FB-DC4 Electric Circuits: Divider Circuits And Kirchhoff's Laws

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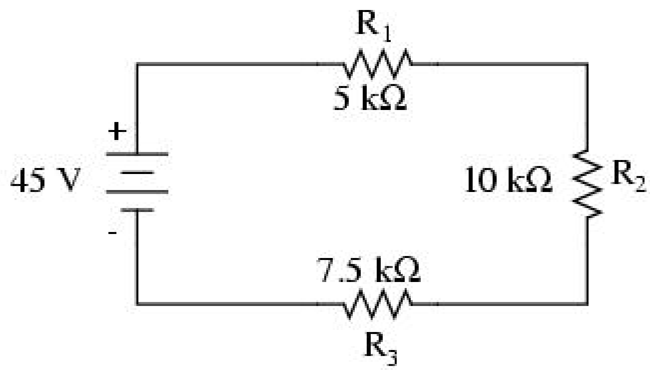
1. Voltage divider circuits

Let's analyse a simple series circuit, determining the voltage drops across individual resistors:



It should be apparent that the voltage drop across each resistor is proportional to its resistance, given that the current is the same through all resistors. For example, the voltage across R_2 is double that of the voltage across R_1, just as the resistance of R_2 is double that of R_1.

The voltage across R_2 is still exactly twice that of R_1's drop, despite the fact that the source voltage has changed. The proportionality of voltage drops (ratio of one to another) is strictly a function of resistance values.
With a little more observation, it becomes apparent that the voltage drop across each resistor is also a fixed proportion of the supply voltage. The voltage across R_1, for example, was 10 volts when the battery supply was 45 volts. When the battery voltage was increased to 180 volts (4 times as much), the voltage drop across R_1 also increased by a factor of 4 (from 10 to 40 volts). The *ratio* between R_1's voltage drop and total voltage, however, did not change:

For this reason a series circuit is often called a *voltage divider* for its ability to proportion -- or divide -- the total voltage into fractional portions of constant ratio. With a little bit of algebra, we can derive a formula for determining series resistor voltage drop given nothing more than total voltage, individual resistance, and total resistance:

Voltage drop across *any* resistor $E_n = I_n R_n$

Current in a series circuit $I_{total} = \frac{E_{total}}{R_{total}}$

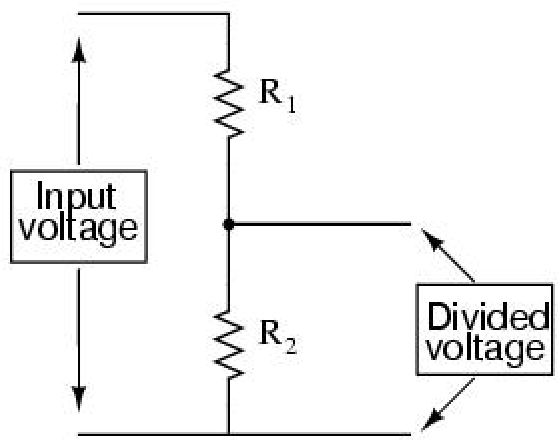
. . . Substituting $\frac{E_{total}}{R_{total}}$ *for* I_n *in the first equation . . .*

Voltage drop across any *series* resistor $E_n = \frac{E_{total}}{R_{total}} R_n$

. . . or . . .

$$\boxed{\mathbf{E}_n = \mathbf{E}_{\mathbf{total}} \frac{\mathbf{R}_n}{\mathbf{R}_{\mathbf{total}}}}$$

Voltage dividers find wide application in electric meter circuits, where specific combinations of series resistors are used to "divide" a voltage into precise proportions as part of a voltage measurement device.



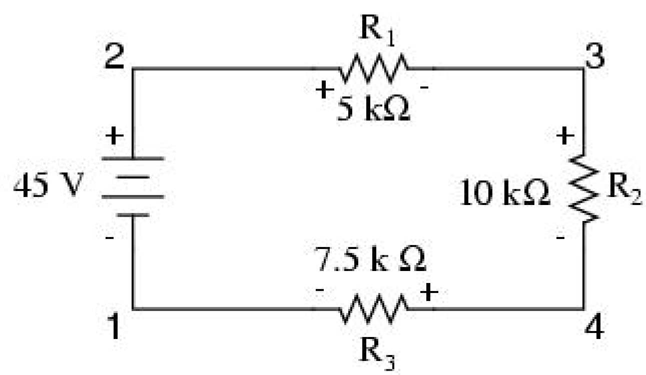
One device frequently used as a voltage-dividing component is the *potentiometer*, which is a resistor with a movable element positioned by a manual knob or lever. The movable element, typically called a *wiper*, makes contact with a resistive strip of material at any point selected by the manual control:

Section Review:

- Series circuits proportion, or *divide*, the total supply voltage among individual voltage drops, the proportions being strictly dependent upon resistances: $E_{Rn} = E_{Total} (R_n / R_{Total})$
- A potentiometer is a variable-resistance component with three connection points, frequently used as an adjustable voltage divider.

2. Kirchhoff's Voltage Law (KVL)

Let's take another look at our example series circuit, this time numbering the points in the circuit for voltage reference:



If we were to connect a voltmeter between points 1 and 2, black test lead to point 1 and red test lead to point 2, the meter would read +45 volts. Typically the "+" sign is not shown (but implied) for positive readings in digital meter displays. However, for this lesson the polarity of the voltage reading is very important:

V21=+45V

This reads: the voltage of point 2 with respect to point 1 is +45V.
We can continue round the loop:

V32=-IR1=10V, V43=-IR2=-20V and V14=IR3=-15V

Adding voltages round the loop:

V21+V32+V43+V14=0

This principle is known as *Kirchhoff's Voltage Law* (discovered in 1847 by Gustav R. Kirchhoff, a German physicist), and it can be stated as such:

"The algebraic sum of all voltages in a loop must equal zero"

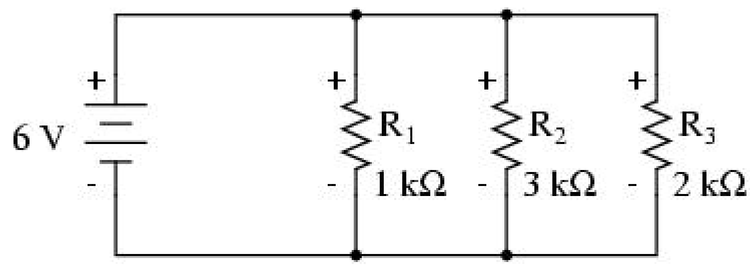
By *algebraic*, I mean accounting for signs (polarities) as well as magnitudes. By *loop*, I mean any path traced from one point in a circuit around to other points in that circuit, and finally back to the initial point. In the above example, the loop was formed by following points in this order: 1-2-3-4-1. It doesn't matter which point we start at or which direction we proceed in tracing the loop; the voltage sum will still equal zero.

Section Review:

- Kirchhoff's Voltage Law (KVL): *"The algebraic sum of all voltages in a loop must equal zero"*

3. Current divider circuits

Let's analyse a simple parallel circuit, determining the branch currents through individual resistors:



Using Ohm's Law (I=E/R) we can calculate each branch current:
Knowing that branch currents add up in parallel circuits to equal the total current, we can arrive at total current by summing 6mA, 2mA, and 3mA:

Once again, it should be apparent that the current through each resistor is related to its resistance, given that the voltage across all resistors is the same. Rather than being directly proportional, the relationship here is one of inverse proportion. For example, the current through R_1 is half as much as the current through R_3, which has twice the resistance of R_1.

Also reminiscent of voltage dividers is the fact that branch currents are fixed proportions of the total current. For this reason a parallel circuit is often called a *current divider* for its ability to proportion -- or divide -- the total current into fractional parts. With a little bit of algebra, we can derive a formula for determining parallel resistor current given nothing more than total current, individual resistance, and total resistance:

Current through *any* resistor $$I_n = \frac{E_n}{R_n}$$

Voltage in a parallel circuit $$E_{total} = E_n = I_{total} R_{total}$$

. . . Substituting $I_{total} R_{total}$ for E_n in the first equation . . .

Current through any *parallel* resistor $$I_n = \frac{I_{total} R_{total}}{R_n}$$

. . . or . . .

$$\boxed{I_n = I_{total} \frac{R_{total}}{R_n}}$$

The ratio of total resistance to individual resistance is the same ratio as individual (branch) current to total current. This is known as the *current divider formula*, and it is a short-cut method for determining branch currents in a parallel circuit when the total current is known.

Using the original parallel circuit as an example, we can re-calculate the branch currents using this formula, if we start by knowing the total current and total resistance:

$$I_{R1} = 11 \text{ mA} \frac{545.45\ \Omega}{1\ \text{k}\Omega} = 6 \text{ mA}$$

$$I_{R2} = 11 \text{ mA} \frac{545.45\ \Omega}{3\ \text{k}\Omega} = 2 \text{ mA}$$

$$I_{R3} = 11 \text{ mA} \frac{545.45\ \Omega}{2\ \text{k}\Omega} = 3 \text{ mA}$$

If you take the time to compare the two divider formulae, you'll see that they are remarkably similar. Notice, however, that the ratio in the voltage divider formula is R_n (individual resistance) divided by R_{Total}, and how the ratio in the current divider formula is R_{Total} divided by R_n:

Voltage divider formula

$$E_n = E_{total} \frac{R_n}{R_{total}}$$

Current divider formula

$$I_n = I_{total} \frac{R_{total}}{R_n}$$

Section Review:

- Parallel circuits proportion, or "divide," the total circuit current among individual branch currents, the proportions being strictly dependent upon resistances: $I_n = I_{Total} (R_{Total} / R_n)$

4. Kirchhoff's Current Law (KCL)

At this point, we know the value of each branch current and of the total current in the circuit. We know that the total current in a parallel circuit must equal the sum of the branch currents.

This fact should be fairly obvious if you think of the water pipe circuit analogy with every branch node acting as a "tee" fitting, the water flow splitting or merging with the main piping as it travels from the output of the water pump toward the return reservoir or sump: so long as there are no leaks in the piping, what flow enters the fitting must also exit the fitting. This holds true for any node ("fitting"), no matter how many flows are entering or exiting. Mathematically, we can express this general relationship as such:

$$I_{exiting} = I_{entering}$$

Mr. Kirchhoff decided to express it in a slightly different form (though mathematically equivalent), calling it *Kirchhoff's Current Law* (KCL):

$$I_{entering} + (-I_{exiting}) = 0$$

Summarized in a phrase, Kirchhoff's Current Law reads as such:

"The algebraic sum of all currents entering and exiting a node must equal zero"

That is, if we assign a mathematical sign (polarity) to each current, denoting whether they enter (+) or exit (-) a node, we can add them together to arrive at a total of zero, guaranteed.

Whether negative or positive denotes current entering or exiting is entirely arbitrary, so long as they are opposite signs for opposite directions and we stay consistent in our notation, KCL will work.

Together, Kirchhoff's Voltage and Current Laws are a formidable pair of tools useful in analysing electric circuits.

Section Review:

- Kirchhoff's Current Law (KCL): *"The algebraic sum of all currents entering and exiting a node must equal zero"*