Control, Modelling and Simulation
24CVC119

Semester 2 2025 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **3 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Answer **ALL QUESTIONS in SECTION A.**
Answer **TWO QUESTIONS in SECTION B.**

All questions carry equal marks.

A 4-page formula sheet, with tables and charts, is provided at the end of this paper.

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SECTION A
(Answer **ALL QUESTIONS** in Section A)

Q1.

a) For the vectors

$$\boldsymbol{a} = \begin{bmatrix} 1 \\ 8 \\ 5 \end{bmatrix}, \ \boldsymbol{b} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}, \ \boldsymbol{c} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \ \boldsymbol{d} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

Calculate: $(\boldsymbol{a} + \boldsymbol{b}) \cdot (\boldsymbol{c} + \boldsymbol{d})$

[3 marks]

b) For the matrices *A* and *B* given by

$$A = \begin{bmatrix} -5 & 4 & 1 \\ 4 & -6 & 0 \\ 8 & -2 & 0 \end{bmatrix}, \ B = \begin{bmatrix} -9 & 3 & 2 \\ 1 & 0 & 0 \\ -7 & -2 & -1 \end{bmatrix}$$

Calculate: *AB*

[4 marks]

c) Calculate the determinant of the following matrix

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 0 & -5 \\ 4 & -5 & 4 \end{bmatrix}$$

[3 marks]

Question 1 continues/...

d) Find the eigenvalues and corresponding eigenvectors for the following matrix:

$$B = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$$

[8 marks]

e) For the function $f(x) = \frac{1}{1-x}$, calculate the first three derivatives of *f*.

Writing $f(x) = (1-x)^{-1}$ and substitution of $u = 1 - x$ may help.

[6 marks]

f) Computational Fluid Dynamics (CFD) is an advanced simulation tool.

(i) Describe how CFD can be used to assist in the design of building ventilation.

[3 marks]

(ii) One of the important inputs for CFD is a boundary condition. Describe a boundary condition you could use at an open window when modelling natural ventilation.

[3 marks]

(iii) Briefly describe the purpose of a mesh in CFD simulations.

[3 marks]

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SECTION B

(Answer **TWO QUESTIONS** in Section B)

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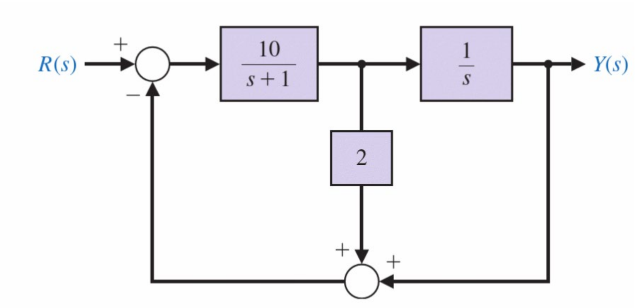
Q2.

a) Show that the closed-loop transfer function of the system shown below is:

$$\frac{Y(s)}{R(s)} = \frac{10}{s^2 + 21s + 10}$$



[5 marks]

b) For the closed-loop transfer function found in part (a), find the poles and plot them in the s-plane. State the type of system (underdamped, overdamped etc) and comment on the stability.

[6 marks]

c) Find the damping ratio ζ and natural frequency ω_n for the system considered in part a) and thus calculate the settling time.

[8 marks]

d) Consider the case when input R(s) is a unit step input. State the Laplace transform of a step input and thus find output Y(s).

[3 marks]

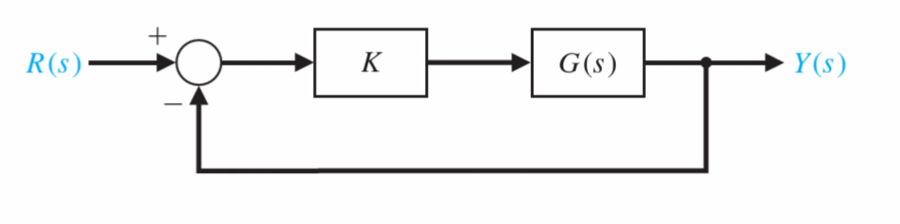
e) Considering the case when R(s) is a unit step input, use partial fractions to separate output Y(s) into parts. Take inverse Laplace transforms of these parts and thus find the response to a unit step input in the time domain, y(t).

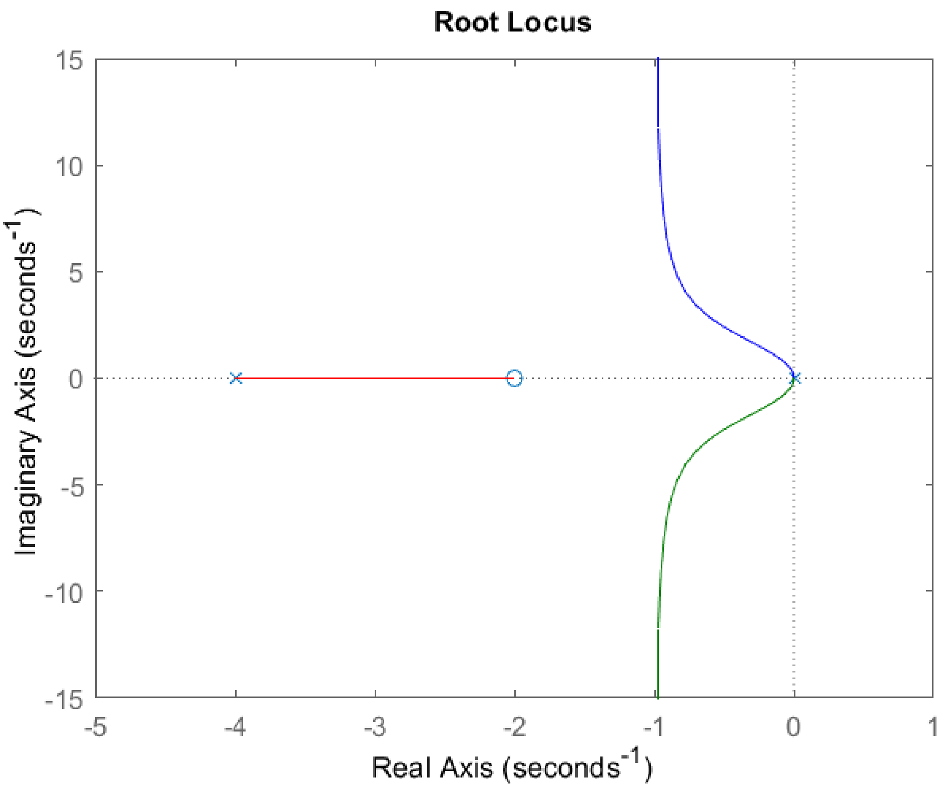
[8 marks]

Question 2 continues/…

f) A root locus plot of a system is shown below, where gain K has been varied from zero to infinity. State whether this is a first-order, second-order or third-order system. State whether the system is stable for all values of gain K, or whether there are certain values of K for which the system is unstable.

[3 marks]





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Q3.

a) Explain the difference between closed-loop and open-loop control systems, and state two benefits of closed-loop feedback control, compared to an open-loop system.

[4 marks]

b) State two objectives of control systems used in building HVAC systems.

[2 marks]

c) Give an example where a PI controller might be used in a building HVAC system and explain why a PI controller is a suitable choice in this case.

[4 marks]

d) Explain why achieving close control of temperature and relative humidity in HVAC systems at the same time is challenging. Describe one possible approach to this challenge, stating its advantages and disadvantages, and give an example of where it might be used.

[6 marks]

e) Explain what is meant by “transport lag” and how this affects the transfer function of sensors.

[2 marks]

f) Explain what is meant by “linearity” and why this is important in the analysis of control systems.

[3 marks]

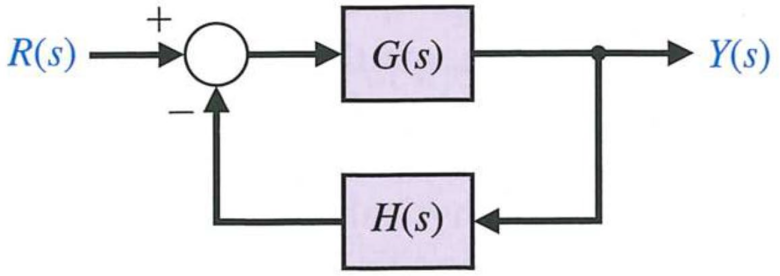
g) The transfer function of the sensor in the system shown below is

$$H(s) = \frac{e^{-0.162s}}{(1.165s + 1)}$$

The Padé approximation states $e^{-as} \approx \frac{1-\frac{a}{2}s}{1+\frac{a}{2}s}$

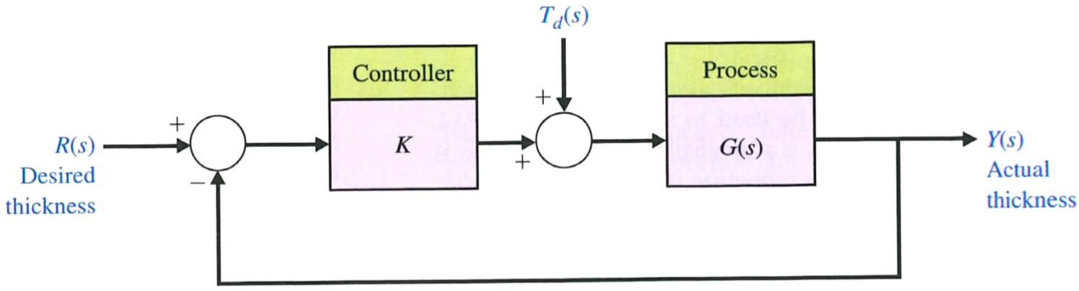
Use the Padé approximation to linearise the sensor transfer function H(s).

[3 marks]



Question 3 continues/…

h) A closed-loop system is used in a high-speed steel rolling mill to control the accuracy of the steel strip thickness. The transfer function for the process shown below can be represented as $G(s) = \frac{1}{s(s+21)}$. Calculate the closed-loop transfer function $\frac{Y(s)}{R(s)}$.



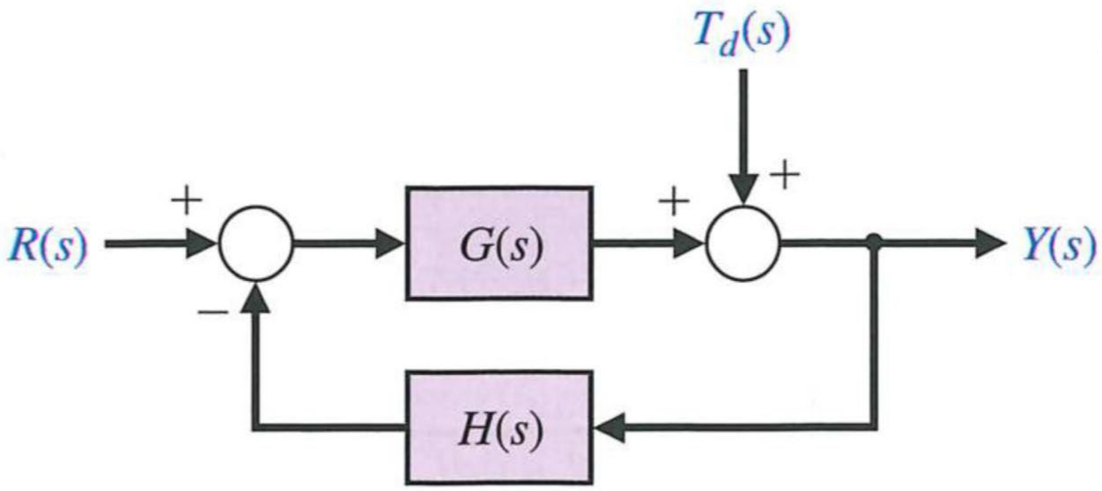
[4 marks]

Calculate the sensitivity of the closed-loop transfer function found in part h) to changes in the controller gain K.

[5 marks]

Q4.

a) In the system shown below, $G(s) = \frac{K}{s+10}$ and $H(s) = \frac{14}{s^2+5s+6}$. Show that the closed-loop transfer function is: $\frac{Y(s)}{R(s)} = \frac{K(s^2+5s+6)}{s^3+15s^2+56s+60+14K}$



[3 marks]

Question 4 continues/...

b) The steady-state error is defined as $E(s) = R(s) - Y(s)$. For the system introduced in part A, determine the steady-state error due to a unit step input, i.e. $R(s) = \frac{1}{s}$

[6 marks]

c) Explain the meaning of the terms "disturbance input" and "measurement noise", and state the desired response of a control system to disturbance inputs and measurement noise.

[4 marks]

d) Compute the transfer function $\frac{Y(s)}{T_d(s)}$ for the system introduced in part a) and determine the steady-state error of the output due to a unit step disturbance input i.e. $T_d(s) = \frac{1}{s}$

[5 marks]

e) In the system introduced in part a), use the Routh Array to find the range of K which gives closed-loop stability.

[8 marks]

f) Consider the system represented in state space form below. Find the poles of this system, assuming there is no pole-zero cancellation. Comment on the stability and the type of system (underdamped, overdamped etc.).

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 3 & 2 \end{bmatrix} \mathbf{x}$$

[7 marks]

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LOUGHBOROUGH UNIVERSITY
SCHOOL OF ARCHITECTURE, BUILDING AND CIVIL ENGINEERING

CVC119: CONTROL, MODELLING AND SIMULATION – FORMULA SHEET AND TABLES

Tables of Laplace Transforms and Theorems

Table 2.3 Important Laplace Transform Pairs

$f(t)$	$F(s)$
Step function, $u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
t^n	$\frac{n!}{s^{n+1}}$
$f^{(k)}(t) = \frac{d^k f(t)}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-) - s^{k-2} f'(0^-) - \ldots - f^{(k-1)}(0^-)$
$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(s)}{s} + \frac{1}{s} \int_{-\infty}^{0} f(t)\, dt$
Impulse function $\delta(t)$	1
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{1}{\omega}\left[(\alpha-a)^2+\omega^2\right]^{1/2} e^{-at} \sin(\omega t+\phi)$, $\phi = \tan^{-1} \frac{\omega}{\alpha - a}$	$\frac{s+\alpha}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$, $\zeta < 1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
$\frac{1}{a^2+\omega^2} + \frac{1}{\omega\sqrt{a^2+\omega^2}} e^{-at} \sin(\omega t - \phi)$, $\phi = \tan^{-1} \frac{\omega}{-a}$	$\frac{1}{s\left[(s+a)^2+\omega^2\right]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi\right)$, $\phi = \cos^{-1} \zeta$, $\zeta < 1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
$\frac{\alpha}{a^2+\omega^2} + \frac{1}{\omega}\left[\frac{(\alpha-a)^2+\omega^2}{a^2+\omega^2}\right]^{1/2} e^{-at} \sin(\omega t+\phi)$.	$\frac{s+\alpha}{s\left[(s+a)^2+\omega^2\right]}$

TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s \to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s \to \infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Performance Criteria

Name	Symbol	First-order systems	Second-order systems
Rise time	T_r	$\frac{2.2}{a}$	$\frac{2.16\zeta + 0.6}{\omega_n}$
Peak time	T_p	-	$\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
Percent Overshoot	$P.O.$	-	$100e^{-(\zeta\pi/\sqrt{1-\zeta^2})}$
Settling time	T_s	$\frac{4}{a}$	$\frac{4}{\zeta\omega_n}$

Second-order systems

$$T(s) = \frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$$

Sensitivity

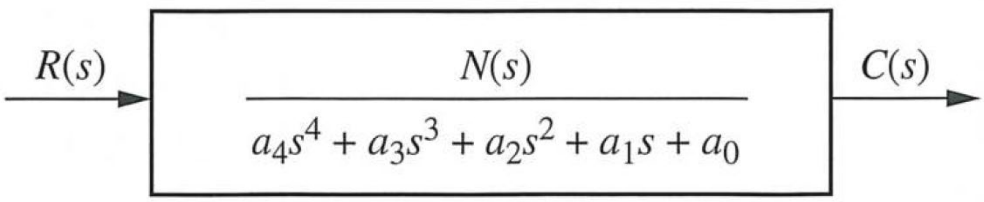
$$S_{F:P} = \frac{P}{F}\frac{\delta F}{\delta P}$$

Quotient Rule

$$h(x) = \frac{f(x)}{g(x)}$$

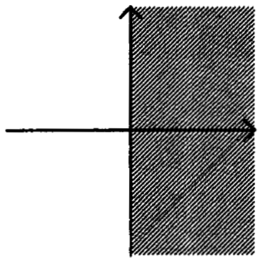
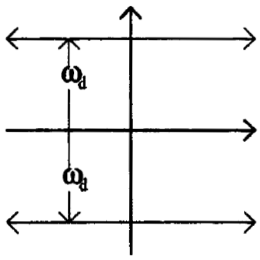
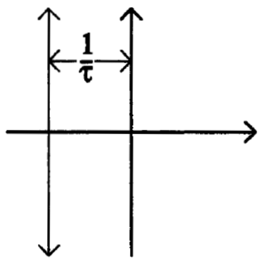
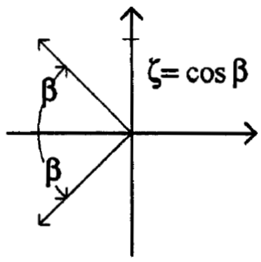
$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}.$$

Routh Array Example



s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Root Locus Plot Interpretation

Name	Formula	S-plane
Region of stability	Re(s) < 0	
Constant damped natural frequency	Im(s) = ω	
Constant time constant	Re(s) = $-\zeta\omega_n = -1/\tau$	
Constant damping ratio	Cos(β)=ζ	
Constant natural frequency	$\mid s \mid = \omega_n$