Fluid Mechanics 2

23WSB802

Semester 2 23/24 In-Person Exam paper

Please fill in:

ID Number: Desk Number:

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may not write in pencil for this exam.

Any additional work must be done in the space provided at the back of this paper.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Please answer all questions.

1. The setup of a typical rotameter used to measure the mass flow rate $\dot{m}$ of an incompressible fluid is show in **Fig.Q1**. It is composed of a vertical conical section full of fluid flowing upwards against the gravity, and a steel sphere (radius $a = 5\ \mathrm{mm}$, density $\rho_s = 7500\ \mathrm{kg\ m^{-3}}$) which is free to move along the z-axis of the cone between to positions $z_{min} = 0\ \mathrm{cm}$ and $z_{max} = 20\ \mathrm{cm}$. The sphere is guided along the cone axis by a thin wire, the effect of which can be neglected.

The sphere will reach different equilibrium position at different flow rates due to the change of the cross-sectional areas. For the sake of simplicity, we consider the fluid velocity is uniform over a given cross section, i.e. the velocity $\vec{\boldsymbol{u}}$ is a function of z only and can be written $u(z) = \dot{V}/A(z)$, where $\dot{V}$ is the volume flow rate of fluid and $A(z)$ is the cross-sectional area at altitude z.

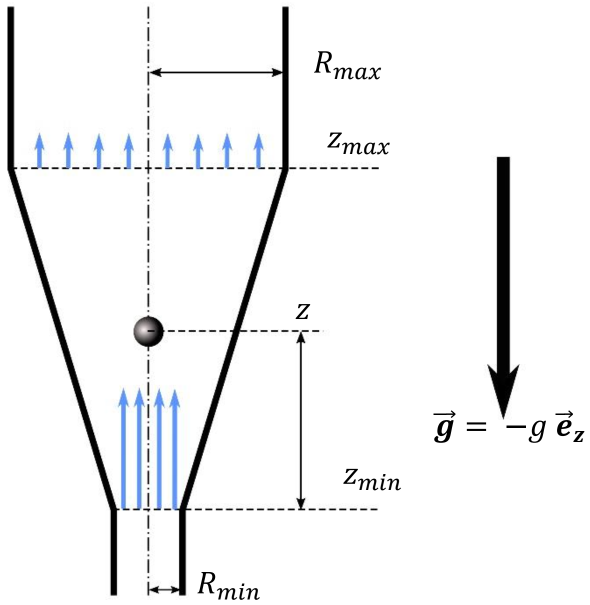

Fig. Q1 Configuration of the system. The flowmeter is placed vertically, with the fluid flowing upwards.

We first consider the case where the fluid is water (density $\rho_w = 10^3 \text{ kg m}^{-3}$, viscosity $\mu_w = 10^{-3} \text{ Pa s}$).

a) Calculate the fluid velocity U_∞ at which the metal bead remains at a steady fixed position, given that the drag coefficient of the bead is $C_D = 0.47$. [5 marks]

Tick here if you continue at the end of the booklet: ☐

b) Calculate the corresponding Reynolds number, based on the sphere radius and the velocity U_∞ previously calculated. Are we dealing with the laminar flow regime, which corresponds to $100 < Re < 5 \times 10^5$? What will happen to the drag coefficient when the flow just reaches the turbulent regime and why? [7 marks]

Tick here if you continue at the end of the booklet: ☐

c) Given that the radius of the cross section of the cone is $R_{min} = 2\ \mathrm{cm}$ at $z = z_{min}$ and $R_{max} = 8\ \mathrm{cm}$ at $z = z_{max}$, calculate the minimum flow rate $\dot{m}_{min}$ and maximum mass flow rate $\dot{m}_{max}$ which can be measured in the case where water is used. What is the largest percentage of error in measuring the mass flow rate if the volume occupied by the stainless-steel sphere is neglected? [6 marks]

Tick here if you continue at the end of the booklet: ☐

d) Derive the expression of the area of the cross section (area of the section of the cone at the altitude z) as a function of R_{min}, R_{max} and z_{max}, knowing that z_{min} is taken as the origin of the z axis. What is the altitude of the sphere for a mass flow rate $\dot{m} = 10\ \mathrm{kg\ s^{-1}}$? Ignore the area occupied by the sphere. [8 marks]

Tick here if you continue at the end of the booklet: ☐

We are now using glycerol (density $\rho_g = 1261 \text{ kg m}^{-3}$, viscosity $\mu_g = 1.24 \text{ Pa s}$), so that the absolute value of the drag force experienced by the sphere is now given by $F_D = 6\pi\mu_g a\, U_\infty$ due to the high viscosity of glycerol (*Viscous Stokes Regime*).

e) Calculate the new velocity U_∞ at which the sphere remains at a fixed position. [4 marks]

Tick here if you continue at the end of the booklet: ☐

f) Calculate the corresponding radius-based Reynolds number. Knowing that the *Viscous Stokes Regime* corresponds to $Re \leq O(1)$, is the *Low Reynolds Number* form of the drag force used in (e) fully justified? Based on the definition of drag coefficient C_D (i.e. defined in the high Reynolds number flow), show the correlation between C_D and Re. [6 marks]

Tick here if you continue at the end of the booklet: ☐

g) What is the altitude of the bead if the mass flow rate $\dot{m} = 1.5\ \mathrm{kg\,s^{-1}}$?
Again, ignore the area occupied by the sphere. [4 marks]

Tick here if you continue at the end of the booklet: ☐

2. Consider a rocket engine burning hydrogen and oxygen. The burnt gas entering the convergent-divergent rocket nozzle is at 3517 K and 25 bar. The flow velocity at the inlet of the nozzle can be assumed to be negligibly small. The specific gas constant for the exhaust gas $R_s = 519.6\ \mathrm{J\ kg^{-1}K^{-1}}$, and the specific heat capacity ratio $k = 1.22$. The pressure at the exit of the nozzle is 0.01174 bar and the area of the throat is 0.4 m^2. Assume the flow through the nozzle is isentropic.

<u>Useful isentropic flow correlations:</u>

Isentropic flow: $\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)\mathrm{M}^2$; $\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)\mathrm{M}^2\right]^{\frac{k}{k-1}}$;

speed of sound in an ideal gas: $c = \sqrt{kR_sT}$.

a) Calculate the exit Mach number and velocity. [10 marks]

Tick here if you continue at the end of the booklet: ☐

b) Calculate the mass flow rate and the required exit area. [10 marks]

Tick here if you continue at the end of the booklet: ☐

c) It can be shown that a normal shock wave occurs at the exit of the nozzle when the environmental pressure is at 0.348 bar. What will happen to the flow inside the nozzle when the rocket is at a low enough latitude where the environmental pressure is 0.5 bar? Please draw the pressure and Mach number distribution inside the nozzle. What can be done to the nozzle to adjust the flow back to isentropic condition? [7 marks]

Tick here if you continue at the end of the booklet: ☐

d) What will happen to the flow inside the nozzle when the rocket is propelled further into the space where the environmental pressure is reduced below 0.01174 bar? [3 marks]

Tick here if you continue at the end of the booklet: ☐

3. Fig Q3 shows an underground research facility that requires fresh air to be supplied from the surface for ventilation. The required flowrate is $0.8\ \mathrm{m^3\ s^{-1}}$. The air may be assumed as incompressible with density $1.2\ \mathrm{kg\ m^{-3}}$ and constant dynamic viscosity $1.85 \times 10^{-5}\ \mathrm{Pa\ s}$.

The flow is generated by a centrifugal blower. All ducting is circular in cross-section and has a roughness height of $0.2\ \mathrm{mm}$.

The inlet and outlet of the system are at an equal height. The air intake system consists of a $0.35\ \mathrm{m}$ diameter duct with a total length of $250\ \mathrm{m}$. The diameter of the exhaust ducting is $0.25\ \mathrm{m}$, and has a total length of $220\ \mathrm{m}$.

The system contains a filter, various sharp 90° bends, abrupt expansions and abrupt entries (or contractions), which all have sharp edges. These features can be identified on the figure, and the corresponding minor loss coefficients are given in the table.

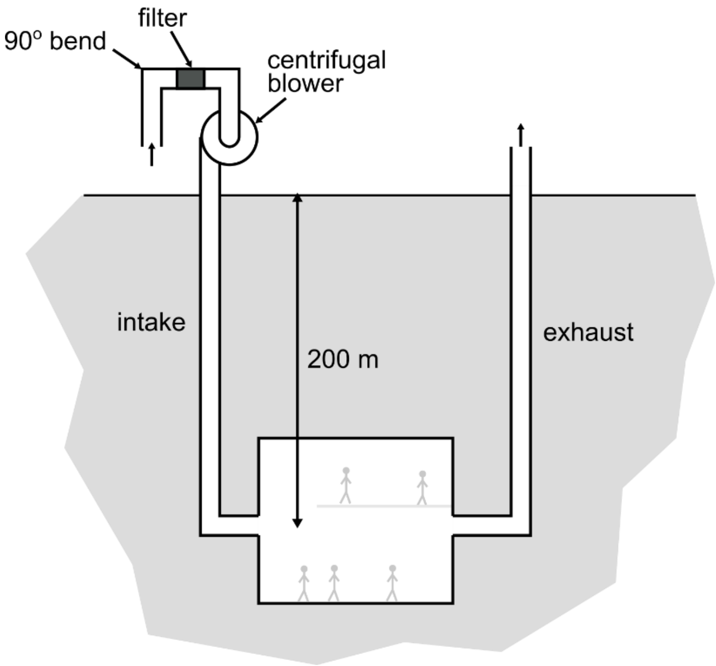

Table of minor loss coefficients	K_L
Sharp edged abrupt entry	0.5
Sharp edged abrupt expansion	1.05
90° bend	0.3
Air filter	4.5

Fig. Q3 Diagram of the underground facility and its ventilation system (not to scale)

<u>Useful correlations</u>

Friction factor (f) :

i. $$\frac{1}{\sqrt{f}} = -1.8 \log\left(\frac{6.9}{Re} + \left(\frac{\varepsilon}{3.7\,D}\right)^{1.11}\right)$$ for turbulent flow;

ii. $$f = \frac{64}{\mathrm{Re}}$$ for laminar flow.

Kinetic energy correction factor :

i. $\alpha = 1.05$ for turbulent flow

ii. $\alpha = 2$ for laminar flow

Conservation of angular momentum for a centrifugal pump (or fan, or blower)

$$T = \rho \dot{V}\left(r_2 u_{2,t} - r_1 u_{1,t}\right)$$

a) Calculate the Reynolds number for both the flow in the intake and exhaust duct. Is the flow laminar or turbulent? [3 marks]

Tick here if you continue at the end of the booklet: ☐

b) Calculate the friction factor for both the intake and exhaust duct (f_{in} and f_{ex}). [3 marks]

Tick here if you continue at the end of the booklet: ☐

c) Calculate the total head loss in the ventilation system. [5 marks]

Tick here if you continue at the end of the booklet: ☐

d) Hence, or otherwise, determine the total useful head that must be supplied by the centrifugal blower, and the power consumed by the blower if it is assumed ideal (100% mechanical efficiency). [5 marks]

Tick here if you continue at the end of the booklet: ☐

e) If atmospheric pressure at the surface is $101.325\ \mathrm{kPa}$, determine the pressure inside the facility, which is at a depth of $200\ \mathrm{m}$ (the variation of pressure within the room Is neglected) [6 marks]

Tick here if you continue at the end of the booklet: ☐

f) The centrifugal blower has radial/straight blades and rotates at a rotational speed of $\omega = 600\ \mathrm{rpm}$, and the flow entering the blower has zero inlet swirl. Given this information, determine the outer radius of the impeller/rotor (assuming the blower is ideal). [6 marks]

Tick here if you continue at the end of the booklet: ☐

g) Give two sources of inefficiency/ losses in centrifugal pumps. [2 marks]

Tick here if you continue at the end of the booklet: ☐

End of questions

Extra Space for answers to any questions:

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