# Engineering Science 2
## 24WSB112

Semester 1 24/25                                         In-Person Exam paper

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**Please fill in:**

ID Number: ☐                    Desk Number: ☐

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This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

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Help during the exam

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

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You may **not** write in pencil for this exam**.**

**Any additional work must be done in the space provided at the back of this paper.**

You may use a calculator for this exam. It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are **not** allowed).

Please answer all questions.

Formula and data sheet is provided at the end of the paper.

**1.** Consider a machine mounted to ground through an isolation system with springs which is deflected by 8 mm under the static load.

Under the free vibration test, it was recorded that the amplitude of motion of the machine decreased with a ratio of 8 after 9 complete cycles.

    a) Find the damped natural frequency.                         [6 marks]

Tick here if you continue at the end of the booklet: ☐

b) Find the mass of the machine if you know that the damping coefficient is equal to $c = 1{,}295.74$ N $s$ m$^{-1}$. [3 marks]

Tick here if you continue at the end of the booklet: ☐

The machine produces a harmonic force of Amplitude $F_0$ during steady-state operating conditions.

c) Determine the force Amplitude $F_0$ if you know that the maximum amplitude at resonance is equal to $X$= 4.4075 mm. [6 marks]

Tick here if you continue at the end of the booklet: ☐

A new isolation with a damping factor $\zeta = 0.8$ has been fitted to reduce the transmitted force to 45% when the machine produced a harmonic force at 105.06 $\text{rad s}^{-1}$ during steady-state operating conditions, as per the safety requirement specification.

d) Determine whether the new isolation damper is appropriate, or it needs to be replaced. [5 marks]

Tick here if you continue at the end of the booklet: ☐

**2.** A machine is mounted on the ground through a spring with a stiffness of 81,738.81 N m$^{-1}$, which deflected under its static load by 12 mm. In the rotating parts of the machine, there is an out-of-balance force created by a mass of 35 g located at a radius $e$, as shown in Fig. Q2.



**Fig. Q2** Vibration of a machine

    a) Prove that the eccentric, $e$, is equal to 14.3 mm if you know that the machine vibrates at 0.005 mm at a frequency more than 10 times higher than the natural frequency.             **[9 marks]**

Tick here if you continue at the end of the booklet: ☐

Tick here if you continue at the end of the booklet: ☐

A damper is fitted with a damping ratio equal to 0.001.

b) Calculate the amplitude of vibration when the out-of-balance mass rotates at $\omega$= 215.5 rpm. [3 marks]

**3.** A rotating shaft supported by bearings 1.2 m apart carries two eccentrically attached masses, $M_1$ and $M_2$, as shown in **Fig. Q3**. The details of the eccentric masses are given in the table below.

|  | Mass (kg) | Radius (cm) | $\theta$ (rad) | $D$ (cm) |
|---|---|---|---|---|
| **M₁** | 1.2 | 12 | $\pi/3$ | 35 |
| **M₂** | 2 | 7.2 | $5\pi/4$ | 75 |



**Fig. Q3** Balance of a rotating shaft

a) Prove how the system is statically balanced but dynamically unbalanced.     [4 marks]

Tick here if you continue at the end of the booklet: ☐

b) If the resulting out-of-balance forces are to be corrected by the addition of two masses attached to the shaft immediately next to each of the bearing locations and at a radius of 10 cm calculate their required masses and attachment angles with respect to the datum direction (given in **Fig. Q3, page 9**). [14 marks]

Tick here if you continue at the end of the booklet: ☐

**4.** Consider the design of a simple air sorting system, shown in **Fig.Q4**, which is used to sort spherical beads of different density. All beads are 1 cm in diameter but can be of two different densities $\rho$: $\rho_1 = 2000$ kg m$^{-3}$; $\rho_2 = 3000$ kg m$^{-3}$. The light beads can be carried away by a properly designed air flow ($\rho_{air} = 1$ kg m$^{-3}$) which can be assumed to be incompressible and with negligible viscosity. The sorting compartment is in cylindrical shape with a diameter of 30 cm. The required air flow is driven by a fan which takes air from a tranquil environment.



**Fig. Q4** An air-assisted sorting system

a) If the drag coefficient for the flow across the beads is 0.5, what is the required air velocity range to carry away a lighter bead but not a heavier bead. [5 marks]

Tick here if you continue at the end of the booklet: ☐

Tick here if you continue at the end of the booklet: ☐

b) What is the required air flow rate and the power input to an 80% efficient fan if the required air velocity in the sorting chamber is 25 m s$^{-1}$? [5 marks]

Tick here if you continue at the end of the booklet: ☐

c) Will the required flow rate increase or decrease when using the system in a very hot environment or at a high altitude? Explain your answer. [5 marks]

Tick here if you continue at the end of the booklet: ☐

**5.** A sealed box containing a drone rest on a set of scales, as shown in **Fig. Q5**. The mass of the box is 2 kg and the mass of the drone is 500 g. Initially, the drone is switched off and at rest, and the scales read a value $m_1$. Next, the drone is switched on and hovers at a constant height, and the scales read $m_2$.



**Fig. Q5** Drone in a box

a) Determine the reading $m_1$ and $m_2$. [5 marks]

Tick here if you continue at the end of the booklet: ☐

b) When hovering, how big is the force exerted onto the air by the drone?
Please clearly indicate the control volume used for analysis. [5 marks]

Tick here if you continue at the end of the booklet: ☐

**6.** Consider a small wind turbine with rotor diameter $D = 50$ cm operating in a location where the wind blows steadily and uniformly at $u_\infty = 8$ m s$^{-1}$ with air density $\rho = 1.25$ kg m$^{-3}$, as in **Fig.Q6**.



**Fig. Q6** Wind turbine

a) Determine the maximum power generation potential under ideal conditions.

[5 marks]

Tick here if you continue at the end of the booklet: ☐

Under real conditions, only a fraction of the available energy can be harvested as shaft power. **Fig. Q6** (page 17) shows points 1 and 2, which are sufficiently far upstream and downstream from the turbine that pressure is atmospheric. The flow at these two points can be simplified as cylindrical uniform flows with diameters $d_1 = 30$ cm and $d_2 = 80$ cm.

b) Determine power generated by the wind turbine and the resultant turbine efficiency (i.e. percentage of the maximum power generation potential). The flow can be approximated as frictionless. [10 marks]

Tick here if you continue at the end of the booklet: ☐

c) Determine the horizontal reaction force $F_R$ on the wind turbine by the base to keep it stationary. [10 marks]

Tick here if you continue at the end of the booklet: ☐

# End of questions

Extra Space for answers to any questions:

Tick here if you continue at the end of the booklet: ☐

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Tick here if you continue at the end of the booklet: ☐

**Equations and other useful information**

**Constants**

The gravitational acceleration $g = 9.81 \text{ m/s}^2$.

**Applied Mechanics**

- SHM: $\omega = 2\pi f = 2\pi \dfrac{1}{T}$

- Natural frequency: $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{d}}$

- Logarithmic decrement: $ln(A_n/A_{n+1}) \approx 2 \times \pi \times \zeta$; $ln(A_n/A_{n+N}) \approx N \times 2 \times \pi \times \zeta$; N=number of cycles (number of periods)

- Critical damping: $c_c = 2.\,m\sqrt{\dfrac{k}{m}} = 2.\,m.\,\omega_n = 2\sqrt{k.\,m}$

- Damping ratio: $\zeta = c/c_C$

- Damped frequency: $\omega_d = \omega_n\sqrt{1 - \zeta^2}$

- Forced vibration equations:

  → Subjected to harmonic force

$$X = \frac{F_0/k}{\sqrt{[(1-r^2)^2+4\zeta^2r^2]}}; \text{ and } \tan(\varphi) = \frac{c.\omega}{k-m.\omega^2} = \frac{2\zeta r}{1-r^2} \text{ ; where } r = \frac{\omega}{\omega_n}$$

  → Subjected to rotating unbalance

$$\frac{X.m}{m\prime.e} = \frac{m.\omega^2}{\sqrt{[(k-m.\omega^2)^2+(c.\omega)^2]}} = \frac{r^2}{\sqrt{(1-r^2)^2+4\zeta^2r^2}} \text{ ; where } r = \frac{\omega}{\omega_n}$$

  → Subjected to base excitation

$$\frac{X}{Y} = \sqrt{\frac{1+4\zeta^2r^2}{(1-r^2)^2+4\zeta^2r^2}}; \text{ where } r = \frac{\omega}{\omega_n}$$

- Transmissibility Ratio:

$$T.R. = \sqrt{\frac{1+4\zeta^2r^2}{(1-r^2)^2+4\zeta^2r^2}}; \text{ where } r = \frac{\omega}{\omega_n}$$

Radius of gyration: $k = \sqrt{\dfrac{I}{A}}$

# Fluid Mechanics

- Hydrostatic force of submerged flat plates: $F_R = P_c A$;

- Position of centre of pressure: $y_{CP} = y_c + \frac{I_{xc}}{y_c A}$; $x_{CP} = x_c + \frac{I_{yc}}{x_c A}$;
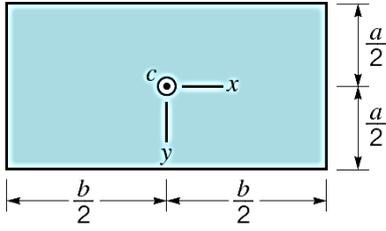


$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$

(a) Rectangle
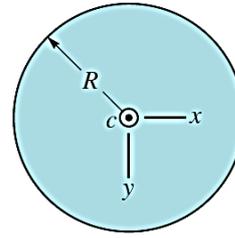
$A = \pi R^2$

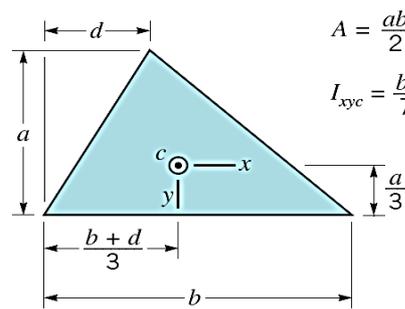$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$

$I_{xyc} = 0$

(b) Circle
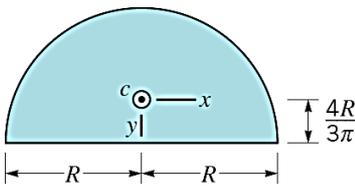
$I_{xc} = 0.1098 R^4$

$I_{yc} = 0.3927 R^4$

$I_{xyc} = 0$

(c) Semicircle

$A = \frac{ab}{2}$    $I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$

(d) Triangle

- Energy Equation for a constant temperature system:

$$\sum_{in} \dot{m}\left(\frac{P}{\rho} + \frac{u^2}{2} + gz\right) + \dot{Q}_{in} + \dot{W}_{in} = \sum_{out} \dot{m}\left(\frac{P}{\rho} + \frac{u^2}{2} + gz\right) + \dot{Q}_{out} + \dot{W}_{out}$$

- Momentum Equation:

$$\sum F = \sum_{out}(\dot{m}\vec{u}) - \sum_{in}(\dot{m}\vec{u}); \quad \sum M = \sum_{out}(\vec{r} \times \dot{m}\vec{u}) - \sum_{in}(\vec{r} \times \dot{m}\vec{u})$$

- Reynolds number: $Re = \frac{\rho u D}{\mu}$

- Haaland equation for frictional factor in turbulent flow:

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

- Pressure loss in pipes: $\Delta P = \left(f\frac{L}{D} + \sum K_L\right)\frac{1}{2}\rho u^2$

- Drag and Lift force: $F_D = C_D A \frac{\rho u^2}{2}$; $F_L = C_L A \frac{\rho u^2}{2}$;