FB-DC2 Electric Circuits: Ohm's Law

Contents

1. How voltage, current, and resistance relate

An electric circuit is formed when a conductive path is created to allow free electrons to continuously move. This continuous movement of free electrons through the conductors of a circuit is called a *current*, and it is often referred to in terms of "flow," just like the flow of a liquid through a hollow pipe.

The force motivating electrons to "flow" in a circuit is called *voltage*. Voltage is a specific measure of potential energy that is always relative between two points. When we speak of a certain amount of voltage being present in a circuit, we are referring to the measurement of how much *potential* energy exists to move electrons from one particular point in that circuit to another particular point. Without reference to *two* particular points, the term "voltage" has no meaning.

Free electrons tend to move through conductors with some degree of friction, or opposition to motion. This opposition to motion is more properly called *resistance*. The amount of current in a circuit depends on the amount of voltage available to motivate the electrons, and also the amount of resistance in the circuit to oppose electron flow. Just like voltage, resistance is a quantity relative between two points. For this reason, the quantities of voltage and resistance are often stated as being "between" or "across" two points in a circuit.
To be able to make meaningful statements about these quantities in circuits, we need to be able to describe their quantities in the same way that we might quantify mass, temperature, volume, length, or any other kind of physical quantity. For mass we might use the units of "pound" or "gram." For temperature we might use degrees Fahrenheit or degrees Celsius. Here are the standard units of measurement for electrical current, voltage, and resistance:

Quantity	Symbol	Unit of Measurement	Unit Abbreviation
Current	I	Ampere ("Amp")	A
Voltage	E *or* V	Volt	V
Resistance	R	Ohm	Ω

The "symbol" given for each quantity is the standard alphabetical letter used to represent that quantity in an algebraic equation. Standardised letters like these are common in the disciplines of physics and engineering, and are internationally recognised. The "unit abbreviation" for each quantity represents the alphabetical symbol used as a shorthand notation for its particular unit of measurement. And, yes, that strange-looking "horseshoe" symbol is the capital Greek letter Ω, just a character in a *foreign* alphabet (apologies to any Greek readers here).

Each unit of measurement is named after a famous experimenter in electricity: The *amp* after the Frenchman Andre M. Ampere, the *volt* after the Italian Alessandro Volta, and the *ohm* after the German Georg Simon Ohm.

The mathematical symbol for each quantity is meaningful as well. The "R" for resistance and the "V" for voltage are both self-explanatory, whereas "I" for current seems a bit weird. The "I" is thought to have been meant to represent "Intensity" (of electron flow), and the other symbol for voltage, "E," stands for "Electromotive force." From what research I've been able to do, there seems to be some dispute over the meaning of "I." The symbols "E" and "V" are interchangeable for the most part, although some texts reserve "E" to represent voltage across a source (such as a battery or generator) and "V" to represent voltage across anything else.

All of these symbols are expressed using capital letters, except in cases where a quantity (especially voltage or current) is described in terms of a brief period of time (called an "instantaneous" value). For example, the voltage of a battery, which is stable over a long period of time, will be symbolised with a capital letter "E," while the voltage peak of a lightning strike at the very instant it hits a power line would most likely be symbolised with a lower-case letter "e" (or lower-case "v") to designate that value as being at a single moment in time. This same lower-case convention holds true for current as well, the lower-case letter "i" representing current at some instant in time. Most direct-current (DC) measurements, however, being stable over time, will be symbolised with capital letters.

One foundational unit of electrical measurement, often taught in the beginnings of electronics courses but used infrequently afterwards, is the unit of the *coulomb*, which is a measure of electric charge proportional to the number of electrons in an imbalanced state. One coulomb of charge is equal to 6,250,000,000,000,000,000 electrons. The symbol for electric charge quantity is the capital letter "Q," with the unit of coulombs abbreviated by the capital letter "C." It so happens that the unit for electron flow, the amp, is equal to 1 coulomb of electrons passing by a given point in a circuit in 1 second of time. Cast in these terms, current is the *rate of electric charge motion* through a conductor.

As stated before, voltage is the measure of *potential energy per unit charge* available to motivate electrons from one point to another. Before we can precisely define what a "volt" is, we must understand how to measure this quantity we call "potential energy." The general metric unit for energy of any kind is the *joule*, equal to the amount of work performed by a force of 1 Newton exerted through a motion of 1 meter (in the same direction). In British units, this is slightly less than 3/4 pound of force exerted over a distance of 1 foot. Put in common terms, it takes about 1 joule of energy to lift a 3/4 pound weight 1 foot off the ground, or to drag something a distance of 1 foot using a parallel pulling force of 3/4 pound. Defined in these scientific terms, 1 volt is equal to 1 joule of electric potential energy per (divided by) 1 coulomb of charge. Thus, a 9 volt battery releases 9 joules of energy for every coulomb of electrons moved through a circuit.

The units and symbols for electrical quantities will become very important to know as we begin to explore the relationships between them in circuits. The first, and perhaps most important, relationship between current, voltage, and resistance is called Ohm's Law. This Law describes how the three quantities interrelate:

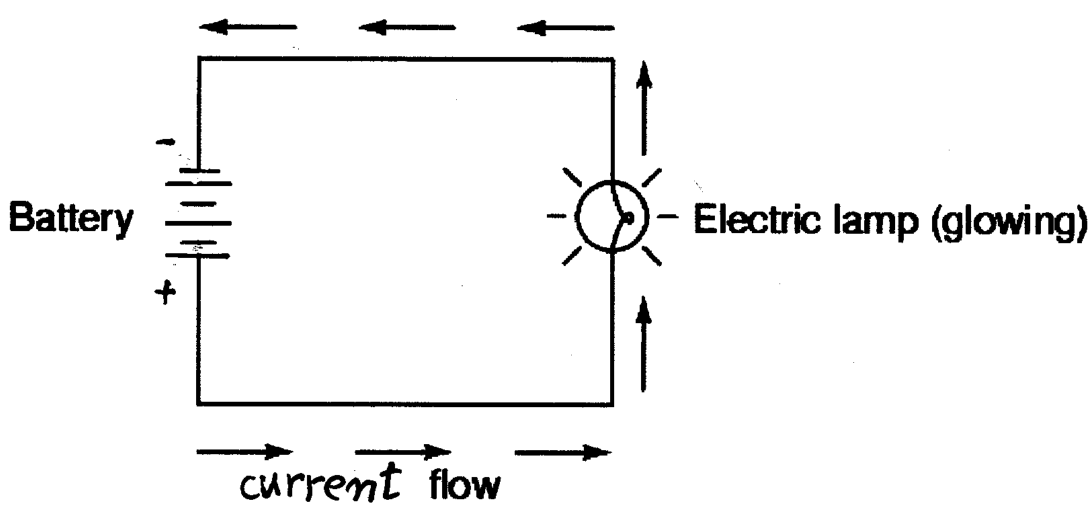
$$E = I\,R$$

In this algebraic expression, voltage (E) is equal to current (I) multiplied by resistance (R). Using algebra techniques, we can manipulate this equation into two variations, solving for I and for R, respectively:

$$I = \frac{E}{R} \qquad\qquad R = \frac{E}{I}$$

Let's see how these equations might work to help us analyse simple circuits:

In the circuit below, there is only one source of voltage (the battery, on the left) and only one source of resistance to current (the lamp, on the right). This makes it very easy to apply Ohm's Law. If we know the values of any two of the three quantities (voltage, current, and resistance) in this circuit, we can use Ohm's Law to determine the third.
In this first example, we will calculate the amount of current (I) in a circuit, given values of voltage (E=12V)) and resistance (R=3 ohm):



What is the amount of current (I) in this circuit?

$$I = \frac{E}{R} = \frac{12\ \text{V}}{3\ \Omega} = 4\ \text{A}$$

Section Review:

- Voltage measured in *volts*, symbolised by the letters "E" or "V"
- Current measured in *amps*, symbolised by the letter "I"
- Resistance measured in *ohms*, symbolised by the letter "R"
- Ohm's Law: E = IR ; I = E/R ; R = E/I

2. An analogy for Ohm's Law

Ohm's Law also makes intuitive sense if you apply if to the water-and-pipe analogy. If we have a water pump that exerts pressure (voltage) to push water around a "circuit" (current) through a restriction (resistance), we can model how the three variables interrelate. If the resistance to water flow stays the same and the pump pressure increases, the flow rate must also increase.

Pressure	=	increase	**Voltage**	=	increase
Flow rate	=	increase	**Current**	=	increase
Resistance	=	same	**Resistance**	=	same

$$\overset{\uparrow}{E} = \overset{\uparrow}{I}\ R$$

If the pressure stays the same and the resistance increases (making it more difficult for the water to flow), then the flow rate must decrease:

Pressure	=	same	**Voltage**	=	same
Flow rate	=	decrease	**Current**	=	decrease
Resistance	=	increase	**Resistance**	=	increase

$$E = \underset{\downarrow}{I}\ \overset{\uparrow}{R}$$

If the flow rate were to stay the same while the resistance to flow decreased, the required pressure from the pump would necessarily decrease:

Pressure	=	decrease	**Voltage**	=	decrease
Flow rate	=	same	**Current**	=	same
Resistance	=	decrease	**Resistance**	=	decrease

$$\begin{array}{ccc} E & = I & R \\ \downarrow & & \downarrow \end{array}$$

As odd as it may seem, the actual mathematical relationship between pressure, flow, and resistance is actually more complex for fluids like water than it is for electrons. If you pursue further studies in physics, you will discover this for yourself. Thankfully for the electronics student, the mathematics of Ohm's Law is very straightforward and simple.

Section Review:

- With resistance steady, current follows voltage (an increase in voltage means an increase in current, and visa-versa).
- With voltage steady, changes in current and resistance are opposite (an increase in current means a decrease in resistance, and visa-verse).
- With current steady, voltage follows resistance (an increase in resistance means an increase in voltage).

3. Power in electric circuits

In addition to voltage and current, there is another measure of free electron activity in a circuit: *power*. First, we need to understand just what power is before we analyse it in any circuits.

Power is a measure of how much work can be performed in a given amount of time. *Work* is generally defined in terms of the lifting of a weight against the pull of gravity. The heavier the weight and/or the higher it is lifted, the more work has been done. *Power* is a measure of how rapidly a standard amount of work is done.

In electric circuits, power is a function of both voltage and current. Not surprisingly, this relationship bears striking resemblance to the "proportional" horsepower formula above:

$$P = I\,E$$

In this case, however, power (P) is exactly equal to current (I) multiplied by voltage (E), rather than merely being proportional to IE. When using this formula, the unit of measurement for power is the *watt*, abbreviated with the letter "W."

It must be understood that neither voltage nor current by themselves constitute power. Rather, power is the combination of both voltage *and* current in a circuit. Remember that voltage is the specific work (or potential energy) per unit charge, while current is the rate at which electric charges move through a conductor. Voltage (specific work) is analogous to the work done in lifting a weight against the pull of gravity. Current (rate) is analogous to the speed at which that weight is lifted. Together as a product (multiplication), voltage (work) and current (rate) constitute power.

Just as in the case of the diesel tractor engine and the motorcycle engine, a circuit with high voltage and low current may be dissipating the same amount of power as a circuit with low voltage and high current. Neither the amount of voltage alone nor the amount of current alone indicates the amount of power in an electric circuit.

In an open circuit, where voltage is present between the terminals of the source and there is zero current, there is *zero* power dissipated, no matter how great that voltage may be. Since P=IE and I=0 and anything multiplied by zero is zero, the power dissipated in any open circuit must be zero. Likewise, if we were to have a short circuit constructed of a loop of superconducting wire (absolutely zero resistance), we could have a condition of current in the loop with zero voltage, and likewise no power would be dissipated. Since P=IE and E=0 and anything multiplied by zero is zero, the power dissipated in a superconducting loop must be zero. (We'll be exploring the topic of superconductivity in a later chapter).

Section Review:

- Power is the measure of how much work can be done in a given amount of time.
- Mechanical power is commonly measured (in America) in "horsepower."
- Electrical power is almost always measured in "watts," and it can be calculated by the formula P = IE.
- Electrical power is a product of both voltage *and* current, not either one separately.
- Horsepower and watts are merely two different units for describing the same kind of physical measurement, with 1 horsepower equalling 745.7 watts.

4. Ohm's Law for calculating power

We've seen the formula for determining the power in an electric circuit: by multiplying the voltage in "volts" by the current in "amps" we arrive at an answer in "watts." Let's apply this to earlier circuit by assuming that the battery voltage is 18V and the lamp resistance 3 ohms:

$$I = \frac{E}{R} = \frac{18\ \text{V}}{3\ \Omega} = 6\ \text{A}$$

Now that we know the current, we can take that value and multiply it by the voltage to determine power:

$$P = I\,E = (6\text{ A})(18\text{ V}) = 108\text{ W}$$

Answer: the lamp is dissipating (releasing) 108 watts of power, most likely in the form of both light and heat.

If, $I = \frac{E}{R}$ *and* $P = I\,E$

Then, $P = \frac{E}{R}E$ *or* $\mathbf{P = \frac{E^2}{R}}$

If we only know current (I) and resistance (R):

If, $E = I\,R$ *and* $P = I\,E$

Then, $P = I(I\,R)$ *or* $\mathbf{P = I^2 R}$

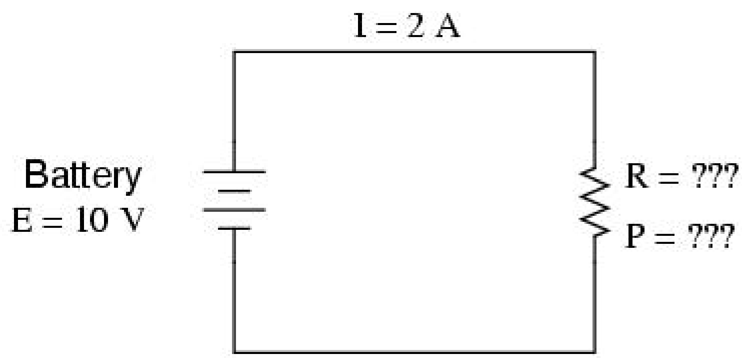
So, in the end, we have these three Ohm's Law equations for electric power in a circuit:

Ohm's Law for Power

$$P = IE \qquad P = \frac{E^2}{R} \qquad P = I^2R$$

5. Resistors

In schematic diagrams, resistor symbols are sometimes used to illustrate any general type of device in a circuit doing something useful with electrical energy. Any non-specific electrical device is generally called a *load*, so if you see a schematic diagram showing a resistor symbol labelled "load," especially in a tutorial circuit diagram explaining some concept unrelated to the actual use of electrical power, that symbol may just be a kind of shorthand representation of something else more practical than a resistor.
To summarise what we've learned in this lesson, let's analyse the following circuit, determining all that we can from the information given:



All we've been given here to start with is the battery voltage (10 volts) and the circuit current (2 amps). We don't know the resistor's resistance in ohms or the power dissipated by it in watts. Surveying our array of Ohm's Law equations, we find two equations that give us answers from known quantities of voltage and current:

$$R = \frac{E}{I} \qquad \textit{and} \qquad P = IE$$

Inserting the known quantities of voltage (E) and current (I) into these two equations, we can determine circuit resistance (R) and power dissipation (P):

$$R = \frac{10\ \text{V}}{2\ \text{A}} = 5\ \Omega$$

$$P = (2\ \text{A})(10\ \text{V}) = 20\ \text{W}$$

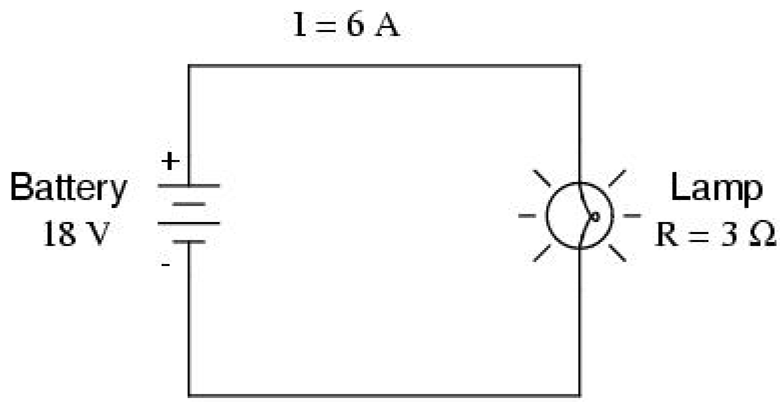
For the circuit conditions of 10 volts and 2 amps, the resistor's resistance must be 5 ohm. If we were designing a circuit to operate at these values, we would have to specify a resistor with a minimum power rating of 20 watts, or else it would overheat and fail.

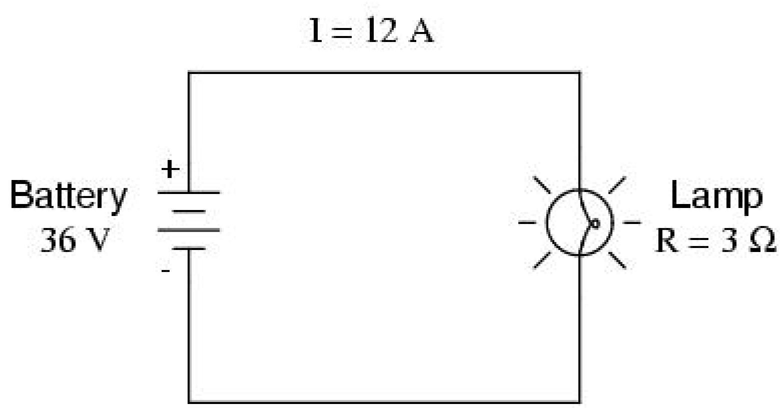
Section Review:

- Devices called *resistors* are built to provide precise amounts of resistance in electric circuits. Resistors are rated both in terms of their resistance (ohms) and their ability to dissipate heat energy (watts).
- Resistor resistance ratings cannot be determined from the physical size of the resistor(s) in question, although approximate power ratings can. The larger the resistor is, the more power it can safely dissipate without suffering damage.
- Any device that performs some useful task with electric power is generally known as a *load*. Sometimes resistor symbols are used in schematic diagrams to designate a non-specific load, rather than an actual resistor.

6. Non-linear conduction

Ohm's Law is a simple and powerful mathematical tool for helping us analyse electric circuits, but we must realise that reality is not always as simple as the mathematical techniques people devise to explain it. For most conductors, resistance is a rather stable property, largely unaffected by voltage or current. For this reason, we can regard the resistance of most circuit components as a constant, with voltage and current being inversely related to each other.
For instance, our previous circuit example with the 3 Ω lamp, we calculated current through the circuit by dividing voltage by resistance (I=E/R). With an 18 volt battery, our circuit current was 6 amps. Doubling the battery voltage to 36 volts resulted in a doubled current of 12 amps. All of this makes sense, of course, so long as the lamp continues to provide exactly the same amount of friction to the flow of electrons through it: 3 Ω.

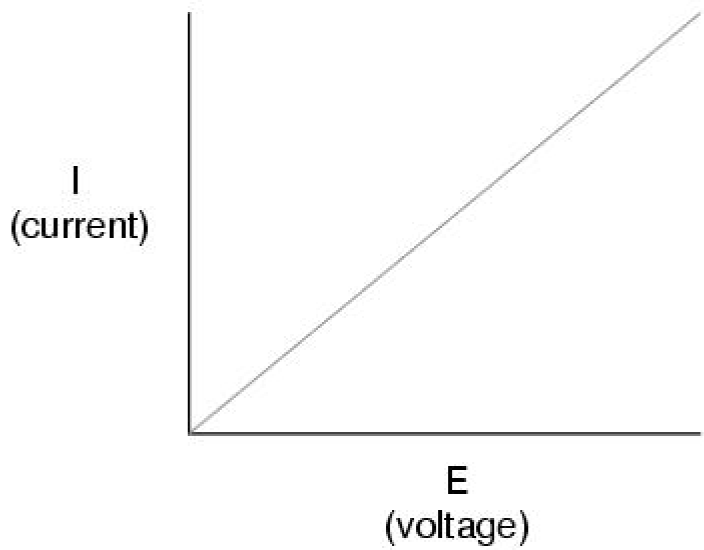




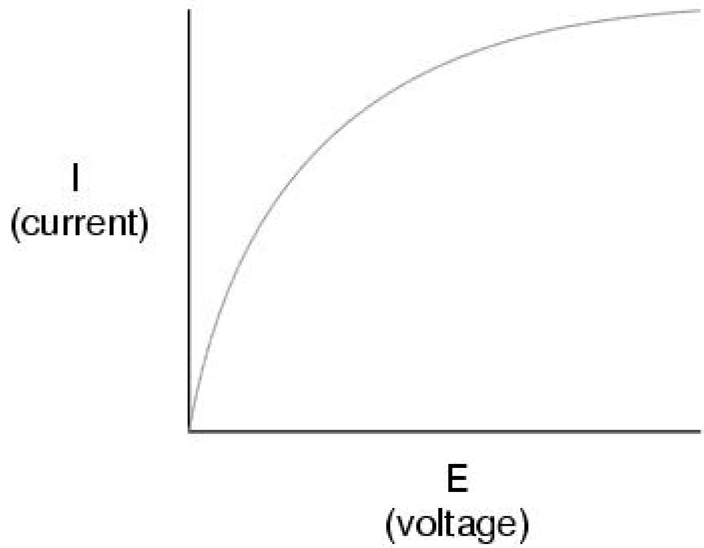
However, reality is not always this simple. One of the phenomena explored in a later chapter is that of conductor resistance *changing* with temperature. In an incandescent lamp (the kind employing the principle of electric current heating a thin filament of wire to the point that it glows white-hot), the resistance of the filament wire will increase dramatically as it warms from room temperature to operating temperature. If we were to increase the supply voltage in a real lamp circuit, the resulting increase in current would cause the filament to increase temperature, which would in turn increase its resistance,

thus preventing further increases in current without further increases in battery voltage. Consequently, voltage and current do not follow the simple model predicted by Ohm's Law in this circuit with a stable resistance value of 3 Ω, because an incandescent lamp's filament resistance does not remain stable for different currents.
The phenomenon of resistance changing with variations in temperature is one shared by almost all metals, of which most wires are made. For most applications, these changes in resistance are small enough to be ignored. In the application of metal lamp filaments, the change happens to be quite large.

This is just one example of "nonlinearity" in electric circuits. It is by no means the only example. A "linear" function in mathematics is one that tracks a straight line when plotted on a graph. The simplified version of the lamp circuit with a constant filament resistance of 3 Ω generates a plot like this:



The straight-line plot of current over voltage indicates that resistance is a stable, unchanging value for a wide range of circuit voltages and currents. In an "ideal" situation, this is the case. Resistors, which are manufactured to provide a definite, stable value of resistance, behave very much like the plot of values seen above. A mathematician would call their behaviour "linear."
A more realistic analysis of a lamp circuit, however, over several different values of battery voltage would generate a plot of this shape:



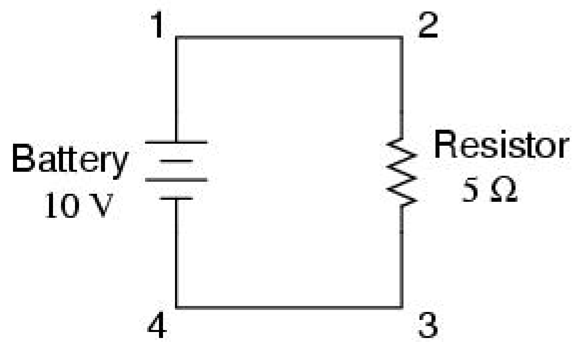
The plot is no longer a straight line. It rises sharply on the left, as voltage increases from zero to a low level. As it progresses to the right we see the line flattening out, the circuit requiring greater and greater increases in voltage to achieve equal increases in current. If we try to apply Ohm's Law to find the resistance of this lamp circuit with the voltage and current values plotted above, we arrive at several different values. We could say that the resistance here is *non-linear*, increasing with increasing current and voltage. The nonlinearity is caused by the effects of high temperature on the metal wire of the lamp filament.

Section Review:

- The resistance of most conductive materials is stable over a wide range of conditions, but this is not true of all materials.
- Any function that can be plotted on a graph as a straight line is called a *linear* function. For circuits with stable resistances, the plot of current over voltage is linear ($I=E/R$).
- In circuits where resistance varies with changes in either voltage or current, the plot of current over voltage will be *non-linear* (not a straight line).

7. Circuit wiring

So far, we've been analysing single-battery, single-resistor circuits with no regard for the connecting wires between the components, so long as a complete circuit is formed. Does the wire length or circuit "shape" matter to our calculations? Let's look at a couple of circuit configurations and find out:



When we draw wires connecting points in a circuit, we usually assume those wires have negligible resistance. As such, they contribute no appreciable effect to the overall resistance of the circuit, and so the only resistance we have to contend with is the resistance in the components. In the above circuits, the only resistance comes from the 5 Ω resistors, so that is all we will consider in our calculations. In real life, metal wires actually *do* have resistance (and so do power sources!), but those resistances are generally so much smaller than the resistance present in the other circuit components that they can be safely ignored. Exceptions to this rule exist in power system wiring, where even very small amounts of conductor resistance can create significant voltage drops given normal (high) levels of current.

If connecting wire resistance is very little or none, we can regard the connected points in a circuit as being *electrically common*. That is, points 1 and 2 in the above circuits may be physically joined close together or far apart, and it doesn't matter for any voltage or resistance measurements relative to those points. The same goes for points 3 and 4. It is as if the ends of the resistor were attached directly across the terminals of the battery, so far as our Ohm's Law calculations and voltage measurements are concerned.

Section Review:

- Connecting wires in a circuit are assumed to have zero resistance unless otherwise stated.
- Wires in a circuit can be shortened or lengthened without impacting the circuit's function -- all that matters is that the components are attached to one another in the same sequence.
- Points directly connected together in a circuit by zero resistance (wire) are considered to be *electrically common*.
- Electrically common points, with zero resistance between them, will have zero voltage dropped between them, regardless of the magnitude of current (ideally).
- The voltage or resistance readings referenced between sets of electrically common points will be the same.
- These rules apply to *ideal* conditions, where connecting wires are assumed to possess absolutely zero resistance. In real life this will probably not be the case, but wire resistances should be low enough so that the general principles stated here still hold.