Loughborough University

Further Structural Analysis and Geotechnical Design
23CVC101

Semester 2 2024 | In-Person Exam Paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

This examination consists of two sections:
Answer **TWO QUESTIONS in Section A**
Answer **TWO QUESTIONS in Section B**

Please use a separate answer book for each section. Print **SECTION A or SECTION B** on the front of the applicable answer books.

All questions carry equal marks.

Formula sheet is attached.

Continues/…

.../continued

SECTION A
(Answer **TWO QUESTIONS in Section A)**

1. a) With the aid of a diagram of Settlement against load explain the difference between ULS and SLS failure for shallow foundations. Indicate how ULS factors of safety may not control for SLS. In your answer describe for a conventional house what SLS design is aimed to prevent occurring.

[7 marks]

b) A house is constructed on strip footings 2m below ground level in a sandy clay soil and the footing is 0.6m wide The water table is at foundation level.
$Cu = 50kN/m^2$ $C_k' = 3\ kN/m^2$ $\varphi_k' = 20^o$ $\gamma_{dry\ k} = 18kN/m^3$ $\gamma_{sat\ k} = 21kN/m^3$

i) Assess the ultimate limit state design capacity of the soil to EC7 DA1 C1, short and long term. Comment on your answer.

[8 marks]

ii) Calculate the maximum design load the foundation could carry (including self-weight).

[4 marks]

iii) Detail three factors that may have influenced the choice of foundation depth and what effect that impact may have on the soil.

[6 marks]

2. a) Explain why groups of piles cannot usually carry the same total load as the sum of individual pile capacities. Detail the factors that will influence group capacity.

[6 marks]

b) A pile near a river extends through layered soil of stiff clay into a layered sand (Details in Table Q2). The piles are precast driven piles 150mm square and 8m long. The water table is initially 10m below ground level.

Assume no load tests, alpha of 0.3, model factor 1.4, K_s= 0.4 and $\delta = 15^o$.

Table Q2

Layer depth (m)	Φ (°)	γ (kN/m³)	Cu (kN/m²)
Ground level to 3m, stiff clay	0	22	Varies linearly 40 at GL to 100 at 3m
3m to 6m	25	17.5 Dry 19 Saturated	
6m to 8m	30	18 Dry 20 Saturated	

Question 2 continues/...

…/question 2 continued

i) Calculate the initial design capacity of the pile to EC7 Design approach 1, case 2.

[10 marks]

ii) From bi) above, recalculate the capacity if the water table rises to the bottom of the clay later.

[6 marks]

iii) Comment on your answer from ii)

[3 marks]

3. An investigation into a new cut slope in Clay (**Figure Q3**) for a highway project found the geometry, loading and soil conditions below (assume all values are characteristic).

Grey London Clay γ_{sat}=20.2kN/m^3 Cu=17kN/m^2 $\varphi p' = 30^o$ Cp'=5kN/m^2 $\varphi_r' = 25^o$ Cr'=0kN/m^2
Weathered London Clay γ_{sat}=19.5kN/m^3. Cu=10kN/m^2

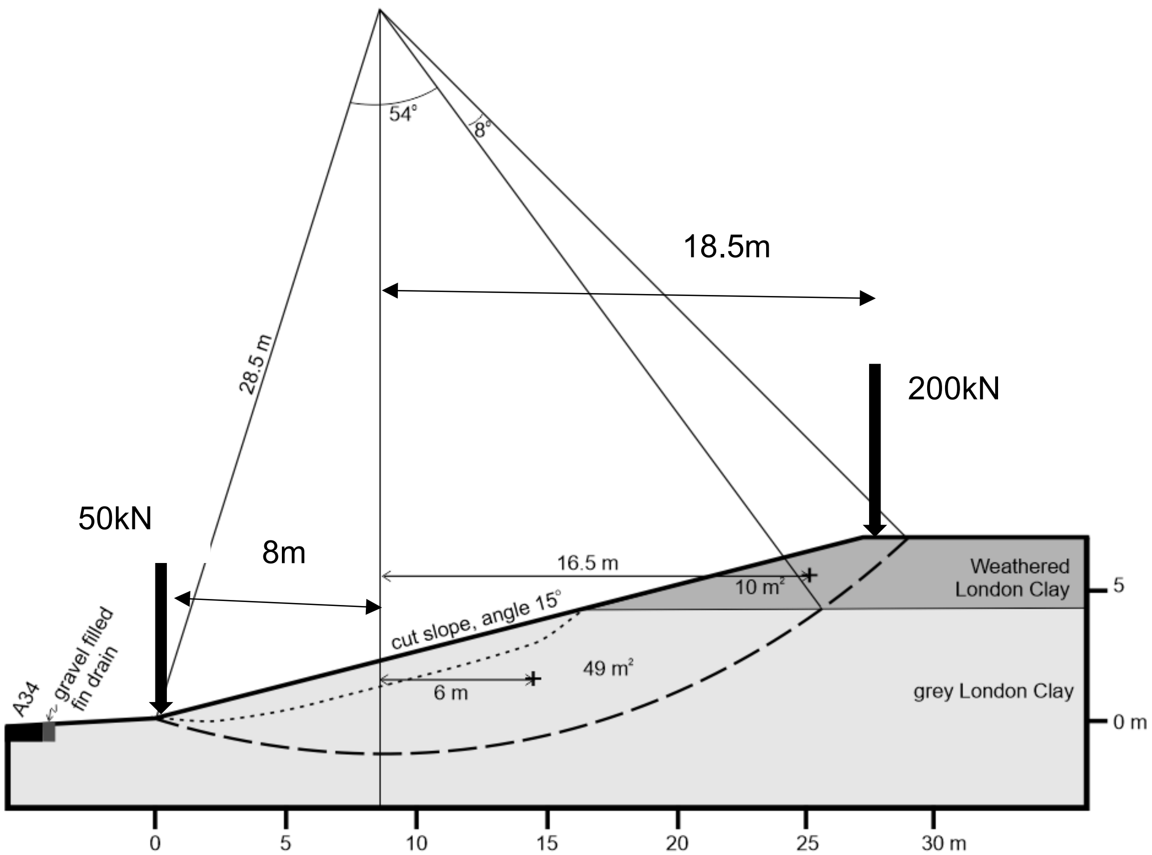


Figure Q3 (Not to Scale)

Question 3 continues/…

…/question 3 continued

a) Assess the basic stability of the slope (Ignoring EC7) when it was created using the trial slip circle shown by the heavy broken line in Figure Q3 and assuming the slope is subject to the two temporary point loads shown.

$$F = \sum Cu\ R^2\Theta \ /\ \sum Wd$$

[13 marks]

b) i) Longer term, there is a deterioration of the slope material near the surface in a zone about 1.2m below the ground surface of the lower section of the slope in the grey London clay, which is showing around 300mm of displacement. The critical slip surface of the slope in its deteriorated state is indicated by the thin dashed line in Figure Q3. Assess the stability of this section of the slope to EC7 DA1 Case 2, assuming the water table is 0.5m below ground level and parallel to the slope.

[7 marks]

ii) Describe and comment on the assumptions needed to be made about the soil properties and water table in a basic translational slide analysis in bi).

[5 marks]

SECTION B
(Answer **TWO QUESTIONS in Section B)**

4. a) In a Finite Element Analysis, explain the difference between “plane stress” and “plane strain” classes of problems.

[3 marks]

b) The frame shown in Figure Q4b is to be analysed using the Stiffness Matrix method. The frame is fixed at joint 1. At joint 3, the horizontal and rotational displacements are totally prevented, but the joint is allowed to move vertically.

i) Draw a diagram showing the restrained structure and the numbering system for the overall degrees of freedom.

[3 marks]

ii) Generate the overall stiffness matrix [SJ] and calculate the overall load vector. Show clearly how the boundary conditions may be incorporated. Assume that the global stiffness matrix for any member *m* is given by

$$[SMG]_m = \begin{pmatrix} S_{11} & \cdots & S_{16} \\ \vdots & \ddots & \vdots \\ S_{61} & \cdots & S_{66} \end{pmatrix}_{6x6}$$

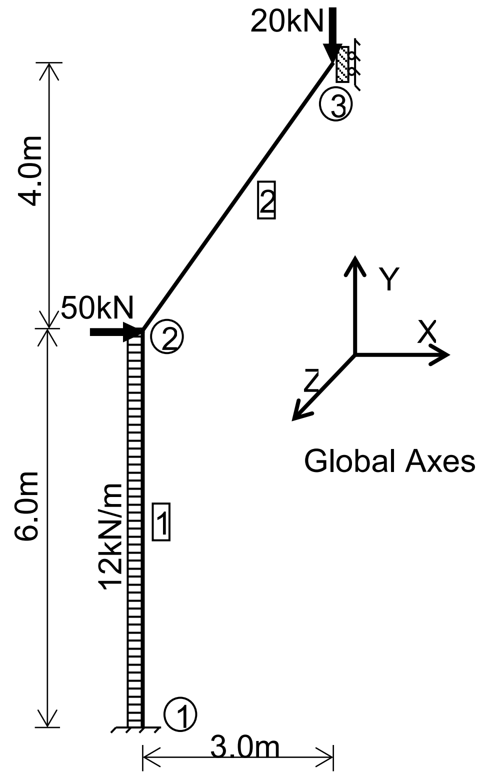
Calculation of the stiffness coefficients S_{ij} **is not** required.

[10 marks]

Question 4 continues/…

iii) Generate the global stiffness matrix [SMG] for member 1. In addition, calculate the reactions at joint 1 assuming that the global displacements at joint 2 are as given in Figure Q4b.

[9 marks]



○ indicates a joint number

□ indicates a member number

Properties of Member 1:

E = 200 kN/mm^2

I = 6750 cm^4

A = 30 cm^2

Displacements at Joint 2:

Horizontal = 28.60 mm

Vertical = - 0.20 mm

Rotation = 4.40E-03 rad

Figure Q4b

Continues/…

…/continued

5. a) State the three conditions which must be satisfied in a plastic collapse mechanism [3 marks]

b) The plastic moments for the members of the frame shown in Figure Q5b are given in the same figure.

i) Calculate the number of elementary mechanisms and sketch the collapse shape for each mechanism. [5 marks]

ii) Assuming that plastic hinges are formed at points A, C and D:

1) Draw a diagram showing the collapse mechanism of the frame. Clearly indicate the key displacements on the diagram. [3 marks]

2) Use the method of virtual displacements to calculate the plastic collapse load. [8 marks]

3) Check whether the position of the hinge assumed at point C is correct. [6 marks]

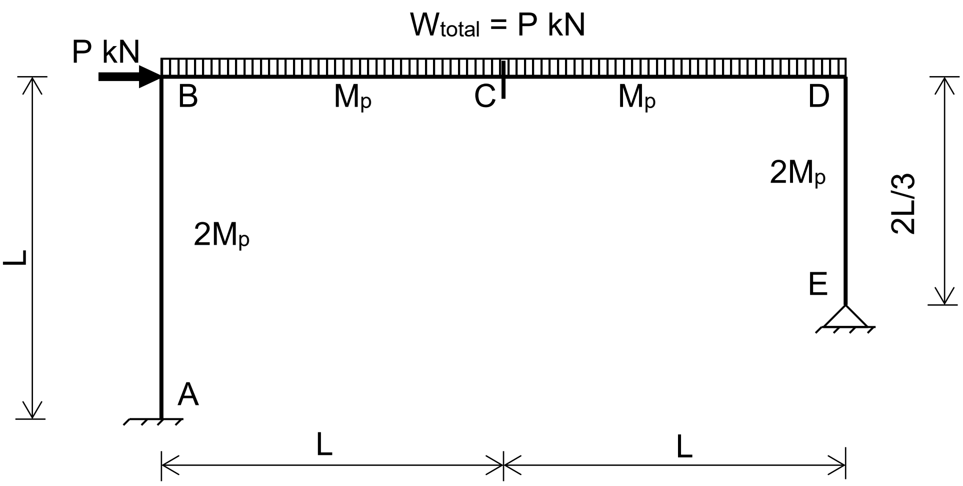


Figure 5b

Continues/…

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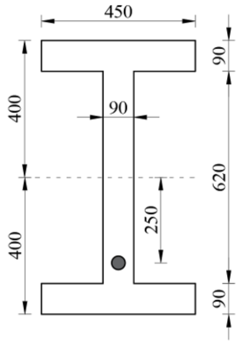
6. a) Calculate the ultimate moment capacity of the cross-section shown in Figure Q6a, displaying the stress and strain distributions. Assume Y_{NA} = 336 mm from the top side of the beam and λ = 0.9. **Perform only one trial**. However, if the forces are not in equilibrium, should the value of Y_{NA} be increased or decreased in the second trial? Provide a reason for your answer.

[15 marks]



Properties of concrete:

- Concrete compressive strength $f_{ck} = 50N/mm^2$
- $\eta = 0.45$

Prestressing force:

- The prestress force $P = 1300kN$
- The total losses = 0.21.

The cable consists of **4 strands** with each strand has the following properties:

- Area of each strand $A_p = 200\ mm^2$
- Yield stress $f_{yp} = 1680N/mm^2$
- Modulus of Elasticity $E_p = 195kN/mm^2$

Figure Q6a

b) The equation of the losses ΔP/P due to friction in post-tensioned beams is given by:

$$\frac{\Delta P}{P} = 1 - e^{-\mu(\theta + kx)}$$

Use the above equation to calculate the average percentage loss of prestress due to the friction component only for the beam shown in Figure Q6b. Assume the coefficient of friction μ = 0.2, and the wobble coefficient per unit length of the cable k = 0.01/m.

[10 marks]

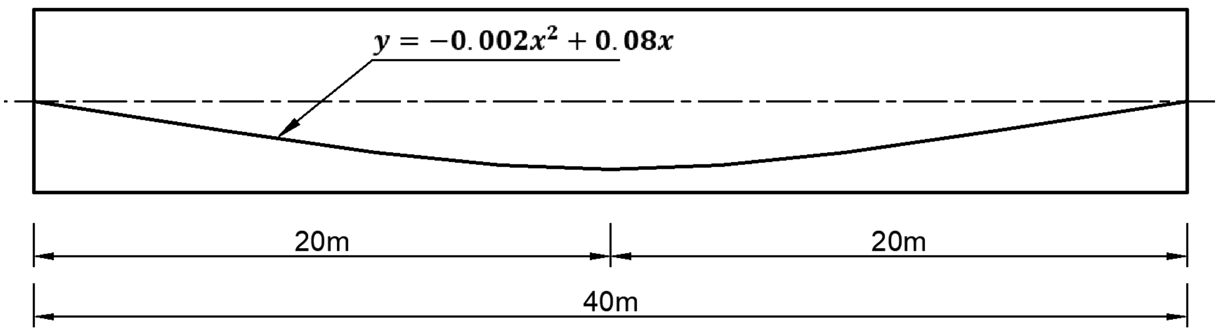


Figure Q6b

A El-Hamalawi
T A Dijkstra
J El-Rimawi
M W Frost
M Shaheen

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Formula Sheet for Further Structural Analysis and Geotechnical Design (CVC101)

Piling:

$$Qult = C_{ud} N_c Sc + \sigma_{vd}' NqSq$$

where q_{ult} = ultimate bearing capacity
B = width of foundation
σ_{vd}' = effective overburden pressure at foundation level
u = ground water pressure at foundation level
c_d = cohesion of soil below foundation
γ' = effective unit weight

$R_{cd} = R_{bk} + R_{sk} = A_b\ q_{ult} + A_s\ C_a$

where Rcd = ultimate characteristic pile resistance, at surface
A_b = area of pile base
q_{ult} = ultimate bearing capacity at base
A_s = area of surface of pile shaft
C_a = ultimate shaft friction

Piles are round or square, so **s_q = 1.2**

Clay:

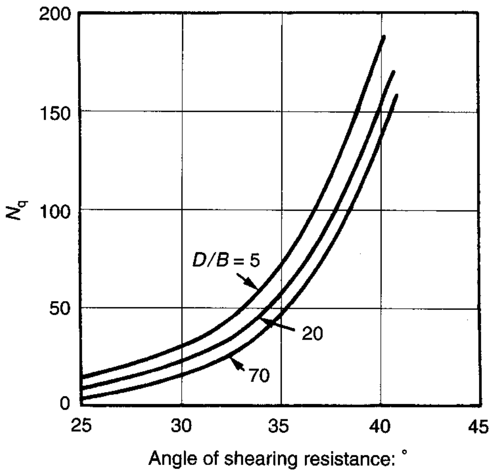
q_{ult} = $9C_b$ where C_b = design shear strength of clay at base ($C_u\omega$ for bored)
C_a = $\alpha\ C_{ave}$ where C_{ave} = average design shear strength of clay adjacent to shaft
α = adhesion factor

Frictional materials:

q_{ultnet} = $q'(N_q)\ s_q$
C_a = $K_s\ p'_{ave} \tan \delta$

where Ks = earth pressure coefficient
q' = effective overburden pressure at the pile base
p'_{ave} or $\overline{\sigma'}_{vd}$ = average effective overburden pressure along pile shaft
δ = angle of pile/soil friction

Pile type	δ	K_s (depending on relative density of soil)
Steel	20°	0.5 - 1.0
Concrete	0.75ϕ	1.0 – 2.0
Timber	0.67ϕ	1.5 – 4.0 (use 2.5)



Berezantsev's Nq Factors

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$R\ group_k = A_{bg}\ N_c\ C_{ud} + Asg\ C_{ud\ ave}$

$R\ group = n\ R_{cd}\ \eta$

Lateral Earth Pressure and Retaining Walls

$K_a = (1 - \sin\phi') / (1 + \sin\phi')$

$K_p = (1 + \sin\phi') / (1 - \sin\phi')$

$p_a = K_a.\ \sigma_z' - 2.c' \sqrt{K_a}$

$p_p = K_p.\ \sigma_z' + 2.c' \sqrt{K_p}$

$R_s = c_w'.\ B + V.\tan \delta'$

$Q = P/A \pm 6M/ B^2L$ When Resultant is in Middle Third

Slope Stability:

Translational slide

$$F.of\ S. = \frac{c' + (\gamma.z - u)\cos^2\beta\tan\phi'}{\gamma.z\sin\beta\cos\beta}$$

Bishop's Method

$$F.of\ S. = \frac{1}{\sum W\sin\alpha}\sum\frac{[c'b + (W - u.b)\tan\phi']\sec\alpha}{1 + \frac{\tan\alpha.\tan\phi'}{F}}$$

$$W = A\frac{\gamma_K}{\gamma_\gamma} \qquad C = \frac{c'_K b}{\gamma_c} + \frac{\tan\varphi'_K}{\gamma_\varphi}\left(A\frac{\gamma_K}{\gamma_\gamma} - ub\right) \qquad D = \frac{\sec\alpha}{1 + \frac{\tan\alpha\tan\phi_K'}{\gamma_\varphi}}$$

Partial factors for the GEO ultimate limit state, **Design Approach 1**.

Combination 1

A1			M1		R1
permanent γ_G	unfavourable	1.35	γ□'	1.0	1.0
	favourable	1.0	$\gamma_{c'}$	1.0	
variable γ_Q	unfavourable	1.5	γ_{cu}	1.0	
	favourable	1.0	γ□	1.0	

Combination 2

A2			M2		R1
permanent γ_G	unfavourable	1.0	γ□'	1.25	1.0
	favourable	1.0	$\gamma_{c'}$	1.25	
variable γ_Q	unfavourable	1.3	γ_{cu}	1.4	
	favourable	0	γ□	1.0	

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Bearing Capacity

$$Qult = C_d N_c S_c + q' N_q S_q + \tfrac{1}{2}\gamma' B N_\gamma S_\gamma$$

where q_{ult} = ultimate bearing capacity
B = width of foundation
q' = effective overburden pressure at foundation level
u = ground water pressure at foundation level
c_d = design cohesion of soil below foundation
γ' = effective unit weight

Bearing Capacity Factors

ϕ	N_c	N_q	N_γ
0	5.14	1.0	0
1	5.4	1.1	0
2	5.6	1.2	0
3	5.9	1.3	0
4	6.2	1.4	0
5	6.5	1.6	0.1
6	6.8	1.7	0.1
7	7.2	1.9	0.2
8	7.5	2.1	0.2
9	7.9	2.3	0.4
10	8.4	2.5	0.5
11	8.8	2.7	0.7
12	9.3	3.0	0.8
13	9.8	3.3	1.1
14	10.4	3.6	1.3
15	11.0	3.9	1.6
16	11.6	4.3	1.9
17	12.3	4.8	2.3
18	13.1	5.3	2.8
19	13.9	5.8	3.3
20	14.8	6.4	4.0
21	15.8	7.1	4.7
22	16.9	7.8	5.5
23	18.1	8.7	6.5
24	19.3	9.6	7.6
25	20.7	10.7	9.1
26	22.3	11.9	10.5
27	23.9	13.2	12.4
28	25.8	14.7	14.5
29	27.9	16.4	17.1
30	30.1	18.4	20.0
31	32.7	20.6	23.6
32	35.5	23.2	27.7
33	38.6	26.1	32.5
34	42.2	29.4	38.4
35	46.1	33.3	45.2
36	50.6	37.8	53.3
37	55.6	42.9	63.2
38	61.4	48.9	74.9
39	67.9	56.0	89.1
40	75.3	64.2	106.0
41	83.9	73.9	126.8
42	93.7	85.4	152.0
43	105.1	99.0	182.8
44	118.4	115.3	220.8
45	133.9	134.9	267.7
46	152.1	158.5	326.3
47	173.6	187.2	399.3
48	199.3	222.3	491.6
49	229.9	265.5	608.5
50	266.9	319.1	758.0

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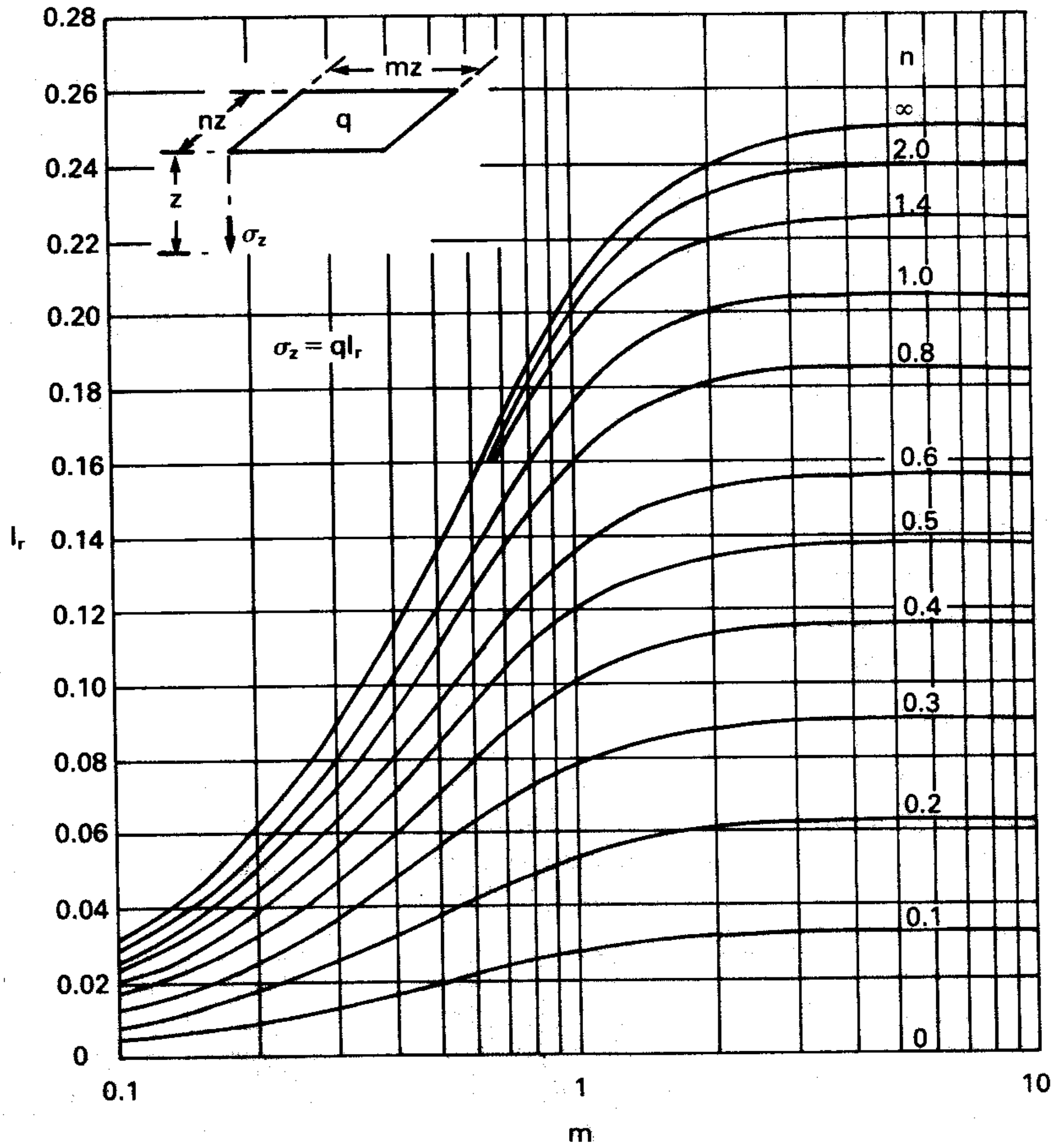
Shape factors

Shape of foundation		s_c	s_q	s_γ
strip		1.0	1.0	1.0
rectangle	Drained	$(s_q N_q - 1)/(N_q - 1)$	$1 + \frac{B'}{L'}\sin\phi$	$1 - 0.3\,\frac{B'}{L'}$
	Undrained	$1 + 0.2\,\frac{B'}{L'}$		
circle or square	Drained	$(s_q N_q - 1)/(N_q - 1)$	$1 + \sin\phi$	0.7
	Undrained	1.2		



Fadum Chart

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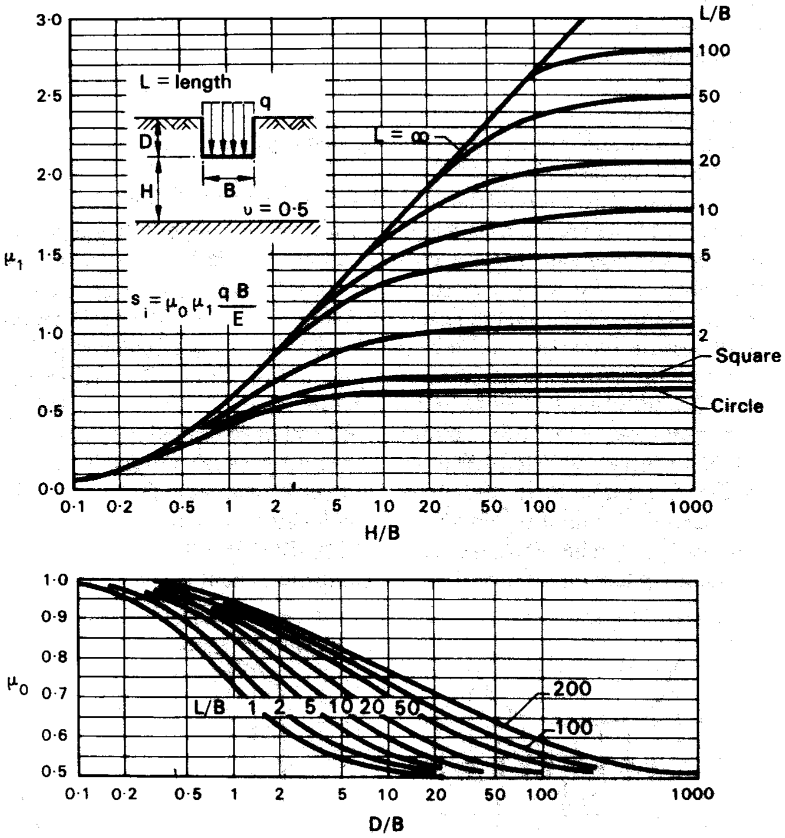

Janbu Chart

Structure	**Design Approach**			
	1		**2**	**3**
	Combination 1	**Combination 2**		
Axially loaded piles and anchors	**A1**+M1+R1	A2+(M1$^{\#}$/**M2**$^{\$}$)+**R4**	**A1**+M1+**R2**	(**A1***/**A2**†)+**M2**+**R3**
Other structures		A2+**M2**+R1		
Slopes			**A1**+M1+**R2**	**A2***†+**M2**+**R3**

$^{\#}$ for calculating resistance; $^{\$}$ for calculating unfavourable actions (e.g. down-drag)
* on structural actions; † on geotechnical actions

- In EN 1997-1, the sets of partial factors are labelled according to whether the partial factors apply to actions (A), material properties (M), or resistances (R). Where factors on actions are applied to the effect of actions rather than the actions themselves, the set is underlined (e.g. for slopes using Design Approach 2, set A1).
- Many of the partial factors given in EN 1997-1 are equal to 1,0 (and therefore can be omitted from calculations). The sets of partial factors that provide the main source of safety (i.e. have values other than 1,0) in a particular combination are shown in **bold**. For example, when using Design Approach 1/combination 1, safety is introduced primarily through factors on actions **A1**. In DA1/combination 2 for slopes, safety is introduced primarily through material factors **M2**.

Extracts from EC7 Design Cases

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Partial factors on actions for different limit states

Duration of action	Effect of action	Symbol γ_F	Limit state/partial factor set				
			EQU	STR/GEO		UPL	HYD
				A1	A2		
Permanent	Unfavourable	$\gamma_{G;dst}$	1,1	1,35	1,0	1,0	1,35
	Favourable	$\gamma_{G;stb}$	0,9	1,0	1,0	0,9	0,9
Variable	Unfavourable	$\gamma_{Q;dst}$	1,5	1,5	1,3	1,5	1,5

Unfavourable actions (with the subscript "dst" above) are those which destabilize the structure and favourable actions (subscript "stb") are those which stabilize the structure. Variable, favourable actions are omitted from the table above because they are deliberately ignored in EN 1997-1 (i.e. $\gamma_{Q;stb} = 0$).

Example (using limit state STR/GEO partial factor set A1)
If the representative vertical load (F_{rep}) on a footing is 100 kN, then the design vertical load (F_d) would be 100 x 1.35 = 135 kN.

Extracts from EC7 Action Factors

Soil parameter	Symbol γ_M	Limit state/partial factor set				
		EQU	STR/GEO		UPL	HYD
			M1	M2		
Angle of shearing resistance	$\gamma_{\phi'}$	1,25*	1,0*	1,25*	1,25*	–
Effective cohesion	$\gamma_{c'}$	1,25	1,0	1,25	1,25	–
Undrained shear strength	γ_{cu}	1,4	1,0	1,4	1,4	–
Unconfined strength	γ_{qu}	1,4	1,0	1,4	1,4	–
Weight density	γ_{γ}	1,0	1,0	1,0	–	–
Tensile pile resistance	$\gamma_{s;t}$	–	–	–	1,4	–
Anchorage	γ_R	–	–	–	1,4	–

*Applied to tan ϕ' not ϕ'

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Extracts from EC7 Material Factors

In equation (7.4), $R_{c;k}$ is the characteristic value of the compressive resistance of the pile and γ_t is a partial factor on that resistance.

In equation (7.5), $R_{b;k}$ is the characteristic base resistance of the pile, $R_{s;k}$ is its characteristic shaft resistance, γ_b is a partial factor on the base resistance and γ_s is a partial factor on the shaft resistance.

Values of γ from the National Annex to BS EN 1997-1 are given below. Please note that these values differ significantly from those given in EN 1997-1 Annex A, and the figures are provisional at the time of writing (July 2007). With these factors, equation 7.4 always gives design resistances equal to or lower than equation 7.5.

Partial factors for piles in compression

Resis-tance	Symbol	Partial factor set for different pile types				
		R1	R4			
			Without load tests*		With load tests*	
		All types	Bored & CFA	Driven	Bored & CFA	Driven
Base	γ_b	1,0	2,0	1,7	1,7	1,5
Shaft	γ_s		1,6	1,5	1,4	1,3
Total	γ_t		2,0	1,7	1,7	1,5

* The lower values of γ_b, γ_s, and γ_t in R4 may be adopted if serviceability is verified by load tests (preliminary and/or working) carried out on more than 1% of the constructed piles to loads not less than 1,5 times the representative load for which they are designed, or if settlement at the serviceability limit state is of no concern.

Extracts from EC7 Pile Resistance Factors
(Note. For design Approach 1, Resistance factors for R1 and R2 are normally = 1)

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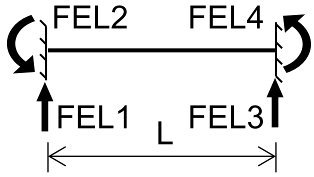
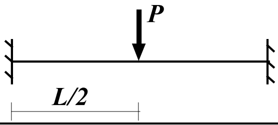
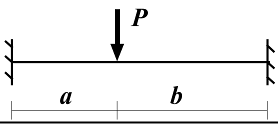
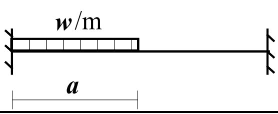
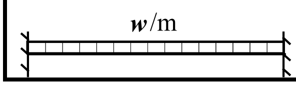
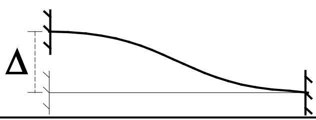
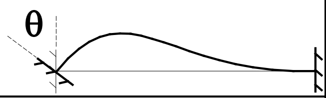
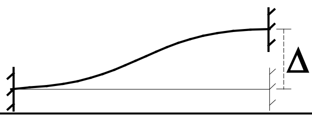
Fixed End Forces

a. Due to in-span loads



	FEL_1	FEL_2	FEL_3	FEL_4
1. 	$\frac{P}{2}$	$\frac{PL}{8}$	$\frac{P}{2}$	$-\frac{PL}{8}$
2. 	$\frac{Pb^2}{L^3}(3a+b)$	$\frac{Pab^2}{L^2}$	$\frac{Pa^2}{L^3}(3b+a)$	$-\frac{Pba^2}{L^2}$
3. 	$\frac{wa}{2L^3}(2L^3-2a^2l+a^3)$	$\frac{wa^2}{12L^2}(6L^2-8al+3a^2)$	$\frac{wa^3}{2L^3}(2L-a)$	$-\frac{wa^3}{12L^2}(4L-3a)$
4. 	$\frac{wL}{2}$	$\frac{wL^2}{12}$	$\frac{wL}{2}$	$-\frac{wL^2}{12}$

b. Due to joint displacement

5. 	$\frac{12EI}{L^3}\Delta$	$\frac{6EI}{L^2}\Delta$	$-\frac{12EI}{L^3}\Delta$	$\frac{6EI}{L^2}\Delta$
6. 	$\frac{6EI}{L^2}\theta$	$\frac{4EI}{L}\theta$	$-\frac{6EI}{L^2}\theta$	$\frac{2EI}{L}\theta$
7. 	$-\frac{12EI}{L^3}\Delta$	$-\frac{6EI}{L^2}\Delta$	$\frac{12EI}{L^3}\Delta$	$-\frac{6EI}{L^2}\Delta$
8. 	$\frac{6EI}{L^2}\theta$	$\frac{2EI}{L}\theta$	$-\frac{6EI}{L^2}\theta$	$\frac{4EI}{L}\theta$
9. 	$FEL_1 = -\frac{EA}{L}\Delta$		$FEL_2 = \frac{EA}{L}\Delta$	