ADVANCED HEAT TRANSFER

24WSC801

Semester 1, 2024 In-Person Exam paper

This examination is to take place in-person at a central University venue under exam conditions. The standard length of time for this paper is **2 hours**.

You will not be able to leave the exam hall for the first 30 or final 15 minutes of your exam. Your invigilator will collect your exam paper when you have finished.

<u>Help during the exam</u>

Invigilators are not able to answer queries about the content of your exam paper. Instead, please make a note of your query in your answer script to be considered during the marking process.

If you feel unwell, please raise your hand so that an invigilator can assist you.

Answer **ALL THREE** questions.

All questions carry equal marks.

Use of a calculator is permitted - It must comply with the University's Calculator Policy for In-Person exams, in particular that it must not be able to transmit or receive information (e.g. mobile devices and smart watches are not allowed).

Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}$ *W/(m^{2}K^4)*

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1. **Figure Q.1** shows a two-dimensional cross-section of a plate. The left side of this plate is at a temperature of T_A (°C). The right side is exposed to convective heat transfer, with a convective heat transfer coefficient of h (W/m^2 °C) and an ambient temperature of T_∞ (°C). The plate is also subjected to internal heat generation at a rate of g (W/m^3). Heat transfer in this one-dimensional situation is governed by:

$$k\frac{d}{dx}\left(\frac{dT}{dx}\right)+g=0$$

The thickness of the plate is L (m), thermal conductivity is k (W/(m.°C)).

Data $L = 60$ mm., $k = 20$ W/(m °C), $T_A = 10$ °C, $h = 50$ W/(m^2 °C), and $T_\infty = 20$ °C, $g = 10^6$ (W/m^3).

a) Using the given boundary conditions and by integrating the governing equation, show that the temperature distribution of the plate is given by:

$$T(x) = -\frac{g}{2k}x^2 + \left[\frac{gL + h[gL^2/2k - T_A + T_\infty]}{(hL+k)}\right]x + T_A$$

[6 marks]

b) Use the energy balance method with a simple grid ($\Delta x = 20\,mm$) as shown in the **Figure Q.1,** to formulate numerical equations for obtaining temperature values at the nodes shown. [6 marks]

c) Using elimination, solve the equations obtained in (b) and calculate the temperature at node 4. [5 marks]

d) Compare your numerical solution obtained in (c) with the temperature obtained from the analytical solution (a) at node 4 and comment on the accuracy of the numerical solution. [3 marks]

2. Shown in **Figure Q.2** is a cross-section of a two-dimensional triangular duct whose surfaces are numbered 1, 2 and 3 as shown. Areas of surfaces 1, 2 and 3 are A_1, A_2 and A_3 respectively.

a) Using the reciprocal and enclosure properties of configuration factors, show that configuration factors F_{1-2} and F_{1-3} are given by

$F_{1-2} = (A_1 + A_2 - A_3)/2A_1$ and $F_{1-3} = (A_1 + A_3 - A_2)/2A_1$

[6 marks]

In usual notations, Reciprocal property: $A_iF_{ij} = A_jF_{ji}$.

Enclosure property: for a geometry forming an enclosure with 1 . … n number of surfaces, for the surface i =1,

$$\sum_{j=1}^{n} F_{i-j} = F_{1-1} + F_{1-2} + F_{1-3} + F_{1-4} \ldots\ldots\ldots\ldots\ldots.. F_{1-n} = 1$$

The dimensions of this geometry are also shown in **Figure Q.2**. Surfaces 1, 2 and 3 are at temperatures of 500 K, 600 K. and 400 K respectively and emissivities are $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.7$ and $\varepsilon_3 = 0.6$. In usual notations for an enclosure with n number of surfaces, the radiosity of the i^{th} surface is related to the other radiosities by the equation:

$$J_i - (1 - \varepsilon_i)\left[\sum_{j=1}^{n} F_{ij}\, J_j\right] = \epsilon_i\, E_{bi}$$

b) Using the equation above, write equations for each surface to connect their own radiosities to the radiosities of the other surfaces for the triangular configuration shown in **Figure Q.2**. [3 marks]

c) Calculate the necessary configuration factors required for the equations obtained in (b), substitute them into the equations, and rearrange your equations into matrix form. [7 marks]

d) If the solution to the matrix equation obtained in (c) is given as $J_1 = 3.6059$ kW/m^2 , $J_2 = 6.1413$ kW/m^2 and $J_3 = 2.4824$ kW/m^2 calculate heat transfer at each surface (per m length). [2 marks]

e) Using the heat transfer values obtained in (d), demonstrate a heat balance for the problem. [2 marks]

3. **Figure Q.3(a)** shows a cross-section of a furnace geometry. The walls and roof of the furnace (surface 1) are at 800 K and have a surface emissivity of 0.45. The bottom surface (surface 2) is at 1000 K with an emissivity of 0.8. The interior volume of the furnace is filled with a mixture of flue gases, comprising 22.45% CO_2, 11.27% H_2O, and 66.28% N_2 by volume at a pressure of 100 kPa. At steady state, the gaseous mixture maintains a fixed temperature of 1000 K.

In this geometry the configuration factor $F_{2-1} = 1$, Configuration factors to the gaseous medium F_{1-g} and F_{2-g} may be taken as 1.0.

a) Using a value of 1.1 m for the path length, estimate the emissivity of the gaseous mixture with the aid of **Figures Q.3(b) and Q.3(c)**. [5 marks]

b) Draw an equivalent electrical network for this configuration, including gaseous radiation. [3 marks]

c) For the network developed in (b), calculate all necessary resistances and write the current balance equations that connect the blackbody emissive powers and radiosities of the surfaces and the gaseous volume. [4 marks]

d) Solve the equations obtained in (c) and calculate the heat transfer at each surface and from the gaseous volume. [6 marks]

e) Demonstrate a heat balance using calculated heat transfer values. [2 marks]

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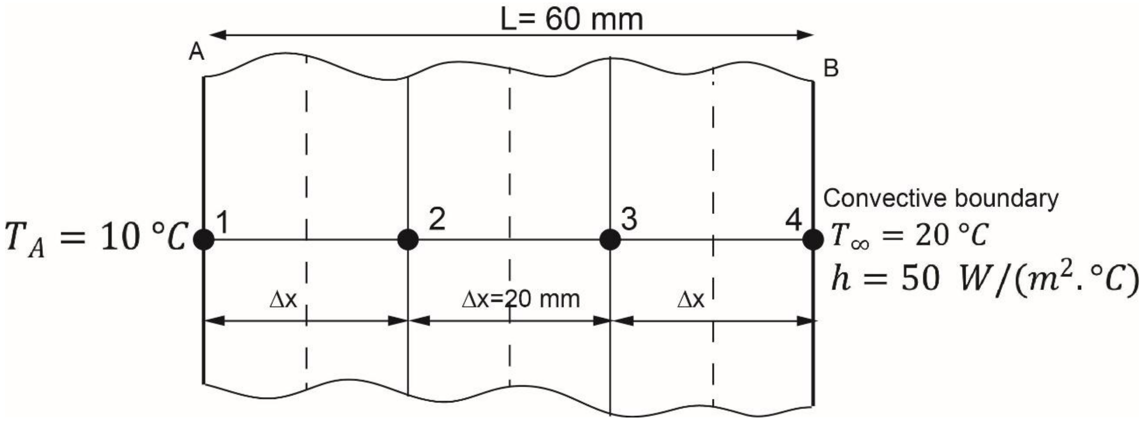

Figure Q.1

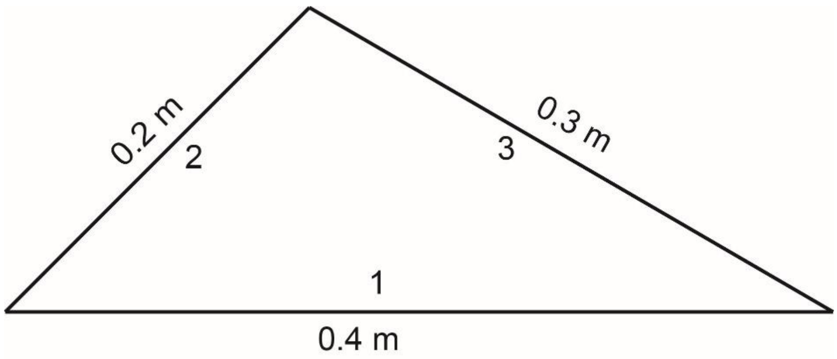

Figure Q.2

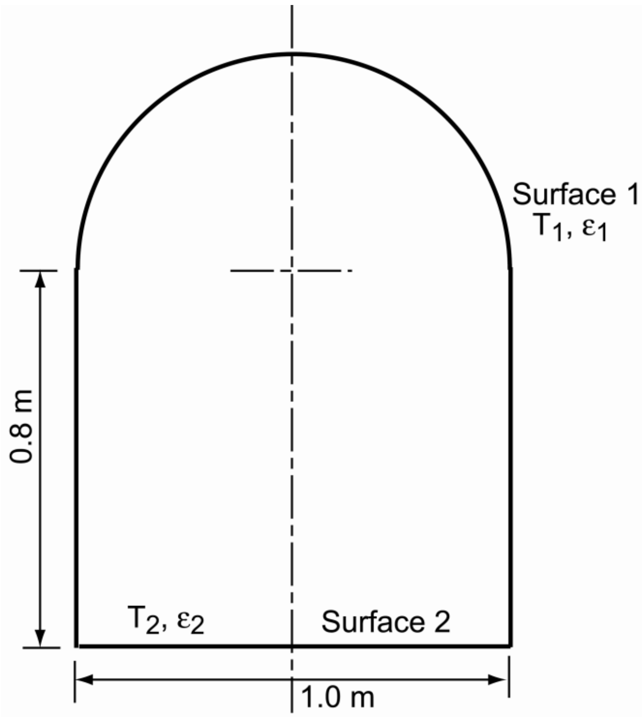

Figure Q.3(a)

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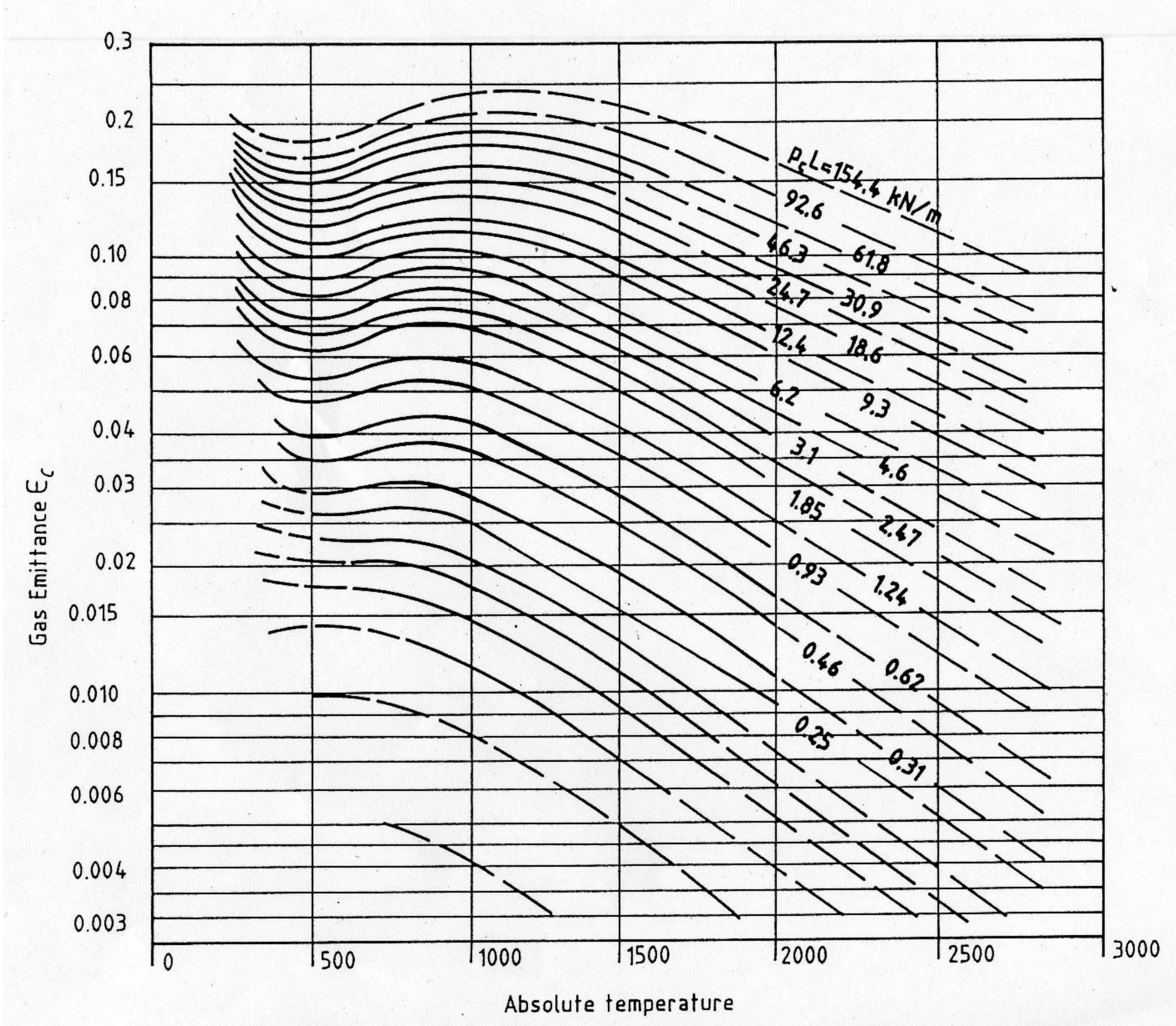

Figure Q.3(b) – Emissivity of CO_2 at a total pressure of 100 kPa

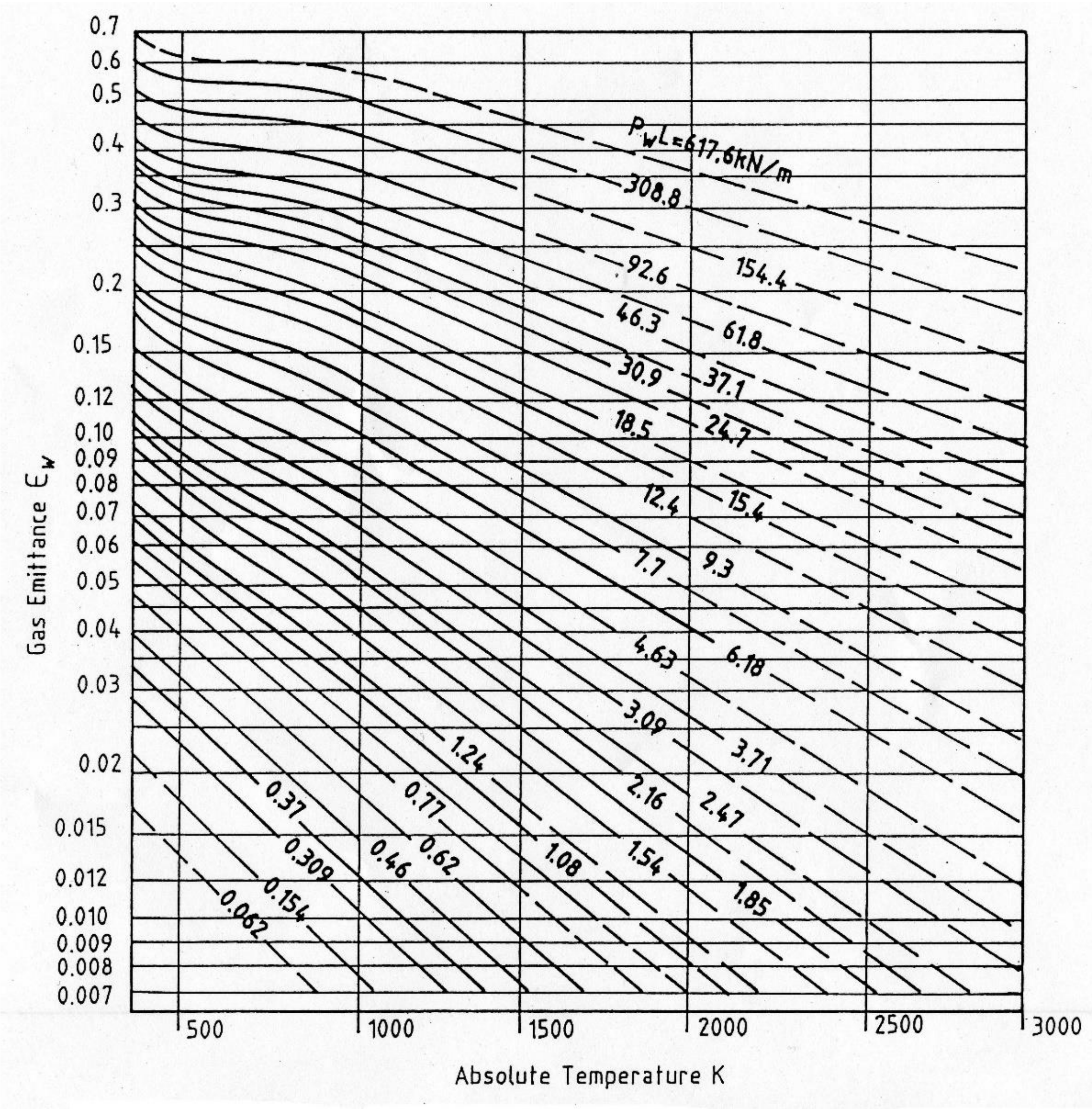

Figure Q.3(c) – Emissivity of H_2O at a total pressure of 100 kPa