Loughborough University

Control System Design 2

24WSB105

Semester 2, 2024-25

In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is 2 **hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

Answer **ALL THREE** questions.

Questions carry the marks shown.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the rear of the paper.

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1. Sketch approximate Bode plots for the following transfer functions, marking relevant frequencies, noting the presence or absence of any resonant peaks, and giving the gradients of any asymptotes.

a)

i. $G_1(s) = \frac{s}{s^2+2s+1}$ [6 marks]

ii. $G_2(s) = \frac{10}{s(s+5)}$ [6 marks]

Figure Q1 shows the (open-loop) Bode plot of a mechatronic positioning system, which is open-loop stable.

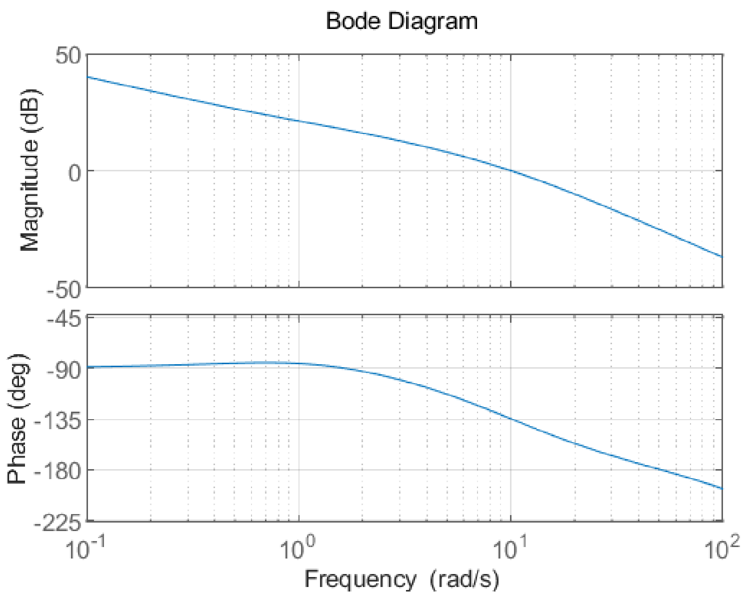

Figure Q1

b) Briefly explain how the Nyquist stability theorem implies that the servomechanism is stable under unity feedback. [4 marks]

c) Estimate the relevant gain and phase margins. [4 marks]

d) Sketch the expected unit step response of the system under unity negative feedback, stating the approximate overshoot, rise time and steady-state error. [5 marks]

2. A process engineer is designing a level controller for a tank between two processing units in a chemical plant. The water level in metres is measured by means of a float and the resulting level $y(t)$ is fed back to a controller implemented on programmable logic controller (PLC), which actuates a valve using a voltage signal $v(t)$ in Volts.

Figure Q2 shows a Bode diagram of the tank, corresponding to the frequency response $Y(j\omega)/V(j\omega)$.

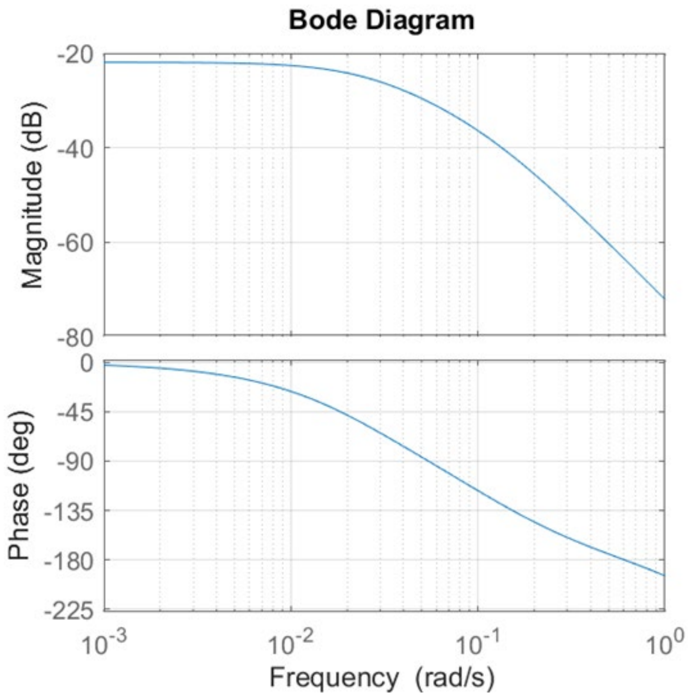

Figure Q2

The engineer wishes to design a feedback controller for the tank that will achieve specifications of less than 10% steady state error and less than 10% overshoot, with a rise time of less than 60 seconds.

a) Estimate the gain required for a proportional i.e. 'P-only' controller to meet the steady-state error specification. [3 marks]

b) Hence, or otherwise, explain why it is not possible to meet the given specifications with a proportional i.e. 'P-only' controller. [3 marks]

The engineer proposes to implement a PI controller that gives a 0dB crossover frequency of 0.05 rad/s and a phase margin of 65 degrees.

c) Show that this choice of controller, crossover frequency and phase margin is suitable, considering the specifications. [3 marks]

d) Calculate suitable values of controller parameters K and T_i for this choice of design crossover frequency and phase margin. [6 marks]

e) Estimate the settling time (to 5%) of the resulting closed-loop system. [2 marks]

When the PI controller is implemented on the PLC, a time delay is introduced due to the sampling process when the PLC reads the water level from the corresponding sensor.

f) Briefly explain the possible effects of this delay on the performance of the control system. [2 marks]

g) If the overshoot of the closed-loop step response must be less than 10%, what is the maximum allowable delay? [3 marks]

In practice, the engineer observes that this controller works well for small changes in set point and small disturbances, but for large changes in setpoint the overshoot is much larger than expected.

h) Suggest a possible explanation, and a possible solution to meet the specifications for large setpoint changes. [3 marks]

3. An automotive engineer is prototyping a lane-keeping assistance system for a motorbike with transfer function $G(s)$, which applies torque to the steering column to give a desired steering angle.

Table Q3 summarises the bike's frequency response function $G(j\omega)$.

Frequency (rad/s)	Magnitude (dB)	Phase angle (deg)
0.1	21.6	-93.4
0.22	14.9	-97.4
0.46	8.38	-106
1	2.14	-123
2.2	-3.07	-155
4.6	-6.96	-201
10	-12.1	-256
22	-21.3	-304
46	-33.4	-333
100	-46.5	-347

Table Q3

The engineer decides to implement a PD controller, choosing a crossover frequency for the design as 2 rad/s and a design phase margin of 45 degrees.

a) Calculate a suitable gain K and derivative time T_d for the PD controller. [6 marks]

In practice, the engineer finds that the overshoot in the step response is too high when using this controller.

b) Estimate the overshoot of the system to a step change in the reference when this PD controller is used. [2 marks]

c) Estimate the minimum achievable rise time for this system if the overshoot must be less than 5%. [3 marks]

As the sensor used for measuring the steering angle is subject to electrical noise, the engineer includes a low-pass filter with a cutoff frequency of approx. 100Hz in the derivative term of the PD controller, so it has a transfer function

$$C(s) = K\left(1 + \frac{T_d s}{1 + 0.019s}\right)$$

where $K = 0.80$ and $T_d = 0.058$.

Figure Q3 shows a step response of the motorcycle, which has transfer function $G(s)$, under unity negative feedback, and when using the PD controller $C(s)$ designed by the engineer.

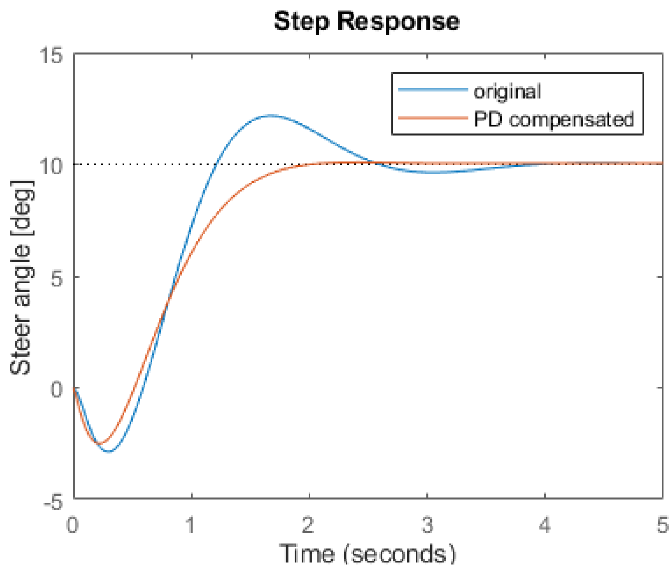

Figure Q3

d) Sketch a Bode plot for this controller, marking any relevant frequencies and the slopes of the asymptotes. [6 marks]

e) Hence, or otherwise, explain the differences and similarities of the step responses in Figure Q3 with reference to the resulting poles and zeros of the closed-loop system. [3 marks]

J Fleming

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