Mathematical Modelling of Structures
24CVA104

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Answer **ALL** questions.
Answer **BOTH** questions in **Section A** – "Mathematics".
Answer **THE QUESTION** in **Section B** – "Analysis of Beams and Frames".

All questions in the same section carry equal marks.

SECTION A – MATHEMATICS
(Answer BOTH Questions in this Section)

Q1. Solve the following 1st order ODEs:

(a) $\frac{dy}{dx} = x^2 y$, subject to the boundary condition: $y(0) = 2$

[7 marks]

(b) $sin(x)\frac{dy}{dx} + cos(x)y = 2xe^{x^2+2}$

[6 marks]

(c) $\frac{dy}{dx} - tan(x)y = 1$

[6 marks]

(d) $x\frac{dy}{dx} + 4y = cos(x^4)$

[6 marks]

Q2. Solve the following 2nd order ODEs:

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$

[6 marks]

(b) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 10y = 0$, subject to the conditions: $y(0) = 1, y\left(\frac{\pi}{2}\right) = -2e^{3\pi/2}$

[6 marks]

(c) $\frac{d^2y}{dx^2} - 16\frac{dy}{dx} + 64y = 0$, subject to the conditions: $y(0) = 3, y(1) = 4e^8$

[6 marks]

(d) $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 10cos(x)$

[7 marks]

SECTION B – ANALYSIS OF BEAMS AND FRAMES
(Answer Q3 in this Section)

Q3. a) The beam shown in Figure Q3a has a frictionless hinge at point C and is subjected to the systems of loads shown.

    i) Calculate all support reactions.

        [8 marks]

    ii) Draw the bending moment and shear force diagrams, clearly indicating all salient values.
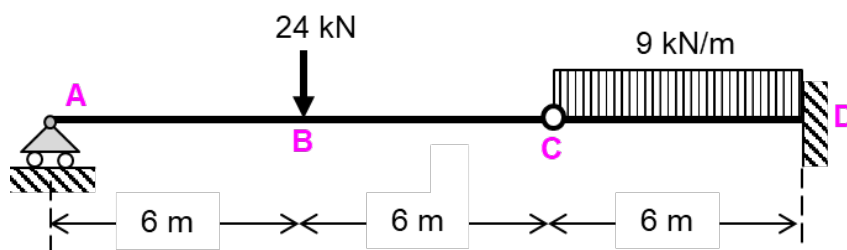
        [10 marks]

24 kN   9 kN/m

A   B   C   D

6 m   6 m   6 m

Figure Q3a

b) The frame shown in Figure Q3b has pinned supports at points A and E, and a frictionless hinge at point B, and is subjected to the system of loads indicated in the figure.

For this frame:
    i) **Calculate** all support reactions.   [10 marks]
    ii) **Draw** the shear force diagram and bending moment diagram for the whole structure, clearly indicating the salient values.   [16 marks]
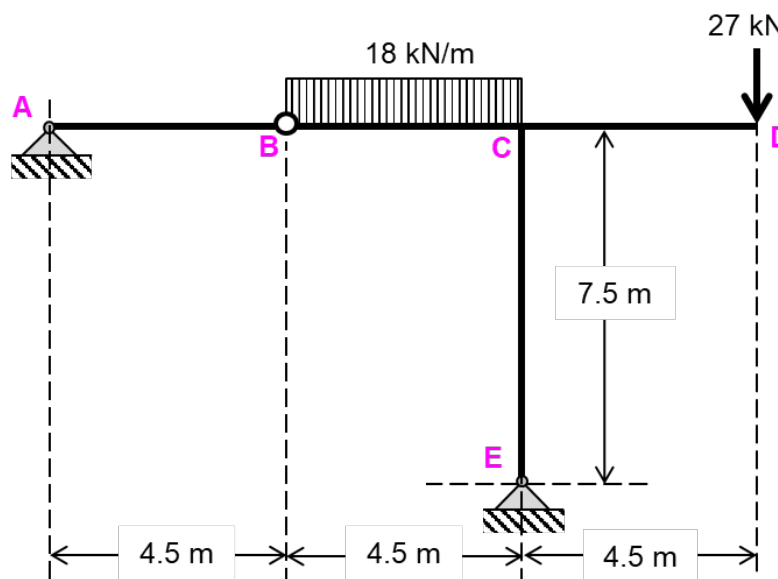   iii) **Sketch** the deflected shape.   [6 marks]

27 kN

18 kN/m

A   B   C   D

7.5 m

E

4.5 m   4.5 m   4.5 m

Figure Q3b

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