Loughborough University

Mechatronics with control

23WSD912

Semester 1 2023 | In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

Answer **ALL FOUR** questions.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular, that it must not be able to transmit or receive information (e.g. mobile devices and smartwatches are not allowed).

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the end of the paper.

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1. On a production line, a multi-jointed robotic arm is used to manipulate and fit complex heavy parts into location on an assembly. Each stage of the robot is actuated by pneumatic cylinders with 320 mm bore and 120 mm diameter rod. Each cylinder is controlled by two electromechanical actuators, consisting of a flow rate controller followed by an actuation controller. The flow rate is controlled by a NC proportional solenoid valve with a proportionality constant of 500 mmA^{-1}. The pneumatic supply is at 8 bar of pressure at 294 K with a density of 1.293 kgm^{-3}.

a. If the desired flow rate and output pressure across the proportional solenoid value is 47.5 m^3h^{-1} and 3 bar respectively, using Figure Q1 (next page), determine the displacement of the proportional solenoid valve, given that the valve has only just been powered on.

[4 marks]

b. To reduce disturbances affecting the pneumatic actuators an upper limit for the ripple current for the proportional solenoid valve has been experimentally determined as $\Delta I \leq$ 500 μA. For a duty cycle of 40%, what is the minimum PWM frequency required by the system if the proportional solenoid valve's resistance and inductance are 7.80 Ω and 11.5 mH respectively?

[6 marks]

c. What is the power efficiency of the pneumatic cylinder on its extension stroke if the cylinder has a packing friction of 1.21 kN?

[6 marks]

d. Draw a fluid power schematic from a pneumatic source to a single pneumatic actuator through the described control valves in the system. Give justification for your choice of pneumatic actuator and actuation controller design with respect to the functional requirements and safety requirements of the robotic system.

[8 marks]

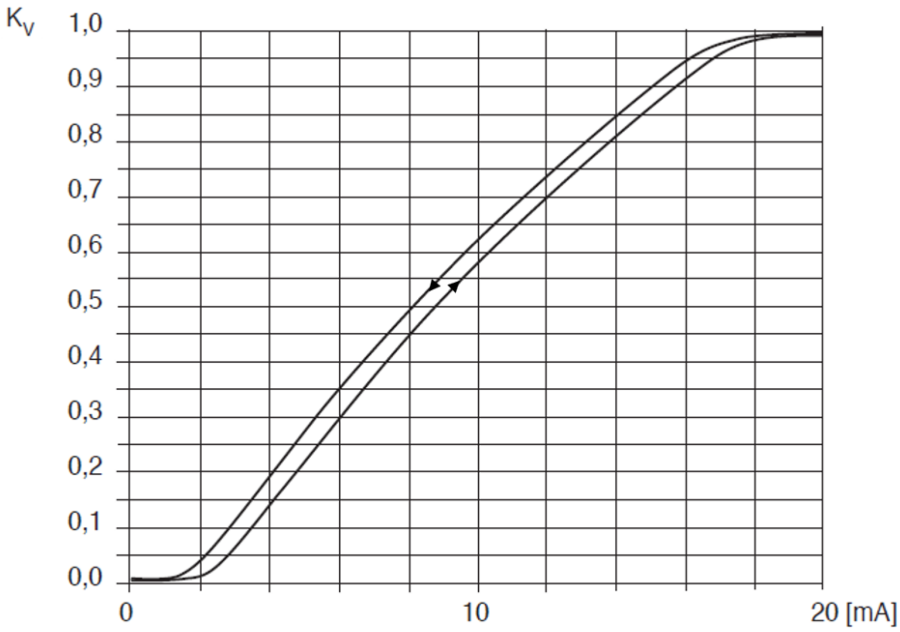

Figure Q1. Proportional solenoid valve hysteresis curve.

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2.

a) An X-Y pick and place gantry unit has its X-axis drive implemented utilising a stepper motor connected to the load via a 5mm pitch toothed belt and two pulleys, as shown in Figure Q2a. Pulley A has a pitch circle diameter of 26.74mm with 28 teeth, and pulley B has a pitch circle diameter of 13.37mm with 14 teeth. The centre distance is 200mm.

Showing all working, calculate the number of teeth required on a belt to fit this arrangement. Using Table Q2a select an appropriate belt and determine if the axis requires a tensioner. [6 marks]

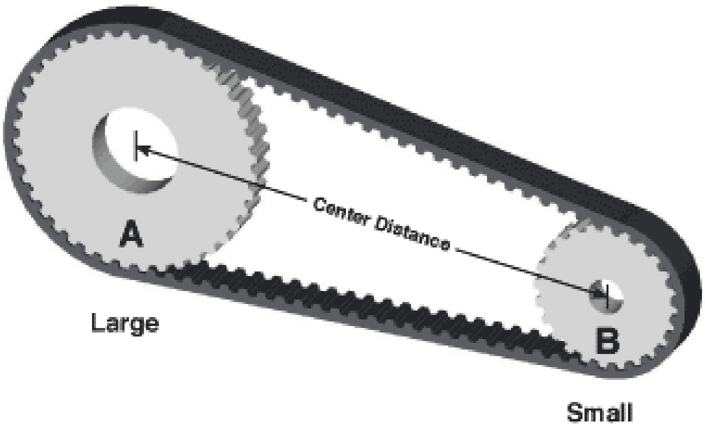

Figure Q2a.

Belt Length mm	No. Teeth	6mm Wide Belt		9mm Wide Belt		15mm Wide Belt	
		Cat. No.	Wt gms	Cat. No.	Wt gms	Cat. No.	Wt gms
276	92	276-3M6	6	276-3M9	9	276-3M15	14
285	95	285-3M6	6	285-3M9	9	285-3M15	15
288	96	288-3M6	6	288-3M9	9	288-3M15	15
291	97	291-3M6	6	291-3M9	9	291-3M15	15
297	99	297-3M6	6	297-3M9	9	297-3M15	15
300	100	300-3M6	6	300-3M9	9	300-3M15	15
312	104	312-3M6	6	312-3M9	10	312-3M15	16
318	106	318-3M6	7	318-3M9	10	318-3M15	16
327	109	327-3M6	7	327-3M9	10	327-3M15	17
330	110	330-3M6	7	330-3M9	10	330-3M15	17
336	112	336-3M6	7	336-3M9	10	336-3M15	17
339	113	339-3M6	7	339-3M9	10	339-3M15	17
345	115	345-3M6	7	345-3M9	11	345-3M15	18
357	119	357-3M6	7	357-3M9	11	357-3M15	18
363	121	363-3M6	7	363-3M9	11	363-3M15	19
375	125	375-3M6	8	375-3M9	12	375-3M15	19
384	128	384-3M6	8	384-3M9	12	384-3M15	20

Table Q2a.

b) An exact match for the required number of teeth often cannot be found in suppliers' catalogues, and in such cases, the belt needs some form of tensioner for correct operation.

Draw and label clear diagrams of two tensioning arrangements that could be utilised for this system. [6 marks]

3.

a) Describe the construction, operation, and characteristics of hybrid stepper motors. You do not need to draw step tables as part of this answer.

Include labelled diagrams showing the general construction of a hybrid stepper motor, an illustration of what happens to the phase current at low, medium, and high speeds, and a typical torque speed curve with short descriptions of what each illustration signifies. [12 marks]

b) Briefly describe, including a diagram for each, single step, half step, and micro-stepping control of the speed of rotation of a hybrid stepper motor. [9 marks]

c) The Y-axis on the machine mentioned in Q2a, is driven by a lead screw connected to a hybrid stepper motor driven from a DM556 micro-stepping drive unit set to 1600 micro steps. The DM556 is supplied with a 30kHz square wave, and the screw's lead is 2mm. What is the velocity of the Y-axis carriage in metres per second? [6 marks]

d) The Y-axis has a 1024 counts per revolution incremental optical encoder connected directly to it. The encoder interfaces to a computer via a quadrature decoding circuit. With the settings given in Q3c, how much will the displayed count increases every second? [3 marks]

4.

a) A controller for a DC motor-controlled system initially utilised a simple proportional controller. The proportional constant was set to Kp=1.4, and the system tested. The motor set-point is 150rpm, however, the controller ran the motor at 106rpm, as shown in Figure Q4a.

What would be the effect of the following changes:

i. Increasing Kp to 32. [4 marks]

ii. Reducing Kp to a low value and adding an integral element to create a PI controller. [4 marks]

iii. Adding a differential control element to the PI controller in section 4aii to make a PID controller. [4 marks]

b) What potential issue might occur by adding the integral component to the controller, and how could this be overcome? [5 marks]

c) The X-axis position for the system in Q2 is controlled using a trapezoidal motion profile. The desired maximum velocity is 1.25 m/s, which is constant for 2.2m. The acceleration and deceleration phases will cover 0.4m for each phase. Determine:

i. The time it runs maximum velocity. [3 marks]

ii. The total distance travelled. [3 marks]

iii. The acceleration and deceleration times. [4 marks

iv. The acceleration and deceleration rate. [3 marks]

d) Draw the trapezoidal motion profile showing all parameters calculated in section Q4b. [4 marks]

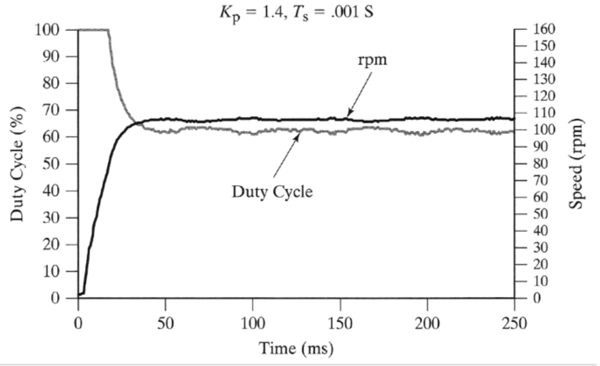

Figure Q4a.

A Sutton

T Williams

Useful formulae

$A * Sin(\omega t + \phi)$

$V_{ref} = Vin * \frac{R2}{R1+R2}$

$V_{out} = A_o(V_{in+} - V_{in-}) = 300000(Vin+ - Vin-)$

Comparator If $V_{in-} > V_{in+}$ V_{out} is –V

Comparator If $V_{in-} < V_{in+}$ V_{out} is +V

$$A_{CL} = -\frac{R_f}{R_{in}}$$

$$A_{CL} = 1 + \frac{R_f}{R_{in}}$$

$$V_{out} = V_{in1}\frac{-R_f}{R_{in1}} + V_{in2}\frac{-R_f}{R_{in2}}$$

$Vin = V_{FS}(B_1*2^{-1}+B_2*2^{-2}+B_3*2^{-3}+...+B_n*2^{-n}) \pm 1/2LSB$

$V_{out} = V_{ref}(B_n*2^{-1} + B_{n-1}*2^{-2} + B_{n-2}*2^{-3} + ... + B_0*2^{-n})$

$$V_{out} = -V_{ref} * \frac{Code}{2^n}$$

$$V_{fso} = V_{ref}\frac{(2^n - 1)}{2^n}$$

$$Sensitivity = \frac{V_{out}}{V_{primary} * Core\ displacement}$$

$$v_{out} = v_1 - v_2 = (k_1 - k_2)\sin(\varpi t - \emptyset)$$

$k_1 > k_2$ then $v_{out} = (k_1 - k_2)\sin(\varpi t - \emptyset)$
$k_1 < k_2$ then $v_{out} = (k_1 - k_2)\sin(\varpi t + (\pi - \emptyset))$

Absolute encoder Resolution° = $360/2^n$

4-bar Grashof criterion s + l ≤ p + q

Module m = PCD/(number of teeth)

$$n = \frac{\omega\ 1}{\omega\ 2} = \frac{\alpha 1}{\alpha 2}\ \frac{r2}{r1} = \frac{N2}{N1}$$

Displacement $r_1\theta_1 = r_2\theta_2$

Velocity $r_1\omega_1 = r_2\omega_2$

Acceleration $r_1\alpha_1 = r_2\alpha_2$

$$\frac{\omega A}{\omega D} = \frac{r_B r D}{r_A r C} = \frac{N_B N_D}{N_A N_C}$$

$$N_c = \frac{2*A_o}{p} + \frac{Z_1+Z_2}{2} + \frac{2.533*p*(Z_2-Z_1)^2}{100*A_0}$$

$$A = \frac{p}{4}(N_A - \frac{Z_1+Z_2}{2} + \sqrt{\left(N_A - \frac{Z_1+Z_2}{2}\right)^2 - \frac{2.027*(Z_2-Z_1)^2}{10}})$$

$$n(rpm) = \frac{travel\ rate(mm/min)}{Lead\ (mm)}$$

$$T_d(Nm) = \frac{F*P}{2*\pi*e}$$

$$n_c = \frac{C_s*1.2*10^8*d_r}{l^2}$$

$$F_c(N) = \frac{C_s*9.687*10^4*d_r^4}{l^2}$$

$E_b = \frac{P\emptyset NZ}{60A}$ and $E_b = V - I_a R_a$

$N = \frac{E_b*60A}{P\emptyset Z}$ let $\mathrm{K} = \frac{60A}{PZ}$ thus $N = \frac{KE_b}{\emptyset}$

Duty Cycle(%) = (T_{ON}/T_S) * 100

Proportional = Error * K_P

PI = Current Integral Error = Δt * [(Error at Last Step + Current Error)/2]

Control Effort = (Error * Proportional Gain) + (Integral Gain * $\sum Error$)

$$u(t) = K_P e(t) + K_i \int_0^t e(\tau)d\tau$$

PID Control Effort = (Error * ProportionalGain)+(IntegralGain * $\sum Error$)+(DerivativeGain * $\frac{dError}{dT}$)

$$u(t) = K_P e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt}e(t)$$

$$\frac{U(s)}{E(s)} = G_{PID}(s) = K_P + K_I\frac{1}{s} + K_D s$$

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PID in time constant format = $u(t) = K_c\left[e(t) + \frac{1}{\tau_i}\int_0^t e(t)dt + \tau_d\frac{de}{dt}\right]$

Integral time constant $\tau_i = \frac{K_P}{K_i}$

Derivative time constant $\tau_d = \frac{K_d}{K_p}$

Proportional solenoid flow coefficient for gases: $k_v = \begin{cases} \frac{Q}{514}\sqrt{\frac{T\rho}{P_2\Delta P}}, & \frac{P_2}{P_1} > \frac{1}{2} \\ \frac{Q}{257P_1}\sqrt{T\rho}, & \frac{P_2}{P_1} < \frac{1}{2} \end{cases}$

Magnetic flux density in the centre of a solenoid: $B = \frac{\mu_0 nI}{l_s}$

LR circuit time constant: $\tau = \frac{L}{R}$

Peak current of an LR circuit: $I_{max} = I_{avg} + \frac{\Delta I}{2}$ $\qquad I_{min} = I_{avg} - \frac{\Delta I}{2}$

LR circuit current discharge: $I_{min} = I_{max}e^{-\frac{T_{OFF}}{\tau}}$

PWM time period: $T_s = T_{ON} + T_{OFF}$

$Duty\ Cycle(\%) = \frac{100T_{ON}}{T_S}$

PWM frequency: $f_s = \frac{1}{T_s}$

Fluid Power: $P_{in} = P_2Q$

Cylinder Force: c $F_c = P_2A$

Piston Force: $F_p = F_c - F_N$

Piston velocity: $V_p = \frac{Q}{A}$

Mechanical Power: $P_{out} = F_pV_p$

Efficiency: $\eta = \frac{P_{out}}{P_{in}}$

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Horizontal travel speed

$$VL = \frac{SL}{tL}$$

Lead screw speed

$$\varpi S = VL \cdot \frac{2 \cdot \pi}{\lambda}$$

rotational speed of motor shaft at maximum speed

$$\varpi M = \varpi S \cdot R$$

motor shaft speed

$$n = \frac{30 \cdot \varpi M}{\pi}$$

Linear accelerating force

$$F = wL(\mu L \cdot Cos(A) + Sin(A))$$

Torque at the leadscrew at maximum speed

$$M = FL \cdot \left(\frac{\lambda}{2 \cdot \pi \cdot \eta s} \right)$$

torque reflected to motor shaft

$$Mo = Ms \cdot \frac{1}{R} \cdot \frac{1}{\eta G}$$

inertia of linear load

$$JL = (ML + MN) \cdot \left(\frac{\lambda}{2 \cdot \pi} \right)^2$$

inertia of feed screw

$$JS = 0.5 \cdot Ms \cdot \left(\frac{ds}{2} \right)^2$$

inertia of loaded machine reflected back to feed screw

$$JR = (JL + JS + JG) \cdot \left(\frac{1}{R^2} \right) \cdot \left(\frac{1}{\eta s} \right) \cdot \left(\frac{1}{\eta G} \right)$$

total inertia of the system

$$JT = JM + JR$$

acceleration of the motor

$$\alpha M = \frac{MM}{JT}$$

acceleration of the leadscrew

$$\alpha S = \frac{\alpha M}{R}$$

Accelerating time

$$tA = \frac{\varpi S}{\alpha S}$$

distance travelled during acceleration and deceleration, from accelerating time

$$SA = \frac{1}{2} \cdot \alpha S \cdot \lambda \cdot (2 \cdot tA)^2$$

total distance travelled in trapezoidal motion

$$ST = SA + SL$$