Loughborough University

Robotics and Control

23WSC104

Semester 1 2023 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

Answer **ALL FOUR** questions.

All questions carry equal marks.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the rear of the paper.

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1. Sketch approximate Bode plots for the following transfer functions, marking relevant frequencies, noting the presence or absence of any resonant peaks, and giving the gradients of any asymptotes.

a)

i. $G_1(s) = \frac{10(s+1)}{s}$ [6 marks]

ii. $G_2(s) = \frac{4}{s^2+0.4s+4}$ [6 marks]

b) State the Nyquist stability criterion in a form suitable for open-loop stable systems. Considering your statement of the Nyquist criterion, are the two transfer functions in part a) closed-loop stable under unity feedback? [4 marks]

A block diagram of a servomotor is shown in Figure Q1(a), where the input $I(s)$ is the applied current in Amps and the output $\Theta(s)$ is motor position in degrees.

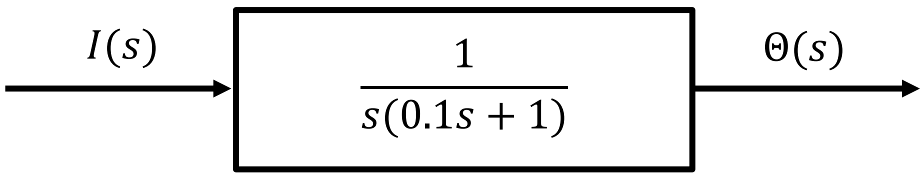


Figure Q1(a)

Figure Q1(b) on the next page shows the Bode plot of this servomotor.

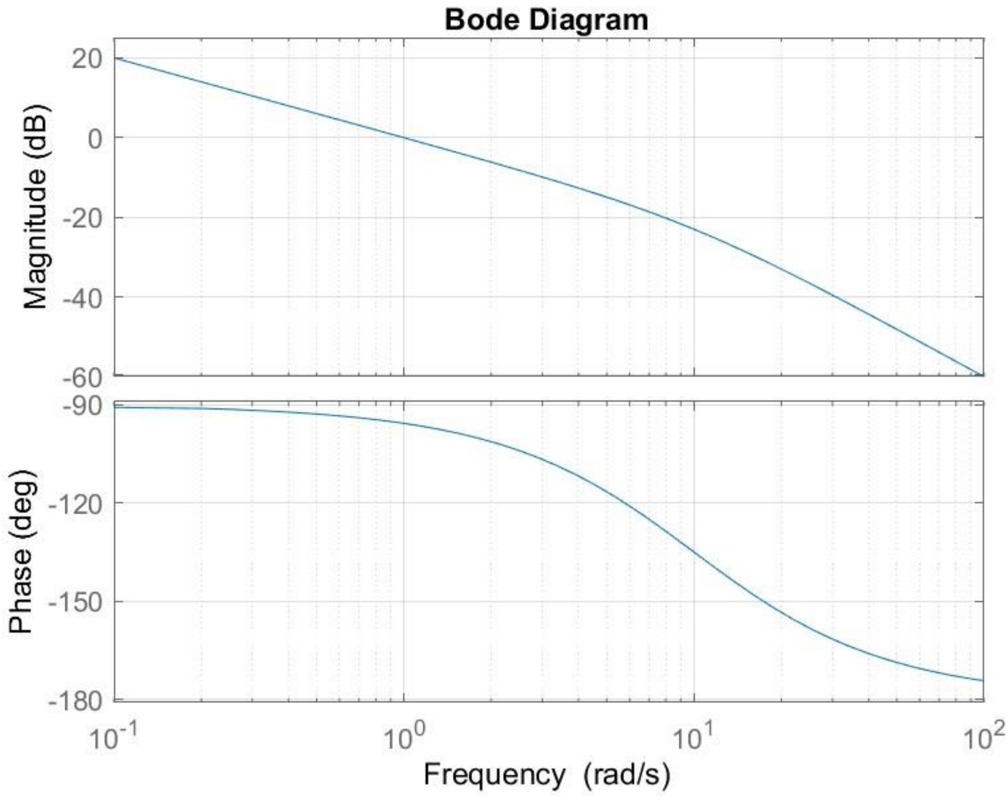


Figure Q1(b)

Considering the Bode plot:

c) If the input $i(t)$ to the servomotor is a sine wave of amplitude 10A, frequency 10 rad/s, what is the amplitude and relative phase of the resulting sine wave in $\theta(t)$? [3 marks]

d) Estimate the gain and phase margins of the servomotor under unity feedback. [3 marks]

e) Estimate the largest allowable proportional feedback gain that will give an overshoot of at most 10% in closed-loop operation. [3 marks]

2. A surge tank between two processing units in a chemical plant has a time constant $T_1 = 20$ seconds, and undergoes a change in water level of $A = 0.01$ metres for each change in input flowrate $U(s)$ of 1 litre/min. The water level is measured by means of a float, which contributes another time constant $T_2 = 4$ seconds, and the resulting level $Y(s)$ is fed back to a controller which implements proportional control of gain $K = 500$ (litres/min)/m.

The transfer function of the surge tank is given by:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{A}{(T_1 s + 1)(T_2 s + 1)}$$

The response to a change in reference from 0.4m to 0.45m is shown in Figure Q2.

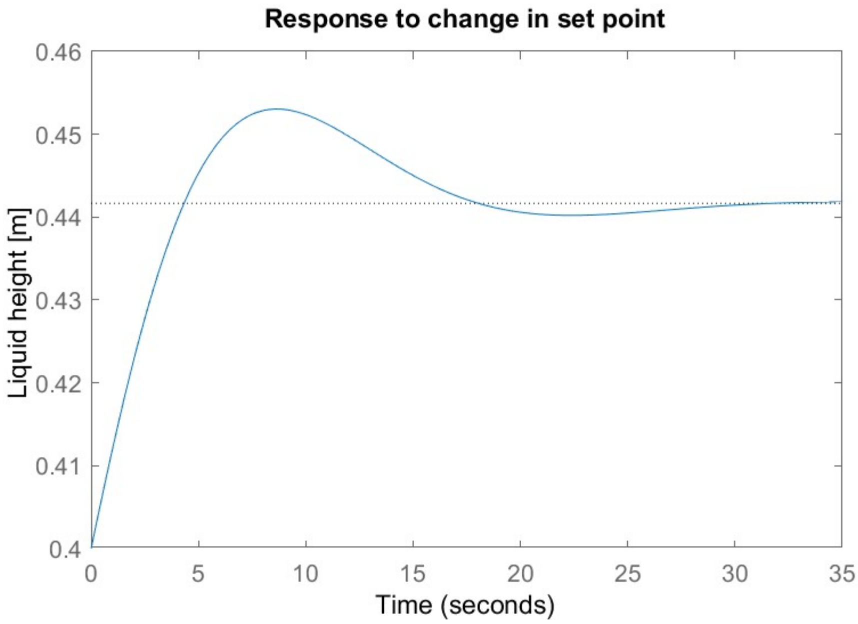


Figure Q2

The specifications for the feedback system are to achieve a rise time of less than 30 seconds, overshoot of less than 10%, and a steady state error of less than 5% to a change in set point.

a) Using Figure Q4, estimate the approximate rise time, overshoot, and steady-state error under proportional control, and state if these specifications are met or not met. [3 marks]

b) Is it possible to meet the specifications using a proportional controller? Briefly explain why. [2 marks]

A process engineer is considering how to improve the response and decides to implement a PI controller $C(s) = K\left(1 + \frac{1}{T_i s}\right)$ instead of the proportional control.

c) What will the steady-state error be with a PI controller, and why? [2 marks]

d) Show that $\omega_d = 0.1$ rad/s and $PM = 65^o$ are suitable target values of 0dB crossover frequency (design frequency) and phase margin for a frequency-domain controller design. [2 marks]

e) Using the given transfer function $G(s)$ to calculate the frequency response of the surge tank at the design frequency, carry out a PI controller design and state the transfer function of your completed PI controller.

NB: To evaluate the frequency response, you may either use direct calculation, or another method such as a Bode plot. [10 marks]

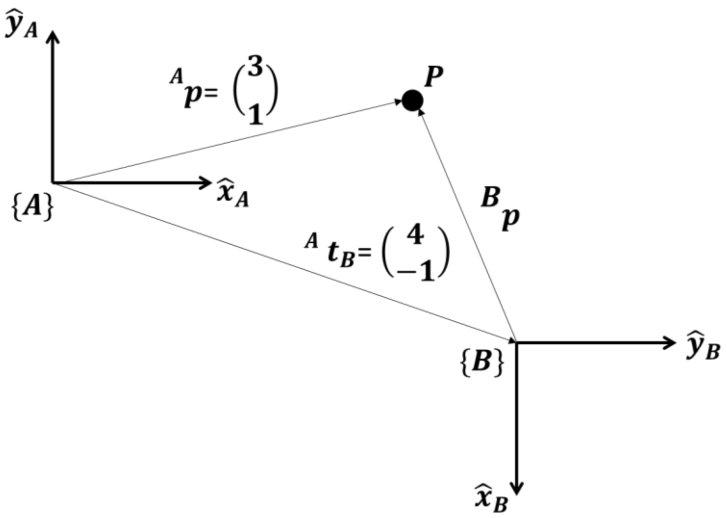
Another process engineer suggests using a PD controller, $C(s) = K(1 + T_d s)$, for the same surge tank.

f) For the PD controller, what is the value of K required to meet the steady-state error specification? [2 marks]

g) Briefly discuss possible advantages and disadvantages of a PD controller design versus a PI controller design for this application. Which do you expect to be more robust to plant uncertainty? [4 marks]

3. Derive the homogeneous transform and related coordinate transformations in the following cases:

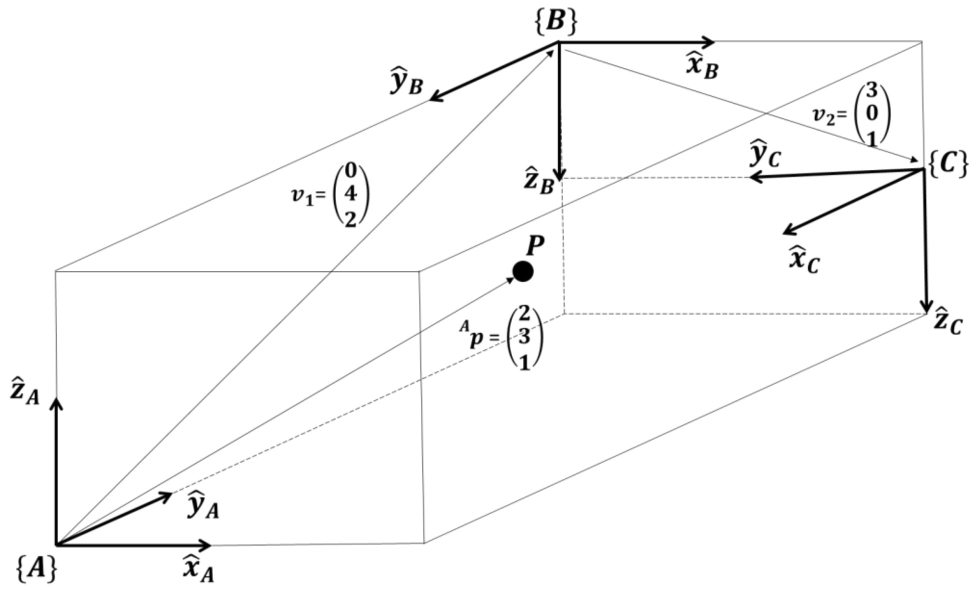
a) Consider the reference frames $\{A\}$ and $\{B\}$ sketched in figure below.



i. Derive the homogeneous transform ${}^{B}T_A$. [4 marks]

ii. Evaluate the coordinates of point ${}^{A}P$ with respect to reference frame $\{B\}$. [4 marks]

b) Consider the reference frames $\{A\}$, $\{B\}$ and $\{C\}$ sketched in figure below.

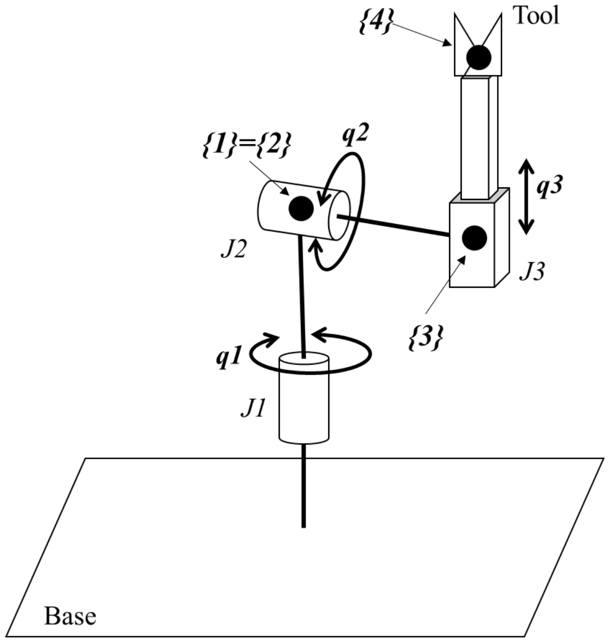


i. Describe the transformation from $\{A\}$ to $\{C\}$ in terms of Roll, Pitch and Yaw angles and in terms of origin translations. [8 marks]

ii. Derive the homogeneous transform ${}^{A}T_{C}$. [6 marks]

iii. Evaluate the coordinates of point ${}^{A}P$ with respect to reference frame $\{C\}$. [3 marks]

4. Consider the RRP serial robot depicted hereafter where all the fixed links have unitary length:



a) Define the robot's Denavit-Hartenberg parameters; to do so, copy the image of the robot in your answer book and sketch the reference frames you are using (note that origins for joint frames {1}, {2} and {3} as well as for tool {4} are indicated in the figure). [8 marks]

b) Derive the forward kinematics when the robot is in the following configuration: $q1 = \pi;\ q2 = \pi\ ; q3 = 2$. [12 marks]

c) Suppose that the robot tool is fed by a pose trajectory that starts in the configuration described by frame $\{A\}$ of Exercise 3(b) and ends in the configuration described by frame $\{C\}$ of Exercise 3(b). Using a linear interpolation, write the homogeneous transform corresponding to the tool pose when the trajectory is 75% of the way along the path. [5 marks]

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Useful information (control systems)

Selected Laplace transforms

t-domain (t>0)	***s-domain***
Unit step	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\ \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\ \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

Some results about Laplace Transforms

$f(t)$	$F(s)$
$f(t-T)$	$e^{-Ts}\,F(s)$
$e^{-at}\,f(t)$	$F(s+a)$
$f'(t)$	$sF(s)-f(0)$
$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0}\ \{s\,F(s)\}$

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Step response of second-order systems *- For systems well approximated by* $\frac{A\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$

Percentage overshoot:

$$O_{\%} = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Peak time:

$$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$$

Settling time to p% of steady-state:

$$T_s = \frac{\ln(100/p)}{\zeta\omega_0}$$

Rise time:

$$T_r \approx \frac{2}{\omega_0}$$

With phase margin ϕ_{PM} in degrees, $\zeta \approx \phi_{PM}/100$ for $\phi_{PM} < 60^o$, and $\phi_{PM} = 65^o$ gives $\zeta \approx 0.7$

Common SISO compensator types (PID and related controllers)

PID controller:

$$C_{pid}(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right)$$

PI controller:

$$C_{pi}(s) = K\left(1 + \frac{1}{T_i s}\right)$$

Lag compensator:

$$C_{lag}(s) = K\frac{s + 1/T_i}{s + p_{lag}}$$

PD controller:

$$C_{pd}(s) = K(1 + T_d s)$$

Lead compensator:

$$C_{lead}(s) = K\frac{T_d s + 1}{\frac{s}{p_{lead}} + 1}$$

Frequency-domain design formulae for PID-type controllers

where G_C, ϕ_C are required controller gain and phase at a chosen design frequency ω_d

PI controller:

$$T_i = \frac{1}{\omega_d \tan(-\phi_C)}$$

PID controller:

$$\tan\phi_C = T_d\omega_d - \frac{1}{T_i\omega_d}$$

PD controller:

$$T_d = \frac{\tan\phi_C}{\omega_d}$$

Controller gain (all cases):

$$K = \frac{G_c}{\sqrt{1 + \tan^2\phi_C}}$$

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Useful information (robotics)

Cross product (or vector product)

A x B = V

Vector **V** is perpendicular to the vectors **A** and **B**

If $A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

then $V = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ with:

$$v_1 = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} = a_2 b_3 - a_3 b_2$$

$$v_2 = -\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} = a_3 b_1 - a_1 b_3$$

$$v_3 = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

Dot product (or scalar product)

$$A \bullet B = \|A\| \, \|B\| \, \cos\theta$$

where $\|A\|$ and $\|B\|$ denote the length (magnitude) of **A** and **B** and θ is the angle between them, or

If $A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ then,

$$A \bullet B = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$\|A\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

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Rotation matrices

3 x 3 rotation matrices around main axes:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & -\sin\vartheta \\ 0 & \sin\vartheta & \cos\vartheta \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\vartheta & 0 & \sin\vartheta \\ 0 & 1 & 0 \\ -\sin\vartheta & 0 & \cos\vartheta \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

General homogeneous transform

4 x 4 homogeneous transform in 3D space:

$$^{A}\boldsymbol{T}_{\boldsymbol{B}} = \begin{pmatrix} & ^{A}\boldsymbol{R}_{\boldsymbol{B}} & & ^{A}\boldsymbol{t}_{\boldsymbol{B}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The inverse matrix can be derived as follows:

$$^{B}\boldsymbol{T}_{\boldsymbol{A}} = \begin{pmatrix} & {^{A}\boldsymbol{R}_{\boldsymbol{B}}}^{\boldsymbol{T}} & & -{^{A}\boldsymbol{R}_{\boldsymbol{B}}}^{\boldsymbol{T}} \cdot {^{A}\boldsymbol{t}_{\boldsymbol{B}}} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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Link parameters (according to the convention of Craig)