Loughborough University

MATHEMATICS FOR MANUFACTURING ENGINEERING (23MAA306)

Semester 1 23/24 | In-Person Exam paper

Please fill in:

ID number: | Desk number:

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **two hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

Answer all questions.

Show working to gain full marks.

State your answers to 3 significant figures.

A Mathematics Formula booklet is provided.

An additional formula sheet is attached at the front of the exam paper.

FORMULA SHEET

Complex Numbers

1. If $z = r\,(\cos\theta + i\sin\theta)$ then

$$z^n = r^n\,(\cos n\theta + i\sin n\theta)$$

for any value of n.

2. The n roots of a complex number $d = r(\cos\theta + i\sin\theta)$ are

$$d^{1/n} = r^{1/n}\ \left(\cos\frac{\theta+2\pi k}{n} + i\sin\frac{\theta+2\pi k}{n}\right)$$

where $k = 0, 1, 2, ..., n-1$ and n is a natural number.

Vectors

1. The area of the triangle ABC is $\frac{1}{2}|\vec{AB}\times\vec{BC}|$.

2. The equation of a plane perpendicular to the vector $\underline{n}$ and passing through the point with position vector $\underline{a}$ is

$$\underline{r}\cdot\underline{n} = \underline{a}\cdot\underline{n}.$$

Applications of Integration

1. Volume of solids of revolution about the x-axis

$$\pi\int_a^b [f(x)]^2\,dx$$

2. If $f(x)$ is defined on the interval $a \le x \le b$, then

(a) the mean value is given by

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

(b) the root mean square (rms) value is given by

$$\sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2\,dx}$$

1. (a) Use Cramer's Rule to solve the following system of linear equations:

$$\begin{aligned} 2x + y - z &= -2 \\ x + 3y + 4z &= -17 \\ 4x - y - z &= -4 \end{aligned}$$

[15]

Tick here if you continue at the end of the booklet: ☐

(b) Use the definitions of $\cosh(x)$ and $\sinh(x)$ to prove that $\dfrac{d}{dx}(\tanh(x)) = \text{sech}^2(x)$.

Hints: (i) Note that $\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$. (ii) Write $\cosh(x)$ and $\sinh(x)$ in terms of exponential functions and differentiate the quotient. [10]

Tick here if you continue at the end of the booklet: ☐

2. (a) A curve is given parametrically by

$$x = \cos(2\theta), \quad y = \sin(2\theta).$$

Find the equation of the tangent to this curve at the point where $\theta = 0.4$ radians. [8]

Tick here if you continue at the end of the booklet: ☐

(b) (i) Use the substitution method to integrate $\int t^2 \cos(t^3)\, dt$. [5]

(ii) Hence, evaluate $\int_1^4 t^2 \cos(t^3)\, dt$ [4]

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Tick here if you continue at the end of the booklet: ☐

(c) Find the vector equation of the plane which is normal to the vector $n = -2\underline{i} - \underline{j} + 5\underline{k}$ and passes through the point with coordinates $(3, -1, 2)$. [4]

Tick here if you continue at the end of the booklet: ☐

3. (a) Given the complex numbers $z = 2 + 5i$ and $w = -3 - i$, find

   (i) $|3z + w|$, [3]

   (ii) $\dfrac{w}{z}$, writing the answer in the form $a + bi$. [4]

Tick here if you continue at the end of the booklet: ☐

(b) (i) Express the complex number $z = -1 + 4i$ in polar form (using degrees). [4]

(ii) Use De Moivre's theorem to find z^7 in polar form (using degrees). [5]

State answers to (i) and (ii) correct to 1 decimal place.

Tick here if you continue at the end of the booklet: ☐

(c) Find the volume of the solid formed if the curve $y = 3x^2 + 1$ is rotated 360° about the $x-$axis between the limits $x = -1$ and $x = 1$. [8]

Tick here if you continue at the end of the booklet: ☐