Loughborough University

Control, Modelling and Simulation
23CVC119

Semester 2 2024 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

THIS PAPER COMPRISES **SECTION A** AND **SECTION B.**

Section A, question 1 is compulsory – you MUST answer this question.
Answer **ONE** question from Section B.

All questions carry equal marks.

A 4-page formulae sheet, with tables and charts, is provided at the end of this paper.

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SECTION A

(Answer **THIS** question)

Q1.

(a) For the vectors

$$\boldsymbol{a} = \begin{bmatrix} 7 \\ 8 \end{bmatrix},\ \boldsymbol{b} = \begin{bmatrix} 5 \\ -2 \end{bmatrix},\ \boldsymbol{c} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}\ \boldsymbol{d} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

calculate: $(\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{c} + \boldsymbol{d})$

[3 marks]

(b) For the matrices *A* and *B* given by

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 7 \end{bmatrix},\ B = \begin{bmatrix} -1 & -4 \\ 5 & 6 \end{bmatrix}$$

calculate: *AB*

[3 marks]

(c) Calculate the determinant of the following matrix

$$\begin{bmatrix} 4 & -1 \\ -2 & -3 \end{bmatrix}$$

[3 marks]

(d) Find the eigenvalues and corresponding eigenvectors for the following matrix:

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$$

[6 marks]

Question 1 continues/…

(e) Obtain the Taylor series expansion for $f(x) = (1+x)^p$ about $x = 0$

[8 marks]

(f) The equation below shows the general form of the governing equation used in CFD. Give the names of each of the four terms in the equation and list four variables that are represented by ϕ (your answers should be in words not symbols).

[4 marks]

$$\underbrace{\frac{\partial(\rho\phi)}{\partial t}}_{\text{term 1}} + \underbrace{div(\rho\phi\mathbf{u})}_{\text{term 2}} = \underbrace{div(\Gamma_\phi \; grad \; \phi)}_{\text{term 3}} + \underbrace{S_\phi}_{\text{term 4}}$$

(g) Turbulence can be modelled using the so-called eddy viscosity concept.

(i) Describe the eddy viscosity concept.

[4 marks]

(ii) Briefly describe the principle of Large Eddy Simulation (LES).

[2 marks]

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SECTION B

(Answer **ONE QUESTION**)

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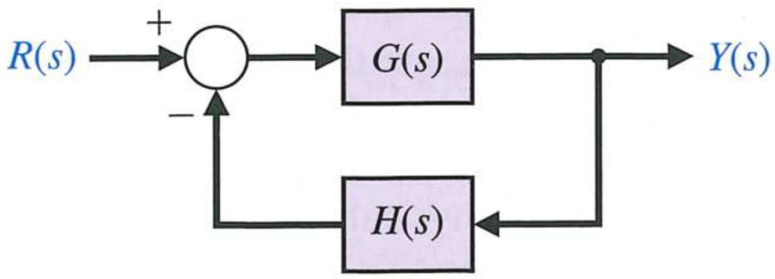
Q2.

(a) A negative feedback system is shown below. State two benefits of closed-loop feedback control, compared to an open-loop system.

[2 marks]



(b) Obtain the closed-loop transfer function for the system shown above, when process transfer function $G(s) = \frac{225}{s(s+12)}$ and sensor transfer function $H(s) = 1$.

[3 marks]

(c) For the closed-loop transfer function found in part (b), find the poles and plot them in the s-plane. State the type of system (underdamped, overdamped etc). Find the damping ratio ζ and natural frequency ω_n.

[8 marks]

(d) Consider the case when input R(s) is a unit step input. State the Laplace transform of a step input, and thus find output Y(s).

[3 marks]

(e) Considering the case when R(s) is a unit step input, use partial fractions to separate output Y(s) into parts. Take inverse Laplace transforms of these parts, and thus find the response to a unit step input in the time domain, y(t).

[8 marks]

(f) The original sensor has transfer function $H(s) = 1$. An alternative sensor is available, with transfer function $H(s) = \frac{1}{0.1s+1}$. State which of these two sensors is preferable and why.

[2 marks]

Question 2 continues/…

…/question 2 continued

(g) Explain what is meant by “linearity” and why this is important in the analysis of control systems.

[3 marks]

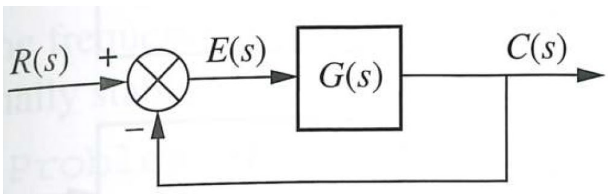
(h) Another sensor is available with transfer function $H(s) = e^{-0.2s}$. Explain why this transfer function cannot be used in its current form. Show that when linearised about s = 0, the transfer function becomes $H(s) = \frac{1-0.1s}{1+0.1s}$. (This is also known as the Padé approximation). Hint: $e^{-0.2s} = \frac{e^{-0.1s}}{e^{0.1s}}$

[4 marks]

Q3.

(a) Find the closed-loop transfer function of the system below.

[4 marks]



Where:

$$G(s) = \frac{K(s^2+1)}{(s+1)(s+2)}$$

(b) Stability is an important criterion that control systems must meet. Explain the meaning of stability, and how this is related to the position of the roots in the s-plane.

[3 marks]

(c) For the system in part (a), use the Routh-Hourwitz criterion to determine the range of values of K which will give stability.

[4 marks]

(d) Find the steady-state error e(∞) when R(s) is a unit step input and K = 3.

[6 marks]

(e) Consider a system with closed-loop transfer function $T(s) = \frac{K}{(s+1)(s+2)+K}$.
Find the sensitivity of the closed-loop transfer function to changes in K.

[6 marks]

Question 3 continues/…

(f) Explain what is meant by the term “hunting” in the context of HVAC control systems. Explain how it might occur, and why it is problematic.

[5 marks]

(g) Explain what is meant by the terms “disturbance input” and “measurement noise”. Sketch a block diagram of a system showing both a disturbance input and measurement noise. State the desired response of a control system to disturbance inputs and measurement noise.

[5 marks]

M Cook
S Watson

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LOUGHBOROUGH UNIVERSITY
SCHOOL OF ARCHITECTURE, BUILDING AND CIVIL ENGINEERING

CVC119: CONTROL, MODELLING AND SIMULATION – FORMULA SHEET AND TABLES

Tables of Laplace Transforms and Theorems

Table 2.3 Important Laplace Transform Pairs

$f(t)$	$F(s)$
Step function, $u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$
$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(s)}{s} + \frac{1}{s} \int_{-\infty}^{0} f(t)\, dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{1/2} e^{-at} \sin(\omega t+\phi)$, $\phi = \tan^{-1} \frac{\omega}{\alpha - a}$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$, $\zeta < 1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t - \phi)$, $\phi = \tan^{-1} \frac{\omega}{-a}$	$\frac{1}{s\left[(s+a)^2+\omega^2\right]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi\right)$, $\phi = \cos^{-1} \zeta$, $\zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega}\left[\frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at} \sin(\omega t+\phi)$.	$\frac{s+\alpha}{s\left[(s+a)^2+\omega^2\right]}$

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TABLE 2.2 Laplace transform theorems

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$		Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s \to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s \to \infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.
[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Performance Criteria

Name	**Symbol**	**First-order systems**	**Second-order systems**
Rise time	T_r	$\frac{2.2}{a}$	$\frac{2.16\zeta + 0.6}{\omega_n}$
Peak time	T_p	-	$\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
Percent Overshoot	$P.O.$	-	$100e^{-(\zeta\pi/\sqrt{1-\zeta^2})}$
Settling time	T_s	$\frac{4}{a}$	$\frac{4}{\zeta\omega_n}$

Second-order systems

$$T(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Continues/…

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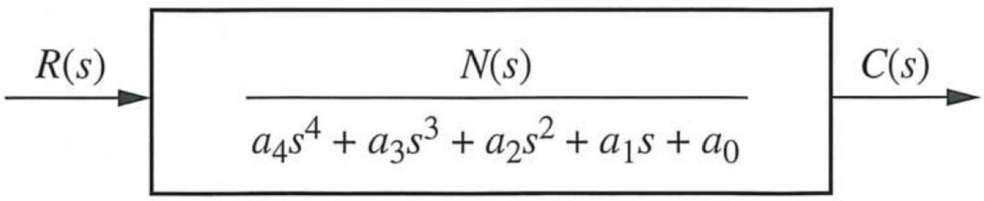
Sensitivity

$$S_{F:P} = \frac{P}{F}\frac{\delta F}{\delta P}$$

Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$
$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}.$$

Routh Array Example



s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

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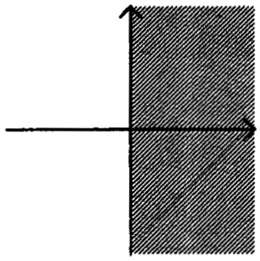
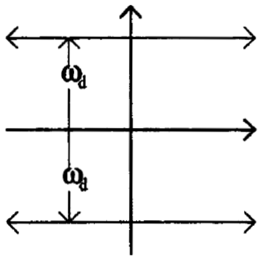
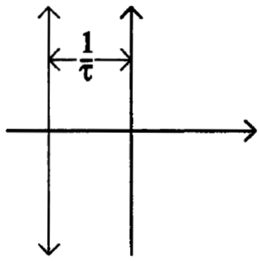
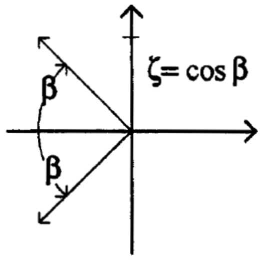
Root Locus Plot Interpretation

Name	Formula	S-plane
Region of stability	Re(s) < 0	
Constant damped natural frequency	Im(s) = ω	
Constant time constant	Re(s) = $-\zeta\omega_n$ = - 1/τ	
Constant damping ratio	Cos(β)=ζ	
Constant natural frequency	| s | = ω_n