Loughborough University

Thermodynamics and Fluid Mechanics

23WSA800

Semester 2 23/24 | In-Person Exam paper

Please fill in:

ID Number:

Desk Number:

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Answer all questions.

A Thermodynamic Tables booklet will be provided.

A Formula sheet is attached to this document.

Answering Guidelines

Please answer the questions by writing your answer in the space provided for each question.

Answer 1a):

20 000

If you make a mistake or want to change your answer, please clearly cross out your original answer and write a new answer in the same box provided.

Answer e.g.:

~~20 000~~ 30000

<u>If more space is required, use extra pages at the end and make a note on the answer that rest of answer is in PAGE X.</u>

1. Figure Q1a shows a cylindrical tank filled with water ($\rho = 1000\ \mathrm{kg\,m^{-3}}$), and open to atmosphere. There is a circular hole in the bottom of the tank with diameter $d = 14\ \mathrm{mm}$. This hole is covered by a plug with negligible mass, which is pressed against the outside of the tank by a float (attached to the plug by a light string). The tank is initially filled to a height of $h_1 = 400\ \mathrm{mm}$, sufficiently high to fully submerge the float. You make take $g = 9.81\ \mathrm{m\,s^{-2}}$.

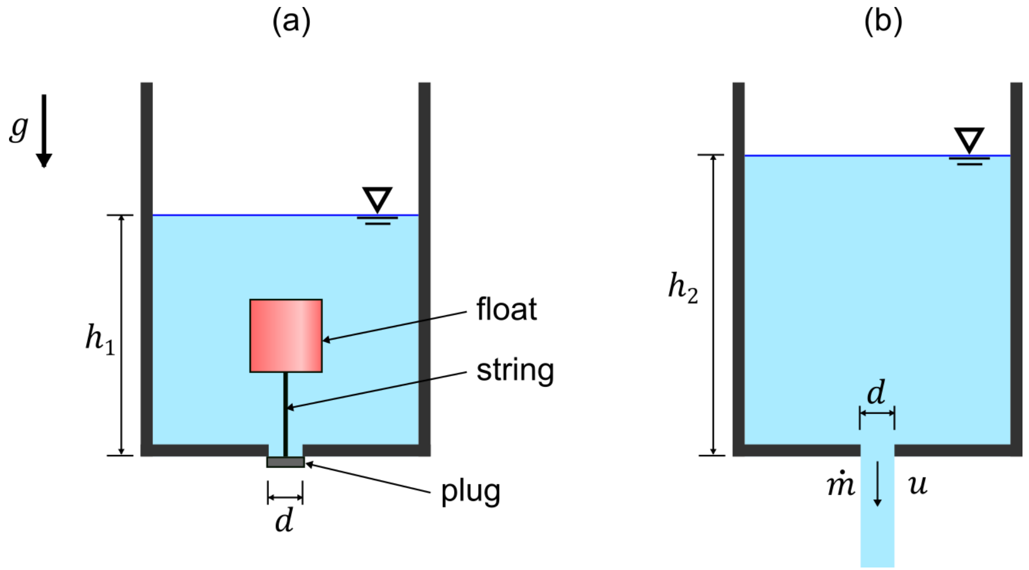


Figure Q1

a) State Archimedes principle (regarding both the magnitude of the buoyant force, and the location of the centre of buoyancy) [3 marks]

Answer 1a):

The float is a cylinder with diameter $40\ \mathrm{mm}$, and height $100\ \mathrm{mm}$, and has average density $130\ \mathrm{kg\ m^{-3}}$.

b) Calculate the buoyant force F_B acting on the float, and the tension T in the string. [5 marks]

Answer 1b):

c) Calculate the gauge pressure at the bottom of the tank, and hence calculate the net pressure force F_P acting on the plug. [5 marks]

Answer 1c):

Now more water is added to the tank, raising the filled height h

d) Calculate the filled height h at which the plug will fall away from the tank, exposing the hole. [6 marks]

Answer 1d):

Now the plug is removed from the tank and the tank is allowed to drain under gravity (see Figure Q1b, page 3)

e) Assuming the flow is ideal and inviscid, apply Bernoulli to show that the velocity of water exiting the hole can be expressed as

$$u = \sqrt{K\,gh}$$

where K is a number, and determine the value of K. [5 marks]

Answer 1e):

f) Hence, calculate the exiting velocity u and mass flowrate $\dot{m}$ of water flowing through the hole when the filled height is $h_2 = 500\ \mathrm{mm}$. [3 marks]

Answer 1f):

2. Figure Q2 shows a block resting on a frictionless surface. The curved surface of the block is impacted by a steady jet of water ($\rho = 1000 \text{ kg m}^{-3}$), that follows the curved surface and is deflected upwards. The jet has a cross-sectional area of $A = 0.04 \text{ m}^2$ and a uniform velocity of $u = 4 \text{ m s}^{-1}$. After being deflected by the angle $\theta = 30^o$, the jet has the same area and uniform velocity. The total mass of the block and fluid in contact with the block is $m = 50 \text{ kg}$. The block is held stationary by applying the force $F_{R,x}$. The flow is assumed to be inviscid. Gravitational acceleration is $g = 9.81 \text{ m s}^{-2}$.

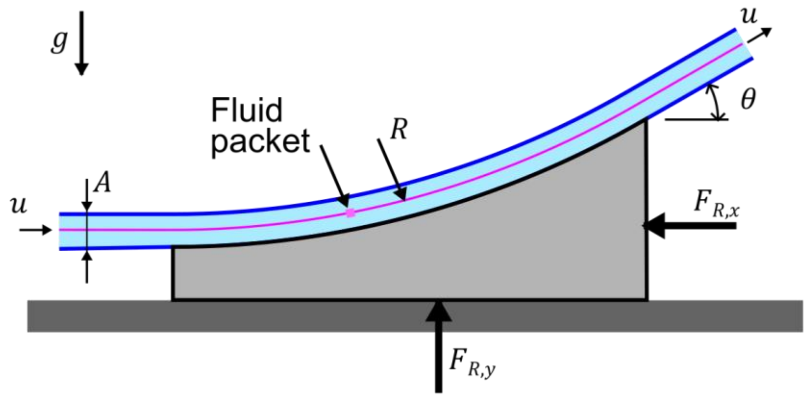


Figure Q2

a) What is the mass flow rate of water impacting the stationary block? [5 marks]

Answer 2a):

b) Calculate the force $F_{R,x}$ to hold the block stationary. [6 marks]

Answer 2b):

c) Calculate the vertical reaction force between the block and the ground $F_{R,y}$ [6 marks]

Answer 2c):

Consider a small fluid packet (or particle) that travels along the streamline in the centre of the jet (highlighted in Figure Q2, page 8). As it is deflected by the block, it follows a circular path with a radius of curvature $R = 5\ \mathrm{m}$.

d) Calculate the acceleration of the fluid packet as moves along the curved path (magnitude only) [3 marks]

Answer 2d):

e) Name the force that is responsible for the acceleration of the fluid packet [2 marks]

Answer 2e):

f) If viscosity were taken into account, what would be the fluid velocity at the surface where the fluid contacts the block and why? [2 marks]

Answer 2f):

3. Figure Q3 shows two large water ($\rho = 1000\ \mathrm{kg\ m^{-3}}$) reservoirs with an elevation difference of $\Delta z = 45\ \mathrm{m}$. It is desired to deliver the water from the lower reservoir to the higher reservoir using a pump as shown. The pump has a shaft input power of $40\ \mathrm{kW}$. Gravitational acceleration is $g = 9.81\ \mathrm{m\ s^{-2}}$.

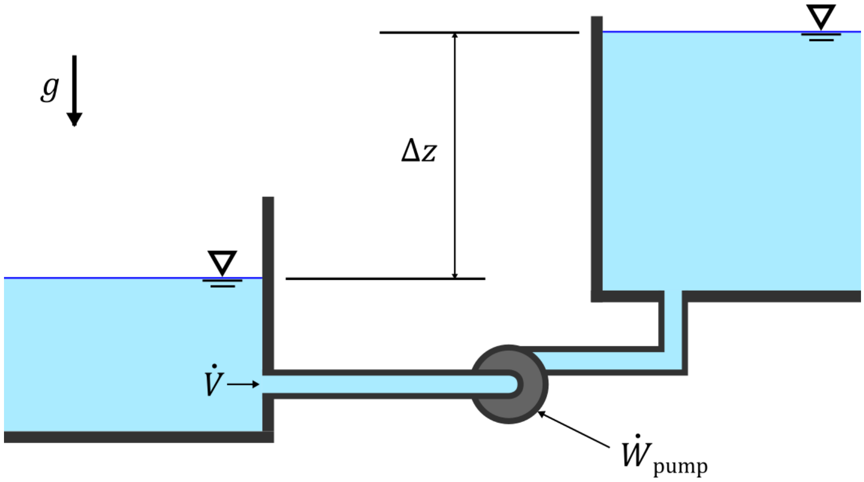


Figure Q3

a) Calculate the maximum volume flowrate $\dot{V}$ that can be delivered if the pump and piping are assumed to be ideal (i.e. pump has 100% mechanical efficiency and the piping has zero head loss) [5 marks]

Answer 3a):

b) In reality, the pump has 70% mechanical efficiency, and the total head loss in the piping is $h_L = 25\ \mathrm{m}$. Calculate the actual volume flowrate $\dot{V}$ taking these non-ideal effects into account. [5 marks]

Answer 3b):

c) Will the temperature of the fluid exiting the piping be greater than, equal to, or less than the temperature of the fluid entering the piping? Explain why. [2 marks]

Answer 3c):

The engineer responsible for this system is interested in increasing the flowrate by modifying the pump. The rotor of the pump has diameter D and rotational speed ω. The pump raises the specific mechanical energy of the fluid by Δe_{mech} and delivers a volume flowrate of $\dot{V}$. (note that $\Delta e_{\mathrm{mech}} = g h_{\mathrm{pump,u}}$).

d) List the fundamental dimensions of the parameters $D, \omega, \Delta e_{\mathrm{mech}}, \dot{V}$. [5 marks]

Answer 3d):

e) Hence, construct two dimensionless groups that describe the pump characteristics, taking D and ω as repeating variables. [5 marks]

Answer 3e):

f) If the dimensionless groups remain constant, and the pumps rotational speed is increased by 50%, by what percentage will $\dot{V}$ and Δe_{mech} change? [2 marks]

Answer 3f):

4. A heat pump (Figure Q4) with refrigerant-134a as the working fluid is used to keep a space at 25°C by absorbing heat from geothermal water that enters the evaporator at 60°C at a rate of 0.465 kg/s and leaves at 40°C. The refrigerant enters the evaporator at 12°C with a quality of 15 percent and leaves at the same pressure as saturated vapor. The compressor consumes 1.6 kW of power. Use data from the tables.

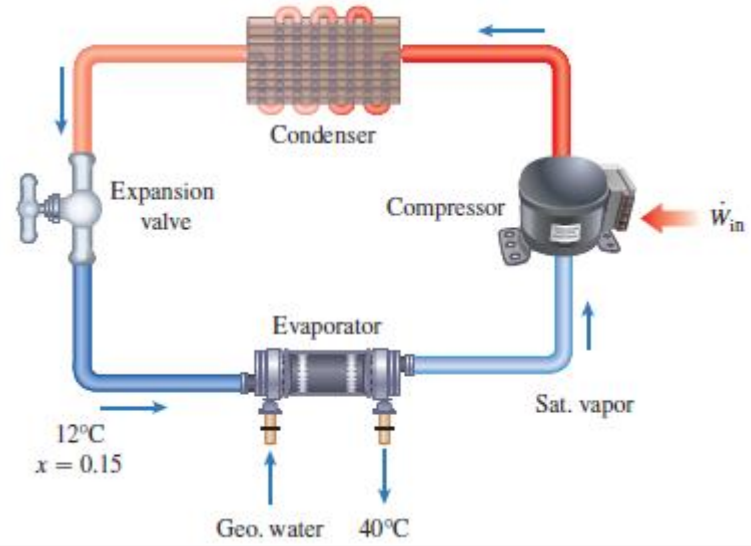


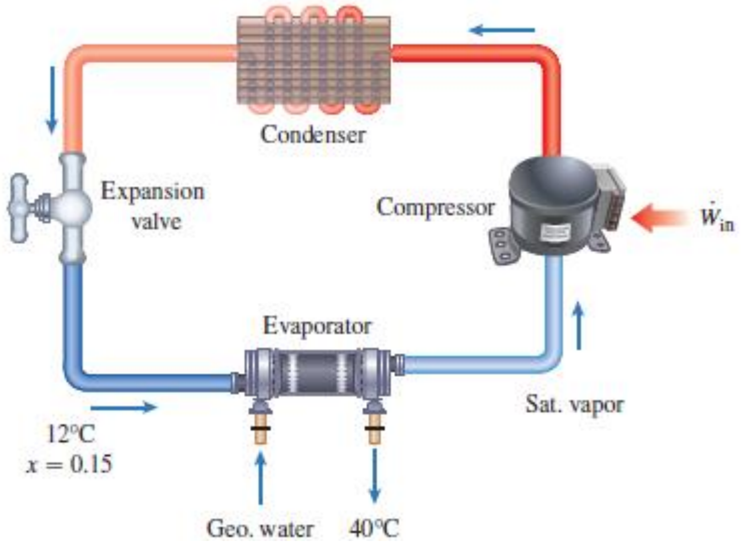
Figure Q4

a) Determine the mass flow rate of the refrigerant. (Round the final answer to three decimal places.)
Hint: As the specific heat capacity of the geothermal water is not available you must use the tables and an incompressible assumption. [10 marks]

Answer 4a):

b) Determine the rate of heat supply (Qh). Draw this heat flow on the diagram. [3 marks]

Answer 4b):



c) Determine the COP. [5 marks]

Answer 4c):

d) Determine the minimum power input to the compressor for the same rate of heat supply. [7 marks]

Answer 4d):

End of exam

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USEFUL FORMULAE - FLUID MECHANICS

Density

$$\frac{\Delta\rho}{\rho_o} = -\beta\Delta T + \alpha\Delta P$$

$$\alpha = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial P}\right)_T \qquad \beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P$$

Ideal gas

$$P = \rho RT$$

Statics

$$\frac{\partial P}{\partial x} = \rho g_x \quad , \quad \frac{\partial P}{\partial y} = \rho g_y \ , \quad \frac{\partial P}{\partial z} = \rho g_z$$

$$\vec{\boldsymbol{F}}_P = -\int_A P\vec{\mathbf{n}}\,\mathrm{d}A$$

$$\vec{\boldsymbol{F}}_B = -\rho_f V\vec{\boldsymbol{g}}$$

Viscosity

$$\tau = \mu\frac{\mathrm{d}u}{\mathrm{d}y}$$

Streamlines

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{u_y}{u_x}$$

Conservation of mass

$$\frac{\mathrm{d}m_{\mathrm{CV}}}{\mathrm{d}t} = \sum_{\mathrm{in}}\dot{m} - \sum_{\mathrm{out}}\dot{m}$$

$$\dot{m} = \int_A \rho\,(\vec{\boldsymbol{u}}_{\mathrm{rel}}\cdot\vec{\mathbf{n}})\,\mathrm{d}A$$

Conservation of energy

$$\frac{\mathrm{d}E_{\mathrm{CV}}}{\mathrm{d}t} = \dot{Q} - \dot{W} + \sum_{\mathrm{in}}\dot{m}\left(e_{\mathrm{int}} + \frac{P}{\rho} + \alpha\frac{u_{\mathrm{avg}}^2}{2} + gz\right) - \sum_{\mathrm{out}}\dot{m}\left(e_{\mathrm{int}} + \frac{P}{\rho} + \alpha\frac{u_{\mathrm{avg}}^2}{2} + gz\right)$$

$$\frac{P_1}{\rho g} + \alpha_1\frac{u_{\mathrm{avg,1}}^2}{2g} + z_1 + h_{\mathrm{pump,u}} = \frac{P_2}{\rho g} + \alpha_2\frac{u_{\mathrm{avg,2}}^2}{2g} + z_2 + h_{\mathrm{turb,e}} + h_L$$

$$\frac{P}{\rho} + \frac{u^2}{2} + gz = C$$

$$\eta_{\mathrm{pump}} = \frac{\rho\dot{V}gh_{\mathrm{pump,u}}}{\dot{W}_{\mathrm{pump}}} \qquad \eta_{\mathrm{turb}} = \frac{\dot{W}_{\mathrm{turb}}}{\rho\dot{V}gh_{\mathrm{turb,e}}}$$

Conservation of momentum

$$\frac{\mathrm{d}(m\vec{\boldsymbol{u}})_{\mathrm{CV}}}{\mathrm{d}t} = \sum_{\mathrm{in}}\beta\dot{m}\vec{\boldsymbol{u}} - \sum_{\mathrm{out}}\beta\dot{m}\vec{\boldsymbol{u}} + \sum\vec{\boldsymbol{F}}$$

$$\frac{\mathrm{d}\vec{\boldsymbol{H}}_{\mathrm{CV}}}{\mathrm{d}t} = \sum_{\mathrm{in}}\vec{\boldsymbol{r}}\times\dot{m}\vec{\boldsymbol{u}} - \sum_{\mathrm{out}}\vec{\boldsymbol{r}}\times\dot{m}\vec{\boldsymbol{u}} + \sum\vec{\boldsymbol{M}}$$

DIMENSIONAL ANALYSIS AND PHYSICAL SIMILARITY

For similarity: $(\Pi_k)_{\mathrm{model}} = (\Pi_k)_{\mathrm{prototype}}$, for k dimensionless groups

Table of fundamental dimensions:

Dimension	Dimension symbol
Length	$\{\mathrm{L}\}$
Mass	$\{\mathrm{m}\}$
Time	$\{\mathrm{t}\}$
Temperature	$\{\mathrm{T}\}$
Amount of matter	$\{\mathrm{N}\}$
Dimensionless	$\{1\}$

Table of common dimensionless numbers:

Name	Definition	Name	Definition
Euler number, Eu	$\Delta P/(\rho u^2)$	Darcy friction factor, f	$8\tau/(\rho u^2)$
Reynolds number, Re	$\rho u L/\mu$	Drag coefficient, C_D	$2F_D/(\rho u^2 A)$
Froude number, Fr	$u/\sqrt{gL}$	Lift coefficient, C_L	$2F_L/(\rho u^2 A)$
Weber number, We	$\rho u^2 L/\sigma_s$	Rayleigh number, Ra	$g\beta\Delta T L^3/(\alpha\nu)$
Mach number,	u/c	Knudsen number, Kn	λ/L

INTEGRATION

Integrand	**Integral**
x^n	$\dfrac{x^{n+1}}{n+1}$
$\dfrac{1}{x}$	$\ln(x)$
$\exp(x)$	$\exp(x)$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\tan(x)$	$-\ln(\cos(x))$
$\dfrac{f'(x)}{f(x)}$	$\ln\big(f(x)\big)$

USEFUL FORMULAE IN THERMODYNAMICS

Basics:

Specific volume $v = \frac{V}{m}$ $\left[\frac{m^3}{kg}\right]$

Specific Internal Energy $u = \frac{U}{m}$ $\left[\frac{kJ}{kg}\right]$

Specific Enthalpy $h = \frac{H}{m}$ $\left[\frac{kJ}{kg}\right]$

Specific Entropy $s = \frac{S}{m}$ $\left[\frac{kJ}{kg.K}\right]$

Constants:

Universal Gas Constant $R_u = 8.314$ $[\frac{kJ}{kmol}.K]$

Specific Gas Constant $R = \frac{R_u}{MW}$ $[\frac{\text{kJ}}{\text{kg}}.\text{K}]$

Quality:

$$u_1 = u_f + xu_{fg}$$
$$h_1 = h_f + xh_{fg}$$
$$s_1 = s_f + xs_{fg}$$
$$v_1 = (1-x)v_f + xv_g$$

1st Law of Thermodynamics

Cycle $\oint \delta Q = \oint \delta W$

Closed System ${}_1Q_2 = U_2 - U_1 + {}_1W_2$

Open System

$$\dot{Q} + \sum \dot{m}_i \left(h_i + (V_i^2/2) + Z_i g\right) = \dot{W} + \sum \dot{m}_e \left(h_e + (V_e^2/2) + Z_e g\right) + \frac{dE_{CV}}{dt}$$

$$\dot{m} = \rho AV = AV/v$$

2nd Law of Thermodynamics

Inequality of Clausius $\oint \frac{\delta Q}{T} \le 0$

Closed System $dS = \frac{\delta Q}{T} + \frac{\delta I}{T}$

Steady State flow Adiabatic Process (single inlet and exit) $s_e \ge s_i$

Enthalpy $h = u + Pv$

Ideal Gases

$$PV = mRT$$
$$C_p - C_v = R$$
$$k = \frac{C_p}{C_V}$$
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = ...$$
$$N = \frac{m}{M}$$
$$u_2 - u_1 = \int_1^2 C_v(T).dT$$
$$h_2 - h_1 = \int_1^2 C_p(T).dT$$
$$s_2 - s_1 = \int_1^2 C_p(T)/T.dT - R\ln(P_2/P_1)$$

For Ideal Gases undergoing Reversible Processes

Isothermal Process : $Pv =$ constant

Polytropic Process : $Pv^n =$ constant

$T_2/T_1 = \left(v_1/v_2\right)^{n-1} = \left(P_2/P_1\right)^{(n-1)/n}$

Adiabatic Process : $Pv^k =$ constant {when constant average specific heats are used}

Boundary (Displacement) Work in Reversible, Closed System Processes

$${}_1W_2 = \int_1^2 PdV$$

Isothermal Process : ${}_1W_2 = PV\ln(V_2/V_1)$

Polytropic Process : ${}_1W_2 = (P_2V_2 - P_1V_1)/(1-n)$

Adiabatic Process : ${}_1W_2 = (P_2V_2 - P_1V_1)/(1-k)$ {when constant average specific heats are used}

Work During Reversible, Steady State, Steady Flow Processes

$$w_{rev} = -\int_i^e vdP + \left(V_i^2 - V_e^2\right)/2 + (Z_i - Z_e)g$$

Efficiency:

$$\eta = \frac{Desired\ Output}{Required\ Input} \qquad \eta_{Heat\ Engine} = 1 - \frac{Q_L}{Q_H} \qquad COP_R = \frac{1}{\frac{Q_H}{Q_L} - 1} \qquad COP_{HP} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$