Heat Transfer

23WSB801

Semester 2 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

Answer **ALL** questions.

Questions carry the marks shown.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the rear of the paper.

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1. A PC CPU chip requires cooling in order to operate effectively. You have been asked to design the cooling system for a chip that is 25 mm x 25 mm and produces 20 W of heat that needs to be rejected to the surrounding air. The maximum temperature that the chip can operate at is 75°C. The ambient air temperature is 35°C. The properties of air are:

$\mu = 2.10^{-5}$ kg.m^{-1}.s^{-1}; $C_p = 1007$ J.kg^{-1}.K^{-1}; $\rho = 1.06$ kg.m^{-3}; $k = 0.028$ W.m^{-1}.K^{-1}

Recall that:

Actual fin with adiabatic tip	Ideal fin	Fin efficiency	Finned Surface
$\dot{Q} = \sqrt{hPkA_c}.\theta_b.\tanh(mL)$ $m = \sqrt{hP/kA_c}$	$\dot{Q} = hA_{fin}\theta_b$	$\eta = \frac{\dot{Q}_{fin\,actual}}{\dot{Q}_{fin\,ideal}}$	$\dot{Q} = h(A_{unfin} + \eta_{fin}A_{fin})\theta_b$

a) For cooling you will use fins mounted on the unit. The fins, 1 mm thick, are manufactured from aluminium ($k_{fin} = 50$ W.m^{-1}.K^{-1}, $\rho = 2500$ kg.m^{-3}); and their dimensions are shown in **Figure Q1**. The heat transfer coefficient for free convection from the finned surface (based on the height of the fins) is 35 W.m^{-2}.K^{-1}. Calculate the efficiency of a single fin, assuming that heat transfer from the fin tip is negligible. [8 marks]

b) Calculate the minimum number of real fins required to keep the chip surface at a temperature at or below 75°C. [8 marks]

c) What other method of cooling could you choose? What are advantages/disadvantages of using fins. [4 marks]

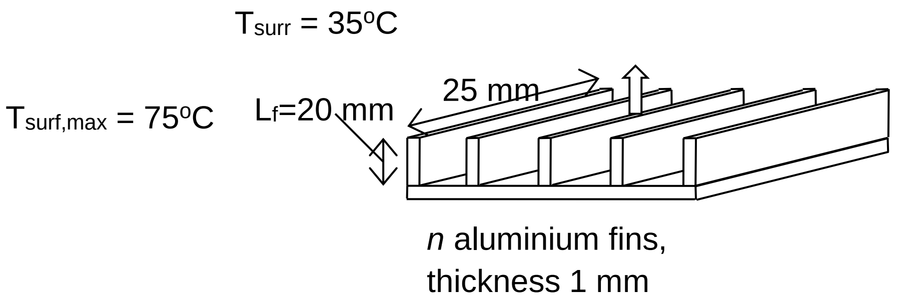

Figure Q1

2. A single shell, two tube pass heat exchanger is used to heat the water entering at $t_{c\text{-}in}$ = 15 °C with the flow rate m_c = 2 kg.s^{-1}. The heating fluid is ethylene glycol entering at $t_{h\text{-}in}$ = 85 °C and mass flow rate m_c = 1 kg.s^{-1}. The overall heat transfer coefficient U_m = 500 W.m^{-2}.K^{-1} and the heat transfer surface is 10 m^2. The specific heat capacity for water is 4180 J.kg^{-1}.K^{-1} , and for ethylene glycol 2600 J.kg^{-1}.K^{-1}. Use the NTU method to calculate:

a) the rate of heat transfer [11 marks]

b) the outlet temperatures of the water and ethylene glycol [4 marks]

3. Consider a steady-state thermal management system, shown in **Figure Q3**. where the heat (with heat flux q_w, $\mathrm{W\,m^{-2}}$) reaching a flat solid surface is dissipated through the evaporation of a thin water film in gases. The right amount of water is fed to the liquid film from the side to maintain a constant thickness δ, during the evaporation process. The top boundary, which is H meters away from the liquid surface, remains at constant pressure p and constant temperature $T_H = T_{sat}(p)$, where T_{sat} is the saturation temperature at p.

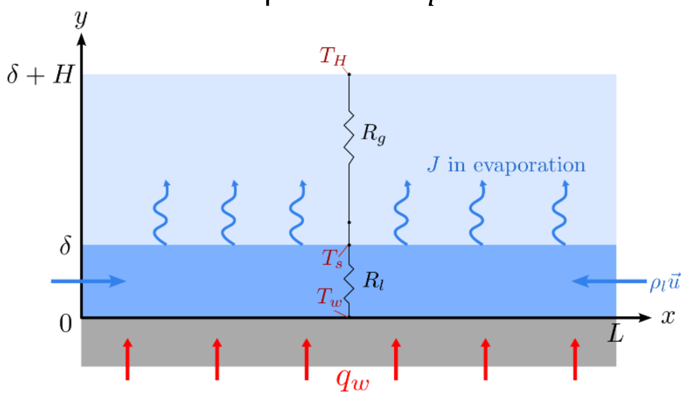

Figure Q3 Heat transfer through a thin water film.

The heat transfer process through the liquid film and the gas domain can be represented by an electrical analogy network, also shown in fig. Q3, where the temperature differences, the heat flux, and thermal resistances (R_l in the liquid film and R_g in the gas domain) depend on the heat transfer process.

Properties of liquid water can be taken as constants: thermal conductivity $k_l = 0.677\ \mathrm{W\,m^{-1}K^{-1}}$, latent heat of evaporation $L = 2257\ \mathrm{kJ\,kg^{-1}}$, thermal diffusivity $\alpha_l = 0.167\ \mathrm{mm^2s^{-1}}$. The Antoine equation can be used to correlate T_{sat} (in the unit of K) and the saturation pressure p_{sat} (in the unit of bar): $\log_{10} p_{sat} = 6.2096 - 2354.7/(T_{sat} + 7.559)$.

Firstly, let's consider the case where the water is evaporating in pure water vapours ($p = 1$ bar).

a) Derive the expression for the thermal resistance in the liquid film R_l as a function of the thermal conductivity k_l and thickness δ. The temperature distribution for a sufficiently thin liquid film can be shown to be linear, even if there is a flow at the side. [2 marks]

b) If the liquid surface is assumed to be at the local thermodynamic equilibrium (i.e. surface temperature T_s equals to T_{sat}), calculate q_w when $\delta = 1$ mm and the wall temperature T_w is 50 K above T_{sat}. [2 marks]

c) If the local equilibrium assumption is replaced by the more accurate Hertz-Knudsen model: $J = \frac{1}{\sqrt{2\pi R_s T_s}}[p_{sat}(T_s) - p]$, where J is the evaporation mass flux (mass flow rate per unit area) and R_s is the specific gas constant for water vapour, derive an expression to calculate T_s based on the conservation of energy across the liquid surface. Assume the heat transfer in the gas domain is negligibly small. [3 marks]

d) Determine whether the heat flux due to thermal radiation q_w^{rad} is negligibly small compared to q_w calculated in b) (i.e. $\leq 5\%$ of the total heat flux q_w). The maximum amount of thermal radiation exchange can be taken as that between two perfect black bodies at T_w and T_H. The viewing factors can be taken as 1 and the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. [2 marks]

Now, let's consider the case where the water is evaporating in nitrogen and the total system pressure p is kept at 1 bar.

e) Assume the liquid surface is at the local thermodynamic equilibrium again and the water vapour mole fraction is 100% at the surface of the liquid film, calculate q_w for a 1 mm thick liquid film when T_w is 50 K above the saturation temperature at 1 bar pressure. [2 marks]

f) If the water vapour mole fraction is now dropped to 50%, calculate q_w while δ and T_w remains the same as in part e). [2 marks]

g) Assume a linear mass fraction distribution for nitrogen, calculate the mass flux of water vapour due to mass diffusion. The mass fraction of nitrogen at the top boundary is 100% and the mass fraction at the liquid surface is 20%. The gas domain height $H = 1\ \mathrm{mm}$ and the mass diffusivity between water vapour and nitrogen $D = 23.5\ \mathrm{mm^2 s^{-1}}$. [2 marks]

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Equations that may be of use:

General conduction equation	One-dimensional	
	Rectangular coordinates	$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+q_g=\rho c\frac{\partial T}{\partial t}$
	Cylindrical coordinates	$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+q_g=\rho c\frac{\partial T}{\partial t}$
Steady state 1-D conduction equation		
	Rectangular coordinates	$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+q_g=0$
	Cylindrical coordinates	$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+q_g=0$
Resistances in electrical analogy method $\dot{Q}=\frac{\Delta T}{R_{equivalent}}$ **Serial** $R_{equivalent}=\sum_i R_i$	Plane slab	Resistance $\frac{L}{kA}$
	Cylindrical thickness	Resistance $\frac{\ln(b/a)}{2\pi k.H}$
	Convection at a boundary	Resistance $\frac{1}{hA}$
Heat transfer from fins	Governing equation	$\frac{d^2T}{dx^2}-\frac{hP}{kA_c}\left(T-T_{ref}\right)=0$ or $\frac{d^2\theta}{dx^2}-m^2\theta=0$ where $m^2=\frac{hP}{kA_c}$ and $\theta=(T-T_{ref})$
	Solution to governing equation	$\theta=C_1e^{mx}+C_2e^{-mx}$ or $\theta=C_1\sinh(mx)+C_2\cosh(mx)$
	Adiabatic tip	Solution: $\frac{\theta}{\theta_b}=\frac{\cosh m(L-x)}{\cosh mL}$ Heat Transfer: $Q=\sqrt{hPkA_c}.\theta_b.\tanh(mL)$
Lumped system analysis	Governing equation	$\frac{dT}{dt}=\frac{Q(t)}{\rho Vc}$
	For cooling objects under Convection	$\frac{dT}{dt}+\frac{hA}{\rho Vc}\left(T-T_a\right)=0$ or $\frac{d\theta}{dt}+m\theta=0$ where $\theta=T-T_a$ and $m=\frac{hA}{\rho Vc}$
	Solution to governing equation	$\theta=\theta_0e^{-mt}$
	Biot number	$Bi=\frac{h}{k/L}$
	For lumped capacity method	$Bi<0.1$

/continued

Equations that may be of use (continued):

Convective heat transfer non-dimensional parameters

	[L] is an appropriate dimension	
	Reynolds number	$\mathrm{Re} = \frac{\rho U[L]}{\mu}$ or $\mathrm{Re} = \frac{U[L]}{\nu}$
	Nusselt number	$Nu = \frac{h[L]}{k}$
	Prandtl number	$\mathrm{Pr} = \frac{C_p \mu}{k}$
	Grashof number	$Gr = \frac{g\beta(T - T_\infty)[L^3]}{\nu^2}$
	Rayleigh number	$Ra = Gr.\mathrm{Pr}$

Forced Convection Correlations - Forced Convection $Nu = C.\mathrm{Re}^m\,\mathrm{Pr}^n\,.K$

	C	M	N	K	Conditions
Circular tube Internal Flow	1.86	1/3	1.3	$(d/l)^{1/3}(\mu/\mu_w)^{0.14}$	Laminar flow short tube Re < 2000
	3.66	0	0	1	Laminar flow long tube Re < 2000
	0.023	0.8	0.4	1	Turbulent flow Re > 2000
Circular Tube Air in cross flow	0.538	0.47	0	1 for air Other fluids multiply Nu by $1.1\,\mathrm{Pr}^{1/3}$	Laminar flow Re < 500
	0.240	0.6	0		Turbulent flow 500 < Re < 50000
Flat plates	0.332 0.644	½ ½	1/3 1/3	(local) 1 (mean)	Laminar flow Re < 5 x 10^5
	0.029 0.037	4/5 4/5	1/3 1/3	(local) 1 (mean)	Turbulent flow Re > 5 x 10^5

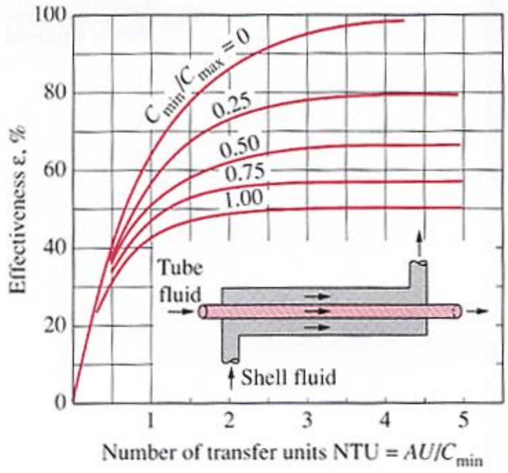
Free Convection correlations $Nu = C.(Gr\,\mathrm{Pr})^n.K$

	C	N	K	Conditions
Horizontal cylinder	0.47	¼	1	Laminar flow $Gr_D.\mathrm{Pr} < 10^9$
	0.10	1/3	1	Turbulent $Gr_D.\mathrm{Pr} > 10^9$
Vertical cylinder D << L	1.37	0.16	$(D/L)^{0.16}$	For $Gr_D.\mathrm{Pr} < 10^4$
	0.60	¼	$(D/L)^{1/4}$	For $Gr_D.\mathrm{Pr} > 10^4$
Vertical cylinder D >> L	0.59	¼	Also applies to Vertical plates	Laminar $Gr_L.\mathrm{Pr} < 10^9$
	0.13	1/3		Turbulent $Gr_L.\mathrm{Pr} > 10^9$
Horizontal Plate (note Le = Plate area/Perimeter)	0.54	¼	Hot-side up	Laminar $Gr_{Le}.\mathrm{Pr} < 10^9$
	0.14	1/3	Hot-side up	Turbulent $Gr_{Le}.\mathrm{Pr} > 10^9$
	0.25	¼	Hot-side down	Laminar $Gr_{Le}.\mathrm{Pr} < 10^9$

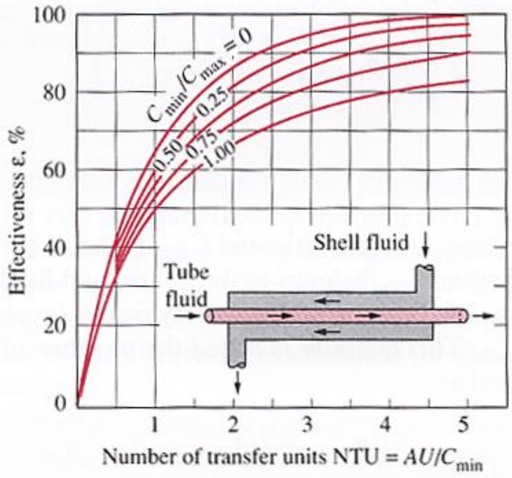
Heat Exchangers

LMTD $\Delta T_m = (\Delta T_1 - \Delta T_2)/\ln(\Delta T_1/\Delta T_2)$	Effectiveness $\dot{Q}^* = [(mC_P)_{\min}(\Delta T)]$ $\varepsilon = \dot{Q}/\dot{Q}^*$
$NTU = AU_m/C_{\min}$	Stream capacity ratio $C = C_{\min}/C_{\max}$

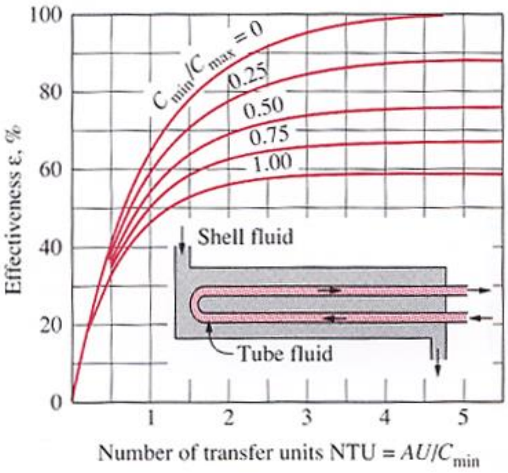
Graphs that may be of use:



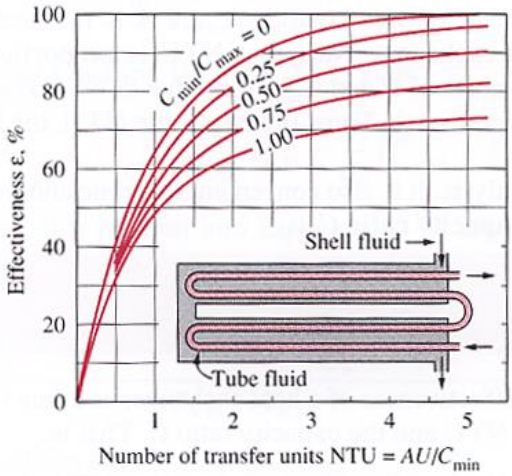
(*a*) Parallel-flow



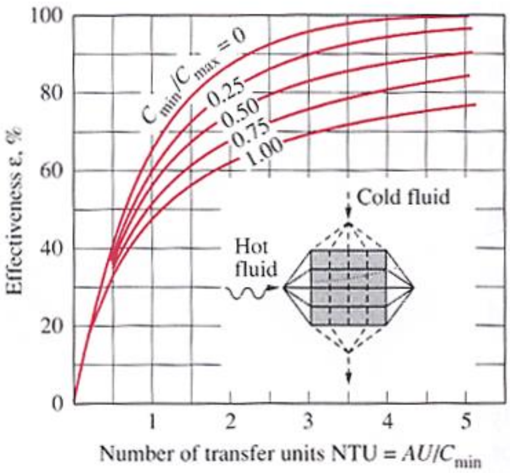
(*b*) Counter-flow



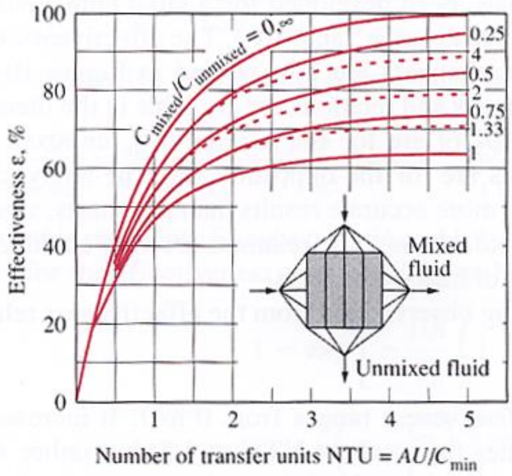
(*c*) One-shell pass and 2, 4, 6, tube passes



(*d*) Two-shell passes and 4, 8, 12, tube passes



(*e*) Cross-flow with both fluids unmixed



(*f*) Cross-flow with one fluid mixed and the other unmixed

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