon_{90}}{2} + \varepsilon_{150}\right) \\ \gamma_{xz} = \dfrac{2}{\sqrt{3}}\left(\varepsilon_{150} - \varepsilon_{30}\right) \end{cases}$$