FINITE ELEMENT ANALYSIS

24WSC106

Semester 2 2024/25 | In-Person Exam paper

Please fill in:

ID Number: | Desk Number:

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

You may not write in pencil for this exam.

Any additional work must be done in the space provided at the back of this paper.

Answer **ALL** questions.

All questions carry equal marks.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

A range of formulae and tables likely to be of benefit in the solution of these questions is provided at the rear of the paper. You can detach the formula sheet for easier viewing

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1. Answer the following individual questions. [Subtotal: 50 marks]

a) List the seven major steps in direct stiffness method. [7 marks]

b) In the Finite Element Method or Direct Stiffness Method, what conditions must be met to satisfy a symmetry boundary condition? [4 marks]

For the two-bar truss shown in **Figure 1**, a force of $P = 1000 \text{ kN}$ is applied at node 1 in the positive y direction, while node 1 settles a displacement of $\delta = 50 \text{ mm}$ in the negative x direction. Let $E = 210 \text{ GPa}$ and $A = 6 \times 10^{-4} \text{m}^2$ for each element. The lengths of the elements are shown in the **Figure 1**.

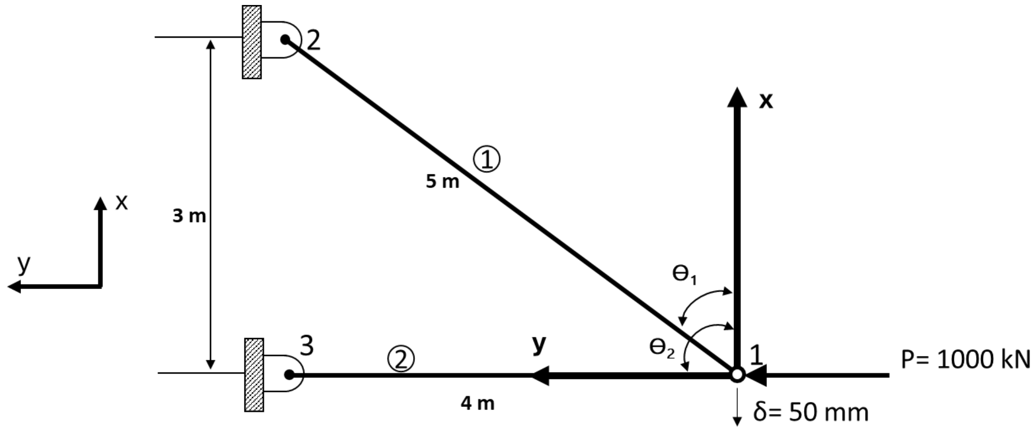

Figure 1 (node: 1, 2, 3; Element ① & ②)

Determine the following (c – h) questions:

c) The degree of freedom (DoF) of the system. [1 mark]

DoF: ___________________

d) The transformation angles (in degrees) for elements 1 and 2 in the given global coordinate system. [2 marks]

Element 1: Θ_1=___________________

Element 2: Θ_2=___________________

e) Element stiffness matrix for both the elements. [4 marks]

k_1=

k_2=

f) The global stiffness matrix [4 marks]

g) The applied boundary condition (B.C.) and reduced global stiffness equation [3 marks]

B.C.: _________________

RGSE:

h) The unknown nodal displacements and reaction forces [7 marks]

A rectangular plane beam section is 6 m long in total. A centrally applied point load $P = 20\ \mathrm{kN}$ and an anti-clockwise moment $M = 20\ \mathrm{kN \cdot m}$ are applied at the node 2 as shown in **Figure 2**. Node 1 and 3 are fixed to the wall. Let $= 210\ \mathrm{GPa}$ and $I = 4 \times 10^{-4}\mathrm{m}^4$. Determine the following using direct stiffness method:

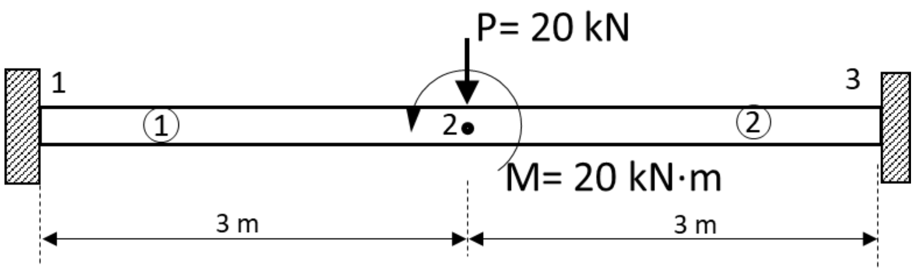

Figure 2 (node: 1, 2, 3; Element ① & ②)

i) The degree of freedom (DoF) per element and the element stiffness matrix for both the elements. [3 marks]

DoF:__________

k1=

k2=

j) The global stiffness <u>equation</u> (GSE) [3 marks]

k) What are the boundary conditions (B.C.) and the reduced GSE? [4 marks]

B.C.: ___________________

RGSE:

l) Unknown displacement and rotation showing the solving process. [4 marks]

m) Reaction force and moment at each node showing solving process. [4 marks]

2. Answer each of the following parts. [Subtotal: 50 marks]

a) Briefly describe the main sources of error in finite element modelling. [4 marks]

b) In a metal forging process simulation (e.g. extrusion, stamping, etc.), describe the sources of nonlinearity and their reasons. [6 marks]

c) Make a list of the main steps of building (pre-processing) a finite element model and briefly describe each step [10 marks]

d) A uniform bar of 100 mm initial length is elongated to a final length of 200 mm in three stages:

Stage 1: 100 mm to 120 mm

Stage 2: 120 mm to 150 mm

Stage 3: 150 mm to 200 mm

Calculate the engineering (e) and true (ε) strains for each stage and compare the sums of three stages with the overall values of the strains. ($\varepsilon = \ln\left(\frac{l_i}{l_i} - 1\right)$; $e_i = \frac{\Delta l}{l_i}$) [14 marks]

e) Using the Gauss quadrature parameters in **Table 1**, calculate the area under $f(x)$ and $g(x)$ functions (**Figure 3**) between $x = -1$ and $x = +1$ using Gauss quadrature method. Compare these answers with those obtained using standard integration techniques. [12 marks]

Numerical integration with Gauss quadrature technique is:

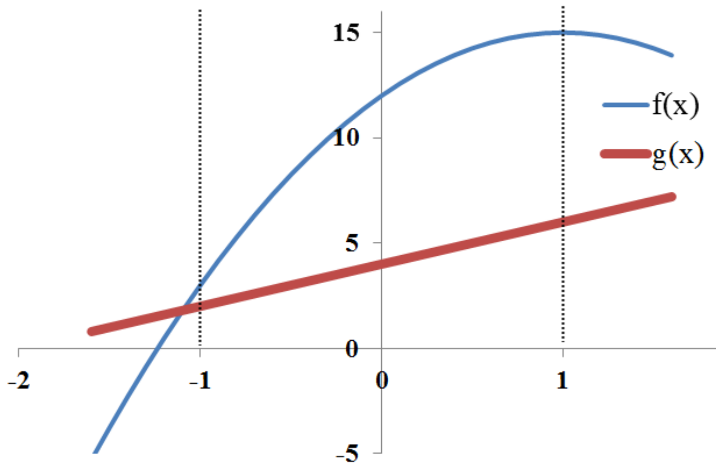

Figure 3. Curves for $f(x) = -2x^2 + 5x + 10$; $g(x) = 3x + 5$

Degree of polynomial	No. of points (n)	Sampling point	Weighting Function (W)
1	1	0	2
2-3	2	±0.577350	1
4-5	3	0 ±0.774597	0.888999 0.555556

Table 1. Sampling points and weights for Gauss quadrature (limits -1 to 1)

f) Briefly explain the methods (with examples) for verifying finite element simulation results. [4 marks]

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Element Stiffness Matrix Formulae

2x2 matrix inversion:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \& \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$$

$$\text{Then } A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

For general bar/truss element:

$$[K] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

Bar/Truss in global coordinates

$$[K] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix} \qquad \begin{matrix} C = \cos\theta \\ S = \sin\theta \end{matrix}$$

For plane beam element:

$$[k] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

In Global Coordinates:

$$\frac{EI}{L^3}\begin{bmatrix} 12S^2 & -12SC & -6LS & -12S^2 & 12SC & -6LS \\ & 12C^2 & 6LC & 12SC & -12C^2 & 6LC \\ & & 4L^2 & 6LS & -6LC & 2L^2 \\ & & & 12S^2 & -12SC & 6LS \\ & & & & 12C^2 & -6LC \\ \text{Symmetry} & & & & & 4L^2 \end{bmatrix}$$

For plane frame element:

$$[K] = \begin{bmatrix} C_1 & 0 & 0 & -C_1 & 0 & 0 \\ 0 & 12C_2 & 6C_2L & 0 & -12C_2 & 6C_2L \\ 0 & 6C_2L & 4C_2L^2 & 0 & -6C_2L & 2C_2L^2 \\ -C_1 & 0 & 0 & C_1 & 0 & 0 \\ 0 & -12C_2 & -6C_2L & 0 & 12C_2 & -6C_2L \\ 0 & 6C_2L & 2C_2L^2 & 0 & -6C_2L & 4C_2L^2 \end{bmatrix} \qquad \begin{matrix} C_1 = \dfrac{AE}{L} \\ \\ C_2 = \dfrac{EI}{L^3} \end{matrix}$$

Beam Theory Equations:

$$M(x) = EI\frac{d^2v}{dx^2}$$

Strain energy U in bending:

$$U = \frac{1}{2}\int_0^L \frac{M^2}{EI}\,dx$$