Loughborough University

Control Systems Design 2

23WSB105

Semester 2

In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

Answer **ALL FOUR** questions.

Questions carry the marks shown.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the rear of the paper.

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1.

Sketch approximate Bode plots for the following transfer functions, marking relevant frequencies, noting the presence or absence of any resonant peaks, and giving the gradients of any asymptotes.

a)

i. $G_1(s) = 100\frac{(1+s)}{s(1+s/0.1)}$ [6 marks]

ii. $G_2(s) = \frac{9}{s(s^2+3s+9)}$ [6 marks]

b) Show that the magnitude and argument of $G_2(j\omega)$ at $\omega = 3$ rad/s is $|G_2(j\omega)| = 1/3$ and $\angle G_2(j\omega) = -180^o$. [4 marks]

Note: You may either use your Bode plot from part a) ii. or calculate these values directly from the transfer function.

c) State and briefly explain if $G_2(s)$ is:

i. Closed-loop stable under unity feedback. [4 marks]

ii. Closed-loop stable under proportional negative feedback with a gain $K = 10$. [5 marks]

2.

An engineer working for a manufacturer of commercial quadrotor drones is tuning an Electronic Speed Controller (ESC), which regulates rotor speed by applying current to a brushless DC motor. Due to the complex aerodynamics, the engineer does not know a transfer function for the rotor, so instead carries out an experiment.

With the rotor operating at a nominal speed, the engineer inputs sine waves of amplitude 1A to the rotor's brushless DC motor circuit and measures the amplitude and phase (relative to the input) of the changes in rotor speed. Their results are shown in Table Q2.

Frequency (Hz)	**Output Amplitude (x100 RPM)**	**Output Phase (deg)**
0.10	10.99	-3.91
0.32	10.86	-12.3
1.00	9.74	-37.2
3.16	5.14	-89.9
10.00	1.01	-145.6
31.62	0.11	-179.9
100.00	0.01	-216.0

Table Q2

As a first attempt at speed control, the engineer sets the ESC to implement proportional feedback of $K = 0.2\ A/(100RPM)$.

a) Calculate the approximate gain and phase margins with this proportional controller. [4 marks]

b) The engineer then tunes the controller by increasing K and observing the closed-loop step response. State and briefly explain the likely effect of increasing K on:

- The rise time and steady-state speed error.
- Closed-loop stability and damping.

[6 marks]

For acceptable performance of the drone in flight, the rise time of the closed-loop step response should be at most 0.04 seconds and the overshoot should be 5% or less.

c) Considering these specifications, choose a target 0dB crossover frequency and phase margin for controller design. You may express the frequency in either rad/s or Hz. [6 marks]

d) Suggest a type of feedback controller to meet this specification, justifying your choice (do not calculate the controller parameters) [5 marks]

e) Do you think it is possible to meet these specifications using only proportional control? Justify your answer. [4 marks]

3.

To regulate the rotation speed $Y(s)$ of a 10MW wind turbine to its rated speed $R(s)$, a closed-loop feedback control scheme is used. The structure of the feedback loop is as shown in Figure Q3(a).

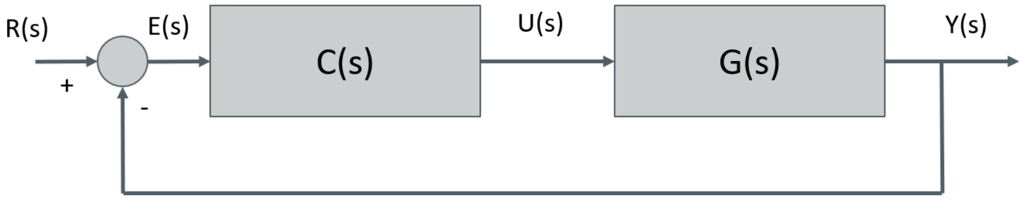

Figure Q3(a)

Using a simulation model of the turbine dynamics, a control engineer investigates the frequency response to changes in the pitch angle $U(s)$ of the blades. The resulting Bode plot of the turbine $G(s)$ is shown in Figure Q3(b).

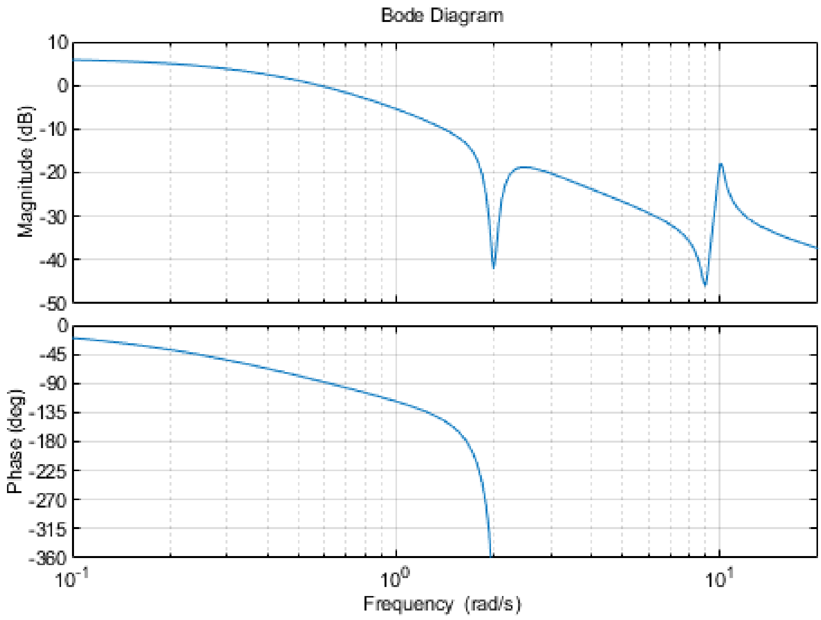

Figure Q3(b)

The engineer first investigates the behaviour of a speed controller using unity feedback, $C(s) = 1$, to set the pitch angle.

a) Estimate the resulting gain and phase margins. [4 marks]

b) Estimate the resulting steady-state error to a unit step change in the reference signal $R(s)$. [3 marks]

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To reduce the steady-state error, the engineer decides to implement a PI controller with a design crossover frequency of 0.6 rad/s and a target phase margin of 60 degrees.

c) For this choice of design frequency and phase margin, calculate suitable PI controller parameters K and T_i. State the transfer function of the completed controller. [8 marks]

d) Estimate the rise time and overshoot of the step response of the PI-controlled system. Using this information, sketch the closed-loop response to a unit step change in $R(s)$, marking the rise time and overshoot on your sketch. [6 marks]

The engineer's line manager suggests using a PD controller instead with an increased design frequency to give a greater value for K, stating that the rise time should be 1 second or less.

e) Is it possible to design a PD controller for a 1 second rise time for this turbine? Explain why using Figure Q3(b).

[4 marks]

4.

The transfer function $G(s)$ of a second-order system incorporating a time delay was derived based on a physical model of the system, then a suitable compensator $C(s)$ was designed. Figure Q4 shows the resulting Bode plot of $C(j\omega)G(j\omega)$ for the compensated system.

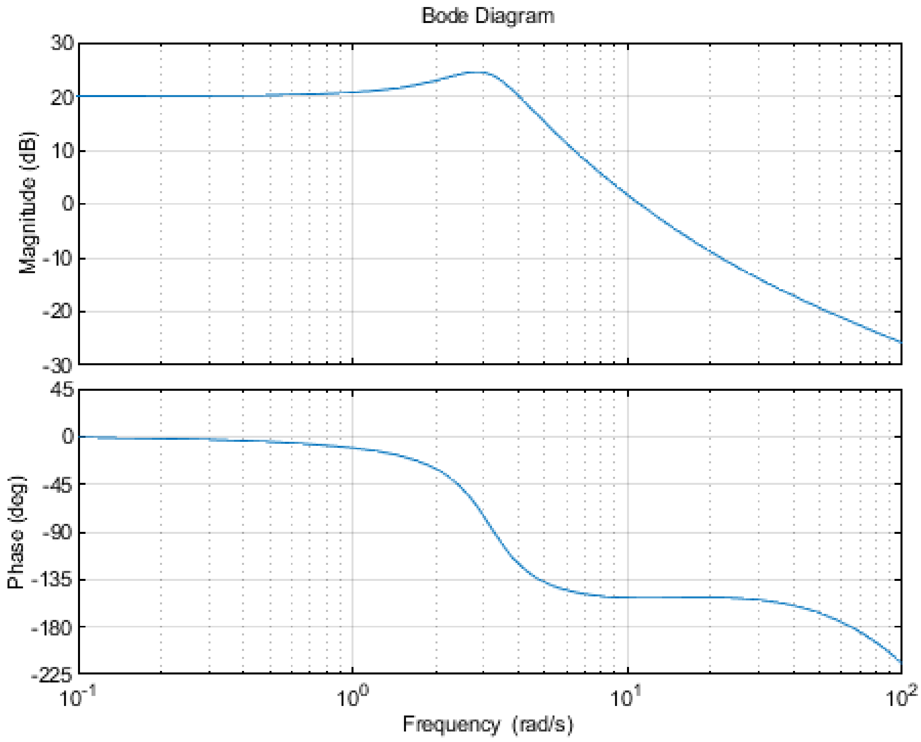

Figure Q4

a) Using the Bode plot, estimate the gain and phase margins for this choice of controller, and comment on the robustness of the design if there is uncertainty about the length of the time delay. [7 marks]

b) Assuming the resulting closed-loop system is well approximated by a second-order system, estimate the overshoot, peak time, and steady-state error of the closed-loop step response. [9 marks]

c) Sketch the closed-loop response to a unit step, indicating the overshoot, peak time, and steady-state error. [9 marks]

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T Harrison

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Useful information (control systems)

Selected Laplace transforms

t-domain (t>0)	***s-domain***
Unit step	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\ \sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\ \cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Some results about Laplace Transforms

$f(t)$	$F(s)$
$f(t-T)$	$e^{-Ts}\,F(s)$
$e^{-at}\,f(t)$	$F(s+a)$
$f'(t)$	$sF(s)-f(0)$
$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0}\,\{s\,F(s)\}$

Step response of second-order systems *- For systems well approximated by* $\frac{A\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$

Percentage overshoot:

$$O_{\%} = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Peak time:

$$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

Settling time to p% of steady-state:

$$T_s = \frac{\ln(100/p)}{\zeta\omega_0}$$

Rise time:

$$T_r \approx \frac{2}{\omega_0}$$

With phase margin ϕ_{PM} in degrees, $\zeta \approx \phi_{PM}/100$ for $\phi_{PM} < 60^o$, and $\phi_{PM} = 65^o$ gives $\zeta \approx 0.7$

Common SISO compensator types (PID and related controllers)

PID controller:

$$C_{pid}(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right)$$

PI controller:

$$C_{pi}(s) = K\left(1 + \frac{1}{T_i s}\right)$$

Lag compensator:

$$C_{lag}(s) = K\frac{s + 1/T_i}{s + p_{lag}}$$

PD controller:

$$C_{pd}(s) = K(1 + T_d s)$$

Lead compensator:

$$C_{lead}(s) = K\frac{T_d s + 1}{\frac{s}{p_{lead}} + 1}$$

Frequency-domain design formulae for PID-type controllers

where G_C, ϕ_C are required controller gain and phase at a chosen design frequency ω_d

PI controller:

$$T_i = \frac{1}{\omega_d \tan(-\phi_C)}$$

PID controller:

$$\tan\phi_C = T_d\omega_d - \frac{1}{T_i\omega_d}$$

PD controller:

$$T_d = \frac{\tan\phi_C}{\omega_d}$$

Controller gain (all cases):

$$K = \frac{G_c}{\sqrt{1 + \tan^2\phi_C}}$$