Loughborough University

MATHEMATICAL MODELLING OF STRUCTURES
(23CVA104)

Semester 2 2024 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours.**

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

Answer **THREE** questions in total:

- **TWO** questions from Section A – "Mathematics", **and**
- **ONE** question from Section B – "Analysis of Beams and Frames".

All questions in the same section carry equal marks.

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SECTION A – MATHEMATICS

(Answer **TWO** questions in this section)

Q1. Solve the following 1st order ODEs:

(a) $\frac{dy}{dx} = \frac{1}{x+3}$, subject to the boundary condition: $y(0) = 1$ [7 marks]

(b) $(x^2 + 1)\frac{dy}{dx} + 2xy = 3x^2 \cos(x^3 + 1)$ [6 marks]

(c) $\frac{dy}{dt} + 2y = e^{3t}$ [6 marks]

(d) $x\frac{dy}{dx} - 2y = x^2$ [6 marks]

Q2. Solve the following 2nd order ODEs:

(a) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$ [6 marks]

(b) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$, subject to the boundary conditions: $y(0) = 1, y\left(\frac{\pi}{2}\right) = 0$ [6 marks]

(c) $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$, with boundary conditions: $y(0) = 0,\ y(1) = 3e^5$ [6 marks]

(d) $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = 144t$ [7 marks]

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Q3. Given the mass-spring oscillator shown in Figure Q3, with mass $m = 2\ kg$, spring constant $k = 200\ N/m$ and external forcing $f(t) = \sin(2t)$, do the following tasks:

i) Determine the equation of motion of the oscillator, assuming there is no friction between the mass and the floor. [5 marks]

ii) Solve the equation of motion of the oscillator, subject to the initial conditions $x(0) = 0$ and $\dot{x}(0) = 30$. [13 marks]

iii) Comment as to whether the system is close to resonance. If the system is not close to resonance, give an example of external forcing that would make it resonate, explaining your reasoning. [7 marks]

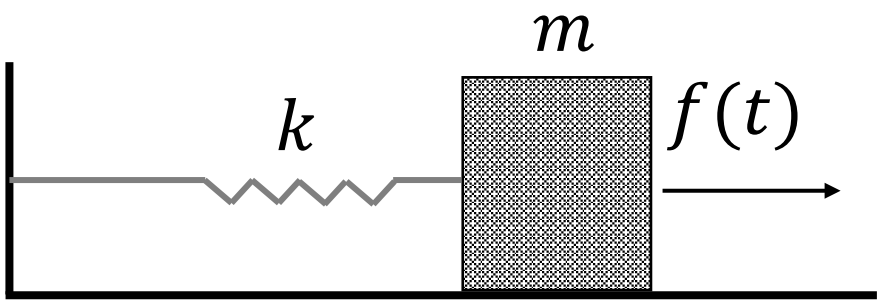

Figure Q3

SECTION B – ANALYSIS OF BEAMS AND FRAMES

(Answer **Question 4** in this section)

Q4. (a) The beam shown in Figure Q4(a) has a frictionless hinge at point B.

i) Calculate all support reactions. [6 marks]

ii) Sketch the bending moment diagram and the shear force diagram, indicating all principal values. [10 marks]

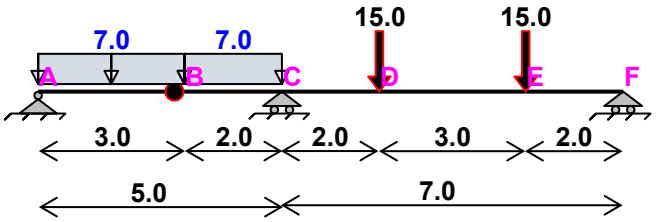

Figure Q4(a)

(b) For the frame shown in Figure Q4(b).

i) Calculate all support reactions. [12 marks]

ii) Draw the axial force diagram and the bending moment diagram for the whole structure, indicating all principal values. [16 marks]

iii) Sketch the deflected shape. [6 marks]

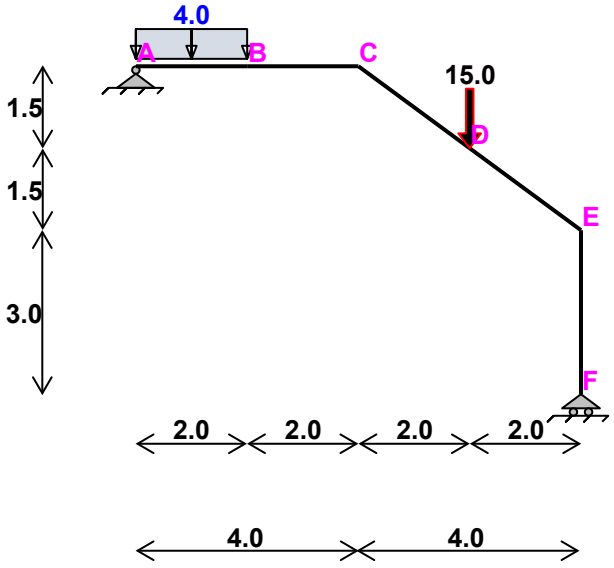

Figure Q4(b)

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