Unit FE-4 Foundation Electricity:

Power in AC circuits. Three-phase systems.

What this unit is about

Power networks consist of resistive and reactive components. This Unit deals specifically with the electrical power aspects in such networks. It is shown that the power associated with a resistor is a double frequency sinusoid displaced with regard to the time axis so that it is always positive. This indicates the irreversible conversion of electrical energy into heat. In contrast, the power associated with reactive components is symmetrically disposed with regard to the time axis with zero average value. This indicates the oscillatory nature of this power. Next the concepts of active, reactive and apparent power as well as power factor are defined and the convention of assigning positive and negative labelling to the active and reactive power is explained. The practical implications of low power factor and the practice of power factor correction are then outlined. It is then shown how reactive and active power can be easily calculated using the complex notation of voltage and current. Finally, the single and three-phase systems for generation, transportation and utilisation of energy are discussed and the advantages of three-phase over other configurations are outlined.

Why is this knowledge necessary

Renewable energy sources are used mainly to generate AC electrical power which is injected into power networks consisting of large number of transmission lines, other conventional generators and consumers. Such power networks, especially in developed countries, are of considerable complexity. To determine the way these injected powers flow from generators to consumers requires complex calculations based on network analysis. This Unit introduces the basic concepts of active, reactive and apparent power. These are the basic tools needed to understand and calculate flows of energy from generators to consumers. In all AC power systems (excluding very low capacity ones) power is generated and transported in three-phase form. The reasons why this is so and the tools to deal with power flows in such systems are then outlined. All this knowledge is an essential prerequisite in the understanding of power flows from renewable generators into networks.

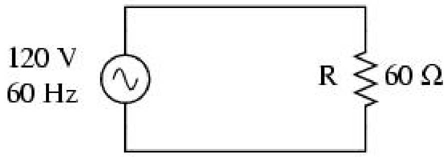
At the beginning of each section the course module(s) that requires the material in this particular section as background knowledge are indicated in bold italics. All sections of this Unit are required as background material for Unit FE-5.

Note that the text uses 60 rather than 50 Hz as the mains frequency. This is of no consequence in the understanding of the material and in its application to 50 Hz systems.

Contents

1. Power in Resistive and Reactive AC Circuits

Consider the circuit of Fig. 1 for a single-phase AC power system, where a 120 volt, 60 Hz AC voltage source is delivering power to a resistive load:



$$Z_R = 60 + j0\ \Omega \quad or \quad 60\ \Omega \angle 0^\circ$$

$$I = \frac{E}{Z}$$

$$I = \frac{120\ \text{V}}{60\ \Omega}$$

$$I = 2\ \text{A}$$

Fig. 1

In this example, the current to the load is 2 amps, RMS. The power dissipated at the load is 240 watts. Because this load is purely resistive, the current is in phase with the voltage, and calculations identical to that in an equivalent DC circuit. The plot of voltage, current, and power waveforms for this circuit are shown in Fig. 2:

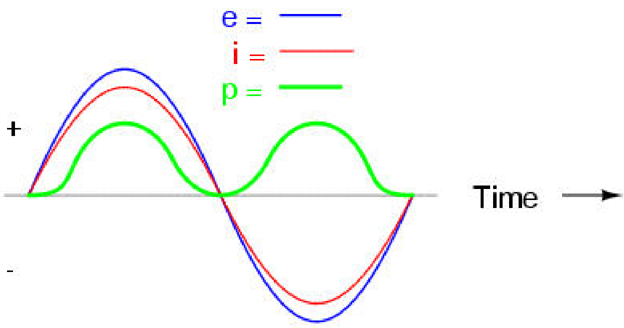


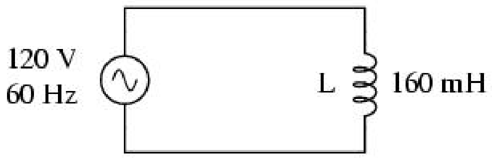
Fig. 2

Note that the waveform for power is always positive, i.e. ***all*** the electrical energy is irreversibly converted into heat. If the source were a mechanical generator of 100% efficiency, it would take 240 watts worth of mechanical power to turn the shaft. Also note that the waveform for power has a frequency which is *double* that of either the voltage or current waveforms.

As we are mostly interested in the mean rate of energy transfer and not in its variability over a cycle, we are only concerned in the average value of this double frequency vertically displaced sinusoid. It can be easily shown mathematically that the average height of the instantaneous power 'p' waveform above the time axis represents the dissipated power of 240W. We conclude that the quantity given by the product of the RMS Voltage V and RMS Current I in a resistive circuit is the scalar quantity P known as **'Active Power'** measured in Watts (W), kilowatts (kW = 10^3W) or megawatts (MW = 10^6W).

In AC circuits the same conventions for the sign of power apply as for DC circuits, positive sign indicating consumption and negative sign generation of power.

Let us now consider a simple AC circuit with a purely reactive load:



$$X_L = 60.319\ \Omega$$

$$Z_L = 0 + j60.319\ \Omega \quad or \quad 60.319\ \Omega \angle 90^\circ$$

$$I = \frac{E}{Z}$$

$$I = \frac{120\ \text{V}}{60.319\ \Omega}$$

$$I = 1.989\ \text{A}$$

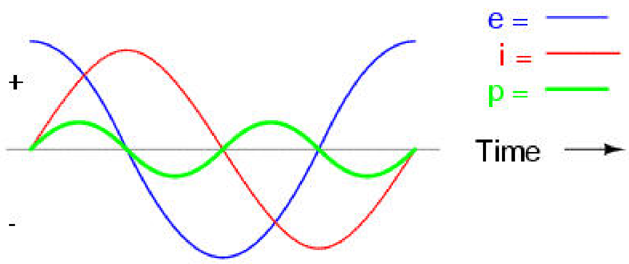


Fig 3

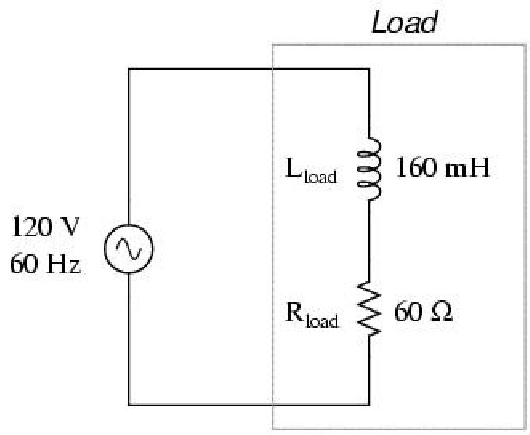
Note that in Fig 3 the power alternates equally between cycles of positive and negative value. This means that power is being alternately absorbed from (positive) and returned (negative) to the source. If the source were a mechanical generator, it would take (practically) no net mechanical energy to turn the shaft, because no power would be used by the load.

In this 'wattless' situation it can be shown that the product of RMS voltage V and RMS current I is equal to the ***peak*** of the green double frequency waveform (it cannot be the average value as this is zero!). This product is a scalar quantity, is known as **'Reactive Power'**, is denoted by Q and is measured in Reactive-Voltamperes (VAR's) or kVAR or MVAR.

In the example above the reactive power is: Q=120x1.989=238.68 VAR. This 'wattles' quantity is a measure of the oscillating power in and out of a reactive elements such as an inductor.

In the case of a capacitor, a mirror image of the inductor waveforms of Fig. 3 will apply with the voltage and the current waveforms interchanged. Whether an inductor is connected in series or in parallel with a capacitor the instantaneous power variations of the two components will be in antiphase. This leads to a very useful convention developed in power systems to mirror the active power situation. The reactive power associated with an inductor is taken to be consumed and therefore positive whilst the reactive power of a capacitor is taken to be generated and therefore negative. This leads to an extremely compact way of analysing power systems to be discussed in the 'Integration' module.

Now, let's consider Fig. 4 in which a load consists of both inductance and resistance:



$$X_L = 60.319\ \Omega$$

$$Z_L = 0 + j60.319\ \Omega \quad or \quad 60.319\ \Omega \angle 90^\circ$$

$$Z_R = 60 + j0\ \Omega \quad or \quad 60\ \Omega \angle 0^\circ$$

$$Z_{total} = 60 + j60.319\ \Omega \quad or \quad 85.078\ \Omega \angle 45.152^\circ$$

$$I = \frac{E}{Z}$$

$$I = \frac{120\ V}{85.078\ \Omega}$$

$$I = 1.410\ A$$

Fig. 4

This is the figure an ammeter would indicate if connected in series with the resistor and inductor.
If we look at the waveform plot of voltage, current, and total power for this circuit (Fig.5) , we see how this combination works:

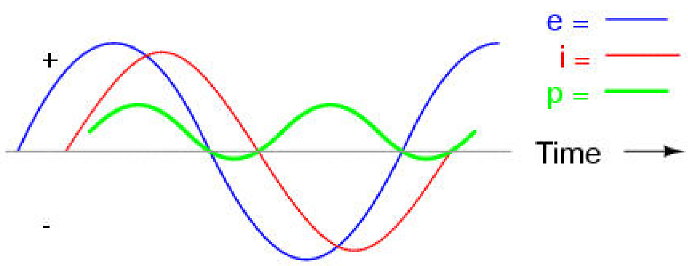


Fig.5

As with any reactive circuit, the power alternates between positive and negative instantaneous values over time. In circuits with mixed resistance and reactance like this one, the amount of positive power will exceed the amount of negative power. In other words, the combined inductive/resistive load will consume more power than it returns back to the source. If the source were a mechanical generator, the amount of mechanical energy needed to turn the shaft (in a system with 100% efficiency) would be the amount of power averaged between the positive and negative power cycles. In the circuit above: P=I^2 R= 1.410^2x60=119.29 W and Q=I^2X=1.410^2x60.319=119.9 VAR

Section Review

- In a purely resistive circuit, all circuit active power is dissipated by the resistor(s).
- The sign of active power indicates whether an element is a consumer (positive) or a generator (negative)
- In a purely reactive circuit (inductive or capacitive), no power is dissipated. Rather, power is alternately absorbed from and returned to the AC source.
- The sign of reactive power indicates whether an element is a consumer (inductive-positive) or a generator (capacitive-negative)
- In a circuit consisting of resistance and reactance mixed, there will be, on average, more power dissipated by the load(s) than returned.Voltage and current in such a circuit will be out of phase by a value somewhere between 0^o and plus-minus 90^o.

2. Active, Reactive and Apparent Power

ELP032

In Fig. 6, a circuit consists of resistance R, and inductive reactance X_L and represents a typical consumer connected to the electrical grid. The grid voltage phasor $\bar{V}$ impressed across the consumer terminals results in a current phasor $\bar{I}$ which when flowing through R and X_L results in voltage drops across these components of $\bar{V}_R$ and $\bar{V}_L$ respectively.

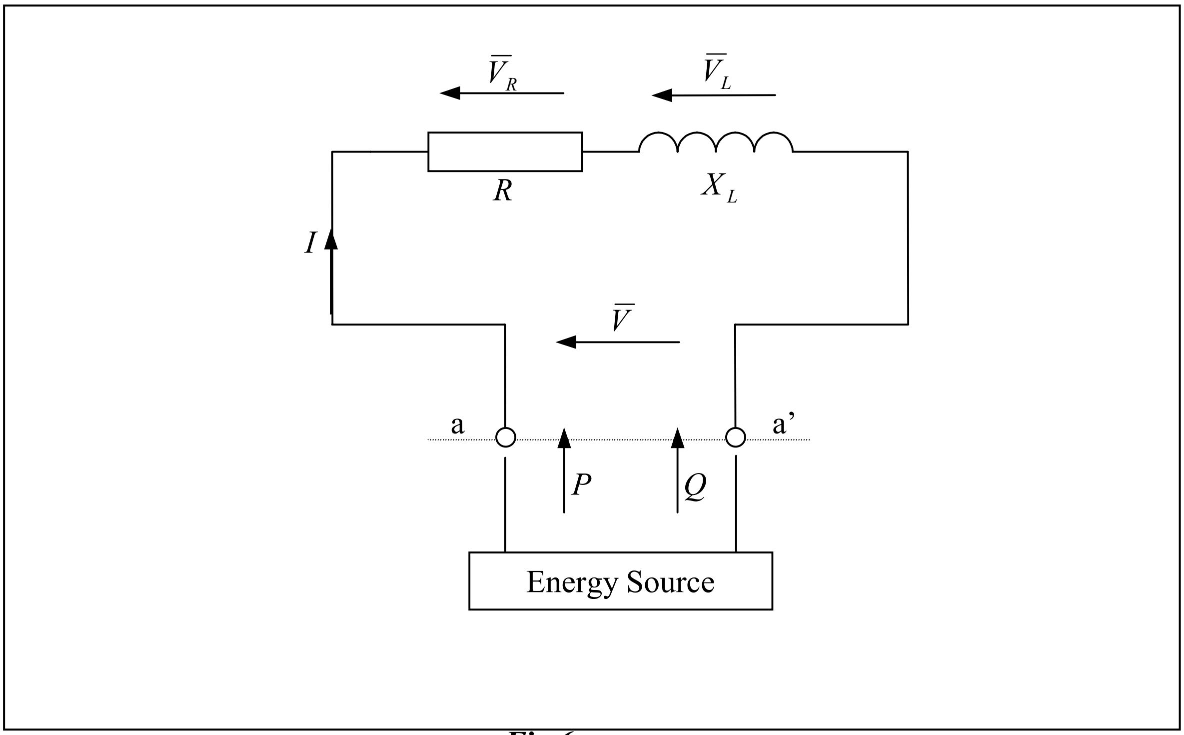


Fig 6

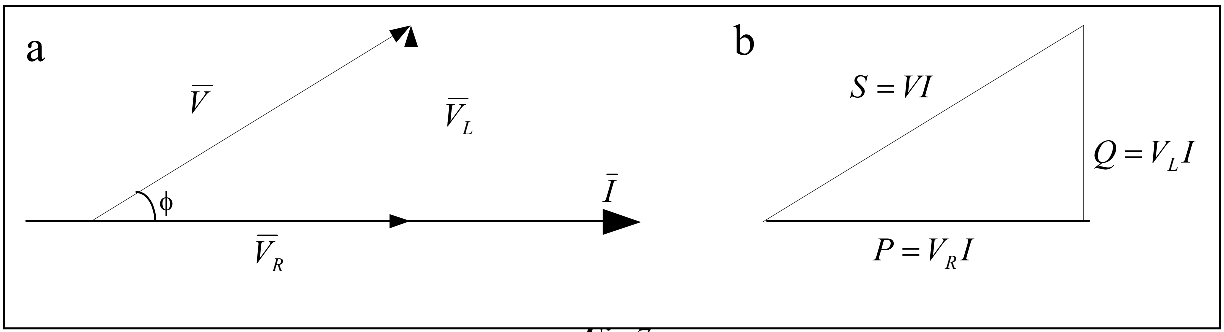


Fig 7

According to Kirchhoff's Voltage law the phasorial sum of voltage drops in Fig. 7 is equal to the applied voltage i.e.

$$\overline{V} = \overline{V}_R + \overline{V}_L$$

This equality is illustrated in phasor diagram (a) where $\overline{V}_R$ is in phase with $\overline{I}$ and $\overline{V}_L$ is leading $\overline{I}$ by 90°. The angle ϕ is the angle by which the current lags the voltage in this circuit and is the same as the impedance angle.

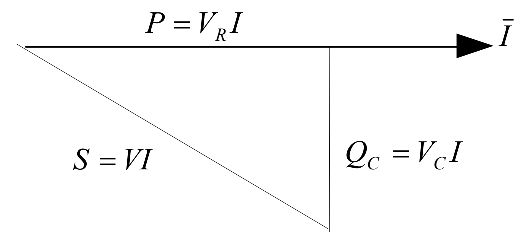
Multiplying the sides of the voltage triangle (a) by I results is the "power triangle" shown in (b). The product $V_R I$ represents the active power P, the product $V_L I$ represents the reactive power Q , and finally the product VI is known as S, the **Apparent Power** with units of Volt-amperes (VA), kilovolt-amperes (KVA) and Megavolt-amperes (MVA). The active and reactive power are obtained from the volt-amperes through the following expressions:

$$P = VI\cos\phi \qquad [1]$$

$$Q = VI\sin\phi \qquad [2]$$

where $\cos\phi$ is known as the circuit "Power Factor" and ϕ is known as the power factor angle. Equation (1) tells us that the power factor is the ratio of active to apparent power.

Redrawing the power triangle (b) for a resistive capacitive circuit we arrive at the power triangle of Fig. 8. Q_C is now negative therefore the reactive power associated with a capacitor is "exported" or "generated".



Apparent, Real and Reactive power for a resistive/capacitive circuit

Fig. 8

As a rule, active power is a function of a circuit's dissipative elements, usually resistances (R). Reactive power is a function of a circuit's reactance (X). Apparent power is a function of a circuit's total impedance (Z).

It should be noted that power factor, like all ratio measurements, is a non-dimensional quantity. For the purely resistive circuit, the power factor is 1 and the power triangle would be a horizontal line For a purely inductive circuit, the power factor is zero and the power triangle would be a vertical line pointing upwards. For a purely capacitive circuit the power triangle would be a vertical line pointing downwards.
There are several power equations relating the three types of power (active or true, reactive and apparent) to resistance, reactance, and impedance (all using scalar quantities):

P = true power $\quad P = I^2R \quad P = \dfrac{E^2}{R}$

Measured in units of ***Watts***

Q = reactive power $\quad Q = I^2X \quad Q = \dfrac{E^2}{X}$

Measured in units of ***Volt-Amps-Reactive (VAR)***

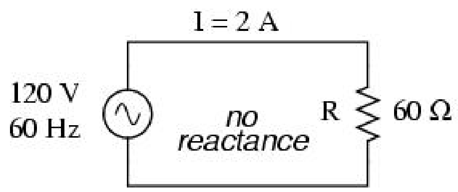
S = apparent power $\quad S = I^2Z \quad S = \dfrac{E^2}{Z} \quad S = IE$

Measured in units of ***Volt-Amps (VA)***

Note that in each case E and I are, respectively, the voltage across and the current through the element in question.

Let us examine the following circuits and see how these three types of power interrelate:

Resistive load only:

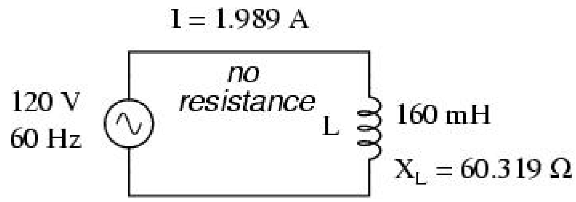


P = true power $= I^2R = 240$ W

Q = reactive power $= I^2X = 0$ VAR

S = apparent power $= I^2Z = 240$ VA

Reactive load only:

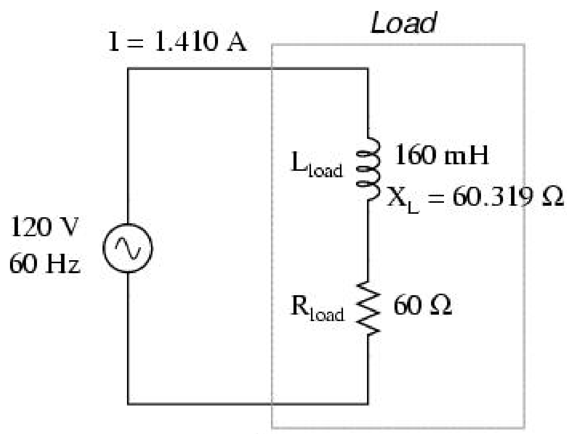


$$P = \text{true power} = I^2R = 0\ \text{W}$$

$$Q = \text{reactive power} = I^2X = 238.73\ \text{VAR}$$

$$S = \text{apparent power} = I^2Z = 238.73\ \text{VA}$$

Resistive/reactive load:



$$P = \text{true power} = I^2R = 119.365\ \text{W}$$

$$Q = \text{reactive power} = I^2X = 119.998\ \text{VAR}$$

$$S = \text{apparent power} = I^2Z = 169.256\ \text{VA}$$

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{169.256\ \text{VA}}$$

$$\text{Power factor} = 0.705$$

$$\cos 45.152^\circ = 0.705$$

Fig 9

The results of this section are summarised in the 'power triangle ' of Fig. 10:

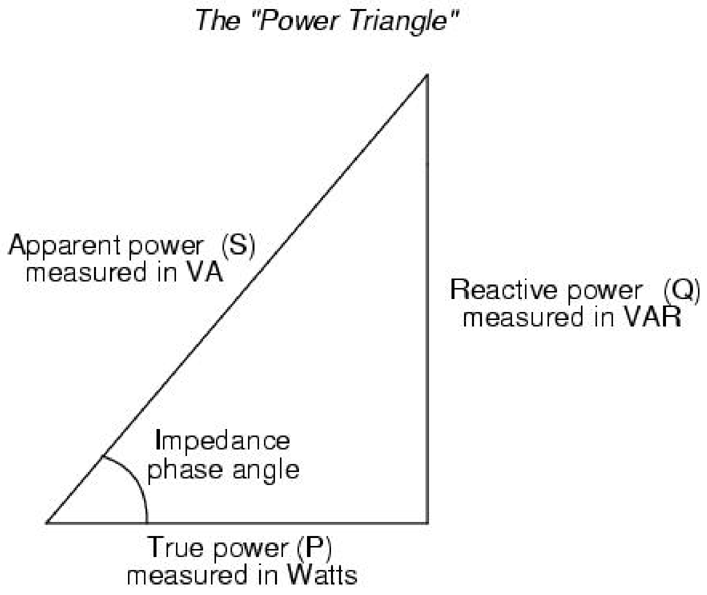


Fig. 10

Using the laws of trigonometry, we can solve for the length of any side given the lengths of the other two sides, or the length of one side and an angle.

Section Review

- Electrical power irreversibly converted into another type of power by a load is referred to as *active power*. Active (true) power is symbolised by the letter P and is measured in the unit of Watt (W).
- Power merely absorbed and returned to the circuit by a load due to its reactive properties is referred to as *reactive power*. Reactive power is symbolised by the letter Q and is measured in the unit of Volt-Amps-Reactive (VAR).
- Inductors and capacitors are associated with positive and negative Q respectively
- Apparent power is symbolised by the letter S and is measured in the unit of Volt-Amps (VA).
- These three types of power are trigonometrically related to one another. In a right triangle, P = base, Q = height, and S = hypotenuse. The angle between P and S is equal to the circuit's impedance (Z) phase angle and is known as the power factor angle.

3. The Significance of Power Factor

ELP032

Figure 11 represents a basic circuit of energy transportation from a generator to a consumer. The transmission line is simply represented by a series inductance and resistance. The consumer is modelled by an inductive resistive impedance.
(The justification for these two assumptions will be discussed in the ‘Integration’ module)
Following the reasoning in section 2, the consumer or load “absorbs” both P_L and Q_L. The generator or “the utility” has to supply both the P of the load and the loss in transmission

given by $I^2 R_t$ where R_t is the resistance of the transmission line. The transmission loss is strongly influenced by the load Q_L.

To appreciate this, assume that the load consists solely of an inductor, which absorbs Q but not P. In this case the energy meter at the consumer premises which records kilowatt-hours (i.e. energy purchased) indicates zero but the finite current in the transmission system results in $I^2 R_t$ which has to be supplied by the utility. Clearly this is a most undesirable scenario which the utility endeavours to discourage through special tariffs which penalise the absorption of Q by large consumers.

Additionally, the utility has the legal obligation to supply power to the consumer at a more or less fixed voltage. It can be shown that for transmission lines, with $X_t > R_t$, the voltage V_L at the consumer terminals is particularly sensitive to changes in Q rather than P. It is now obvious that the utility is keen to encourage consumers to draw P at minimum Q i.e. with $\cos\phi$ close to unity.

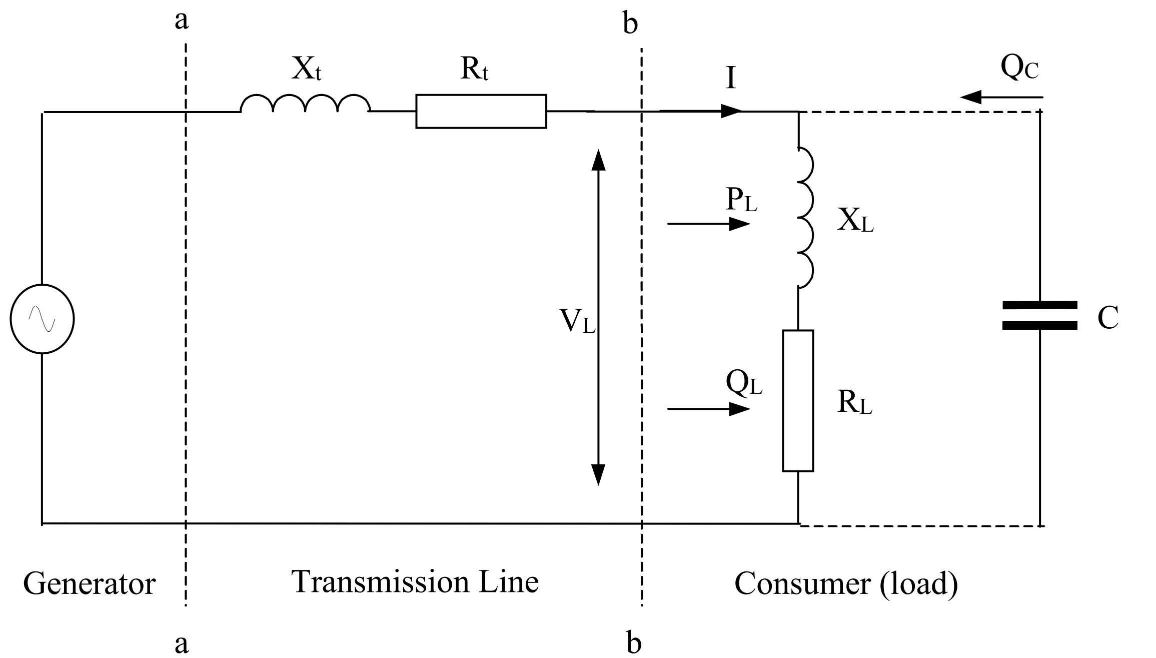


Figure 11

4. Power Factor Correction

ELP032,ELP041

Capacitors are used extensively in power systems to generate or “inject” reactive power at strategic points of the network. Their importance is illustrated in the transmission circuit of Fig 11. If a capacitor C is connected across the load terminal and C is sized to generate Q_C locally so that $Q_C = Q_L$, the reactive power absorbed by the load, then the consumer will appear to the utility as having a unity power factor. The utility will be truly satisfied with such an arrangement as this will minimise losses in the transmission line resistance R_t and will ensure little variation in consumer voltage.

In the circuit of Fig 9 we know that the reactive power is 119.998 VAR (inductive), we need to calculate the correct capacitor size to produce the same quantity of (capacitive) reactive power. Since this capacitor will be directly in parallel with the source (of known voltage), we'll use the power formula which starts from voltage and reactance:

$$Q = \frac{E^2}{X}$$

. . . solving for X . . .

$$X = \frac{E^2}{Q}$$

$$X = \frac{(120\ \text{V})^2}{119.998\ \text{VAR}}$$

$$X = 120.002\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

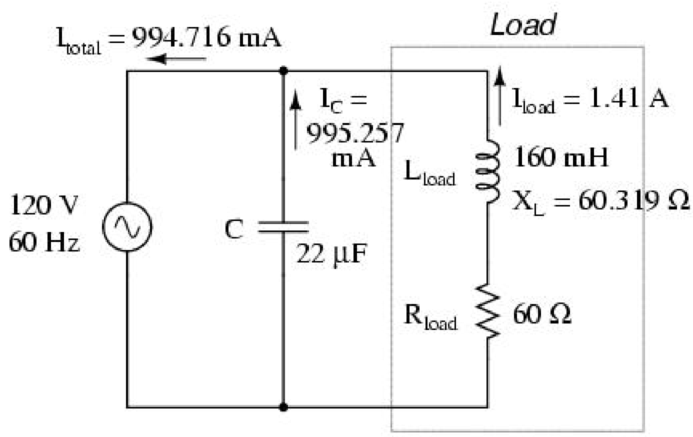
. . . solving for C . . .

$$C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{2\pi(60\ \text{Hz})(120.002\ \Omega)}$$

$$C = 22.105\ \mu\text{F}$$

Let us use a rounded capacitor value of 22 µF and see what happens in the circuit of Fig 12:



$Z_{total} = Z_C / (Z_L - Z_R)$

$Z_{total} = (120.57\Omega\angle -90^o) / (60.319\ \Omega\angle\ 90^o - 60\ \Omega\angle\ 0^o)$

$Z_{total} = (120.64 - j573.58m\ \Omega$ or $120.64\ \Omega\angle 0.274^o$

P = true power = I^2R = 119.365 W
S = apparent [per = I^2Z = 119.366 VA

Fig 12

The power factor for the circuit, overall, has been substantially improved. The main current has been decreased from 1.41 amps to 994.7 milliamps, while the power dissipated at the load resistor remains unchanged at 119.365 watts. The power factor is nearly unity.

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ \text{W}}{119.366\ \text{VA}}$$

$$\text{Power factor} = 0.9999887$$

$$\text{Impedance (polar) angle} = 0.272^o$$

$$\cos 0.272^o = 0.9999887$$

If we had added too large of a capacitor in parallel, we would have ended up with an impedance angle that was negative, indicating that the circuit was more capacitive than inductive. It should be noted that too much capacitance in an AC circuit will result in a low power factor.

If a circuit is predominantly inductive, we say that its power factor is *lagging* (because the current wave for the circuit lags behind the applied voltage wave). Conversely, if a circuit is predominantly capacitive, we say that its power factor is *leading*. Thus, our example circuit started out with a power factor of 0.705 lagging, and was corrected to a power factor of 0.999 lagging.

Section Review

- Poor power factor of a load in an AC circuit (invariably inductive) may be "corrected" and brought close to unity by adding a parallel capacitance.

5. Complex Power

ELP032

Network calculations in power systems are invariably carried out through complex numbers. It would be very desirable if the scalar quantities P and Q could be derived directly from the complex expressions of voltage and current. The object of this section is to show how this can be done.

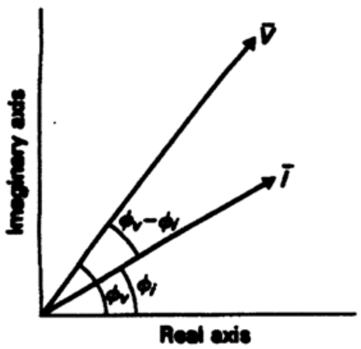
$\bar{V}$ and $\bar{I}$ are drawn at angles ϕ_v and ϕ_i, respectively, with regard to the real axis. Resolving $\bar{I}$ along and at right angles to $\bar{V}$ we get

$$P = VI\cos(\phi_v - \phi_i)$$

and

$$Q = VI\sin(\phi_v - \phi_i)$$

The intention now is to see whether the above two quantities could be derived from the multiplication of the complex numbers $\bar{V}$ and $\bar{I}$.



$$\bar{V} = V(\cos\phi_v + \mathrm{j}\sin\phi_v)$$

and

$$\bar{I} = I(\cos\phi_i + \mathrm{j}\sin\phi_i)$$

Our natural tendency would be to evaluate $\bar{V}\bar{I}$ hoping that something useful might emerge. In fact, for reasons that will be clear shortly, let us evaluate $\bar{V}\bar{I}^*$ where

$$\bar{I}^* = I(\cos\phi_i - \mathrm{j}\sin\phi_i), \text{ the conjugate of } \bar{I}$$

This gives

$$\bar{V}\bar{I}^* = VI(\cos\phi_v + \mathrm{j}\sin\phi_v)(\cos\phi_i - \mathrm{j}\sin\phi_i)$$
$$= VI[\cos\phi_v\cos\phi_i + \sin\phi_v\sin\phi_i + \mathrm{j}(\sin\phi_v\cos\phi_i - \cos\phi_v\sin\phi_i)]$$

Thus

$$\bar{V}\bar{I}^* = VI[\cos(\phi_v - \phi_i) + \mathrm{j}\sin(\phi_v - \phi_i)] = P + \mathrm{j}Q \qquad [3]$$

We have established that to arrive at P and Q and to satisfy the agreed sign convention, it is necessary to use the conjugate of the current in the complex multiplication.

Equation (3) can be rewritten as

$$\bar{S} = P + \mathrm{j}Q$$

where $\bar{S}$ is, with good justification, known as 'complex volt amperes' because the magnitude of $\bar{S}$ is given by $S = \sqrt{(P^2 + Q^2)}$, which is indeed the value of volt amperes.

Let us apply this to the circuit of Fig 9:

$$\bar{S}=\bar{V}\bar{I}^{*}=(120\angle 0^\circ)(1.410\angle -(-45.15^\circ))=168\angle +45.15^\circ=119+j119 \text{ i.e } P=119\text{ W}, Q=119\text{ VAR}$$

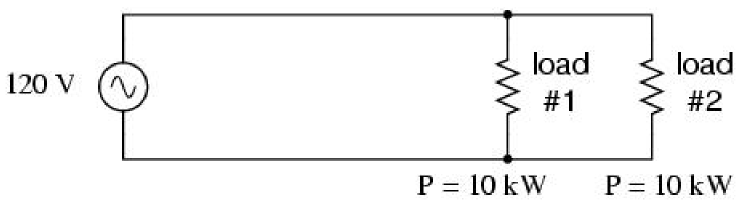
Section Review

- A very convenient way of calculating P and Q when the voltage and current are in complex form is to multiply the voltage by the conjugate of the current. The real and imaginary parts of the resulting complex voltamperes are the active and reactive powers respectively.

6. Single Phase Power Systems

Figure 13 shows a very simple AC circuit. If the load resistor's power dissipation were substantial, we might call this a "power circuit" or "power system" instead of regarding it as just an ordinary circuit. The distinction between a "power circuit" and an "ordinary circuit" may seem arbitrary, but the practical concerns are definitely not.

One such concern is the size and cost of wiring necessary to deliver power from the AC source to the load. If we give the source in the circuit a voltage value and also give power dissipation values to the two load resistors, we can determine the wiring needs for this particular circuit:



$$I = \frac{P}{E}$$

$$I = \frac{10\text{ kW}}{120\text{ V}}$$

$$I = 83.33\text{ A} \quad \textit{(for each load resistor)}$$

$$I_{total} = I_{load\#1} + I_{load\#2} \qquad P_{total} = (10\text{ kW}) + (10\text{ kW})$$

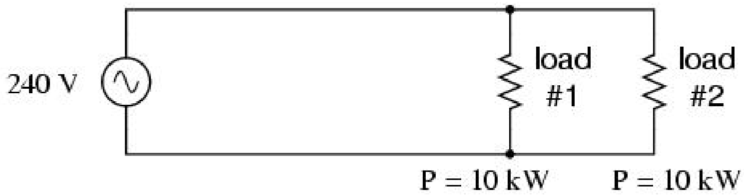
$$I_{total} = (83.33\text{ A}) + (83.33\text{ A}) \qquad P_{total} = 20\text{ kW}$$

$$I_{total} = 166.67\text{ A}$$

Fig. 13

Around 170A is no small amount of current, and would necessitate copper wire conductors with well over 6.5 mm in diameter, weighing over 360 kg per km. Bear in mind that copper is not cheap either! It would be in our best interest to find ways to minimise such costs if we were designing a power system with long conductor lengths.

One way to do this would be to increase the voltage of the power source as in Fig. 14 and use loads built to dissipate 10 kW each at this higher voltage. The loads, of course, would have to have greater resistance values to dissipate the same power as before (10 kW each) at a greater voltage than before. The advantage would be less current required, permitting the use of smaller, lighter, and cheaper wire:



$$I = \frac{P}{E}$$

$$I = \frac{10\ \text{kW}}{240\ \text{V}}$$

$$I = 41.67\ \text{A} \qquad \textit{(for each load resistor)}$$

$$I_{total} = I_{load\#1} + I_{load\#2} \qquad P_{total} = (10\ \text{kW}) + (10\ \text{kW})$$

$$I_{total} = (41.67\ \text{A}) + (41.67\ \text{A}) \qquad P_{total} = 20\ \text{kW}$$

$$I_{total} = 83.33\ \text{A}$$

Fig 14

Now our *total* circuit current is 83.33 amps, half of what it was before. The appropriate wire would weigh considerably less than in the previous case. The reason why Europe has adopted a 240V distribution system rather than the 120V USA practice is now obvious. It should also be obvious why Utilities transmit electric power using very high voltages (many thousands of volts) to capitalise on the savings realised by the use of smaller, lighter, cheaper wire.

However, this solution is not without disadvantages. The gain in efficiency realised by stepping up the circuit voltage presents us with increased danger of electric shock and insulation breakdown. Power distribution companies tackle this problem by stringing their power lines along high poles or towers, and insulating the lines from the supporting structures with large, porcelain insulators.

At the point of use (the electric power customer), there is still the issue of what voltage to use for powering loads. High voltage gives greater system efficiency by means of reduced conductor current, but it might not always be practical to keep power wiring out of reach at

the point of use the way it can be elevated out of reach in distribution systems. This trade-off between efficiency and danger is one that European power system designers have decided to risk.
Is there any way to realise the advantages of both increased efficiency and reduced safety hazard at the same time? One solution would be to install step-down transformers at the end-point of power use. However, this would be expensive and inconvenient for anything but very small loads (where the transformers can be built cheaply) or very large loads (where the expense of thick copper wires would exceed the expense of a transformer).

An alternative solution would be to use a higher voltage supply to provide power to two lower voltage loads in series as shown in Fig. 15. This approach combines the efficiency of a high-voltage system with the safety of a low-voltage system:

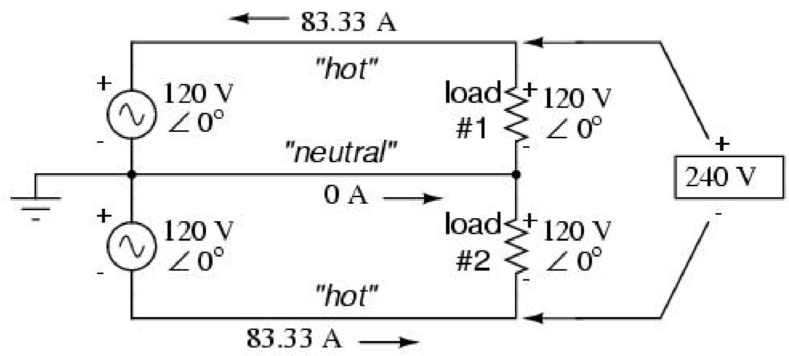


Fig. 15

Here. instead of a single 240 volt power supply, we use two 120 volt supplies (in phase with each other!) in series to produce 240 volts, then run a third wire to the connection point between the loads. This is called a *split-phase* power system. Three smaller wires are still cheaper than the two wires needed with the simple parallel design, so we're still ahead on efficiency. The astute observer will note that the neutral wire only has to carry the *difference* of current between the two loads back to the source. In the above case, with perfectly "balanced" loads consuming equal amounts of power, the neutral wire carries zero current.

Notice how the neutral wire is connected to ground at the power supply end. This is a common feature in power systems containing "neutral" wires, since grounding the neutral wire ensures the least possible voltage at any given time between any "hot" wire and earth.

The term "single phase" is a counterpoint to another kind of power system called "polyphase" which we are about to investigate in detail.

Section Review

- *Single phase* power systems have one AC source supplying a number of consumers.
- A *split-phase* power system is one with multiple (in-phase) AC voltage sources connected in series, delivering power to loads at more than one voltage, with more than two wires. They are used primarily to achieve balance between system efficiency (low conductor currents) and safety (low load voltages).

7. Three Phase Power Systems

ELP032

Split-phase power systems achieve their high conductor efficiency *and* low safety risk by splitting up the total voltage into lesser parts and powering multiple loads at those lesser voltages, while drawing currents at levels typical of a full-voltage system.
But we know from our experience with phasors and complex numbers that AC voltages don't always add up as we think they would if they are out of phase with each other. This principle, applied to power systems, can be put to use to make power systems with even greater conductor efficiencies and lower insulating requirements than with split-phase.

Suppose that we set up the arrangement shown in Fig 16 where a supply with three voltage sources of same magnitude but having successive phase-shifts of 120 degrees supply a set of balanced loads. This arrangement although apparently complicated it can be shown to combine the greatest benefit in terms of maximum power transfer with lowest transmission line costs. It also results in other major benefits that will be discussed later. More phases could be added but it can be shown that three-phases provide the optimum benefits.

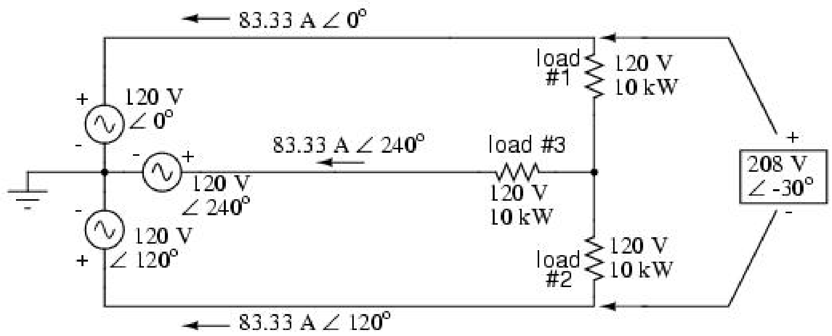


Fig. 16

The 'three-phase' circuit of Fig. 16 has been decomposed in Fig. 17 (a) into three separate single–phase circuits each with its own source, load and transmission line Irrespective of the power factor of the balanced loads, the phasor diagram in (b) indicates that the sum of the three currents in the three circuits add up phasorially to zero. The important consequence of this is that if nodes o and n where to be connected and the return paths joined together in one common return, this return line could be eliminated altogether.

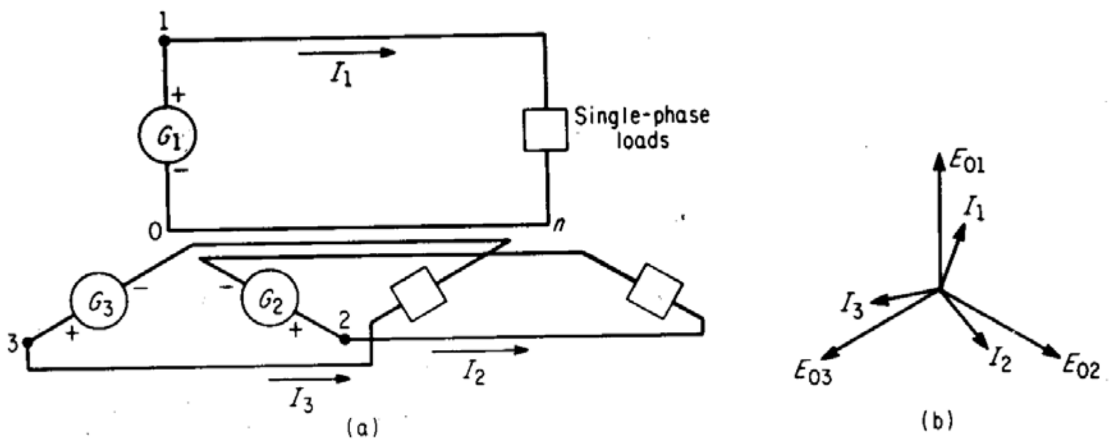


Fig. 17

The advantages of the three-phase system should now be obvious. Three loads consume 10kW each with three wires of the same diameter compared to six wires for the equivalent three separate single-phase systems! This method of transmission is used exclusively in power system networks. Power delivery to big (e.g. industrial) consumers is done at high voltage and on a three-phase basis. However at the lowest voltage level (e.g. domestic consumers) the power is distributed on a single-phase basis. The balance of domestic loads is maintained by connecting in each phase a large number of individual consumers that statistically, behave in a similar way. More will be said on this in the Integration module. A load imbalance in Fig 16 will result in slightly different currents in the three lines and therefore different voltages across the three consumers. To ensure load voltage stability in the event of occasional load imbalances, a neutral wire is connect between the source node and load node as shown in Fig 18.

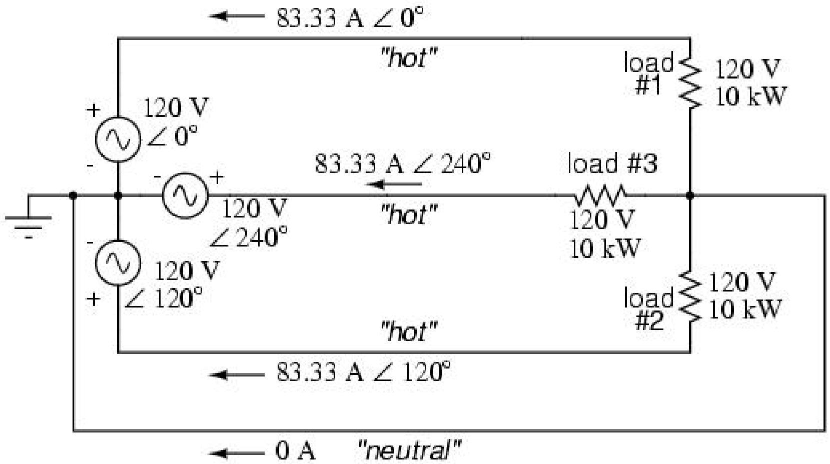


Fig. 18

So long as the loads remain balanced (equal impedances, equal currents), the neutral wire will carry no current at all. It will only carry current in case of load imbalances.

Three-phase power systems are designed to operate in a balances mode therefore most of the analysis of such systems assumes balanced conditions. For this reason, it is convenient to simplify 3-phase circuits by showing only one phase as in Fig. 19.

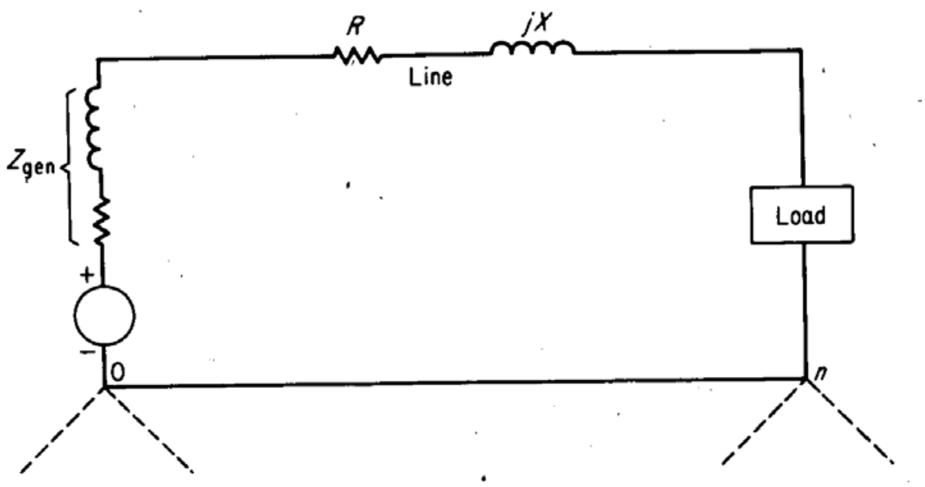


Fig. 19

Here the generator plus its lumped source impedance as well as the transmission line impedance are shown. It is understood that the other two phases carry currents and contain impedances of the same magnitude. In a balanced system the currents in one phase return through the other two phases, therefore the 'neutral' conductor between nodes o and n is redundant as it carries no current. This reasoning leads to an even more convenient representation of three-phase systems whereby only one line is shown, the one spanning nodes o and n through the generator, transmission line and load. All power system network diagrams are shown on a ***one-line*** basis unless unbalanced studies are carried out.

Section Review

- A ***three-phase*** power system uses three voltage sources successively phase-shifted by 120 degrees from each other. It can deliver more power at less voltage with smaller gauge conductors than single- or split-phase systems.
- A ***neutral*** conductor is necessary to ensure voltage stability to the three consumers under unbalanced conditions.
- For the diagrammatic depiction and analysis of 3-phase power system networks a ***one-line*** diagram is used.

8. Three Phase "Y" and "Δ" Configurations.

ELP032

Initially we explored the idea of three-phase power systems by connecting three voltage sources together in what is commonly known as the "Y" (or "star") configuration. This configuration of voltage sources is characterised by a common connection point joining one side of each source:

If we draw a circuit, as in Fig 20, showing each voltage source to be a coil of wire (alternator or transformer winding) and do some slight rearranging, the "Y" configuration becomes more obvious:

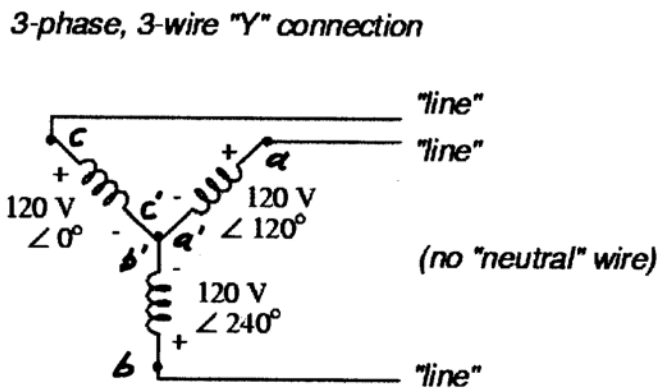


Fig. 20

The three conductors leading away from the voltage sources (windings) toward a load are typically called *lines*, while the windings themselves are typically called *phases*.
In a Y-connected system, there may or may not be a neutral wire attached at the junction point in the middle.

When we measure voltage and current in three-phase systems, we need to be specific as to *where* we're measuring. *Line voltage* refers to the amount of voltage measured between any two line conductors in a balanced three-phase system. The terms *line current* and *phase current* follow the same logic: the former referring to current through any one line conductor, and the latter to current through any one component.

Y-connected sources and loads always have line voltages greater than phase voltages, and line currents equal to phase currents.

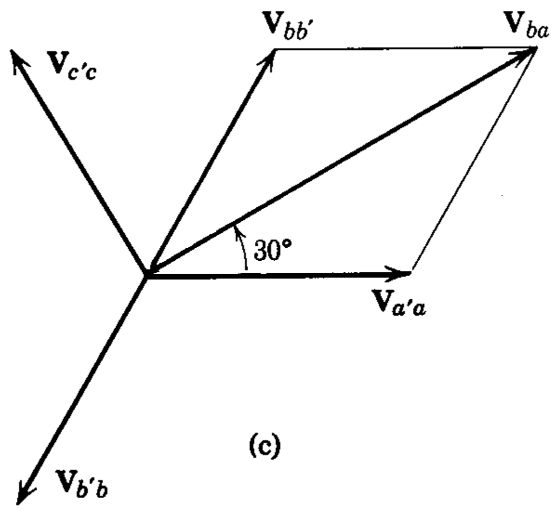


Fig. 21

If the Y-connected source or load is balanced the phasor diagram of Fig. 21 shows that the line voltage of Fig. 20, will be equal to the phase voltage times the square root of 3:

For "Y" circuits:

$$E_{line} = \sqrt{3}\ E_{phase}$$

$$I_{line} = I_{phase}$$

However, the "Y" configuration is not the only valid one for connecting three-phase voltage source or load elements together. Another configuration, shown in fig. 22, is known as the "Delta," (or mesh) for its geometric resemblance to the Greek letter of the same name.

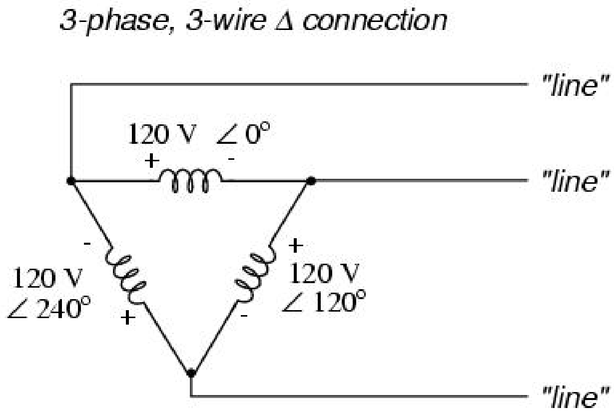


Fig. 22

At first glance it seems as though three voltage sources like this would create a short-circuit current flowing around the triangle with nothing but the internal impedance of the windings to limit it. Due to the phase angles of these three voltage sources, however, this is not the case

One quick check of this is to use Kirchhoff's Voltage Law to see if the three voltages around the loop add up to zero.

$$(120\text{ V} \angle 0^\circ) + (120\text{ V} \angle 240^\circ) + (120\text{ V} \angle 120^\circ)$$

Does it all equal 0?

Yes!

This is also obvious from phasor diagram of Fig.21.

Because each pair of line conductors is connected directly across a single winding in a delta circuit, the line voltage will be equal to the phase voltage. Conversely, because each line conductor attaches at a node between two windings, the line current will be the phasor sum of the two joining phase currents. Not surprisingly, the resulting equations for a delta configuration are as follows:

For Δ ("delta") circuits:

$$E_{line} = E_{phase}$$

$$I_{line} = \sqrt{3}\ I_{phase}$$

Let's see how this works in the example circuit of Fig. 23:

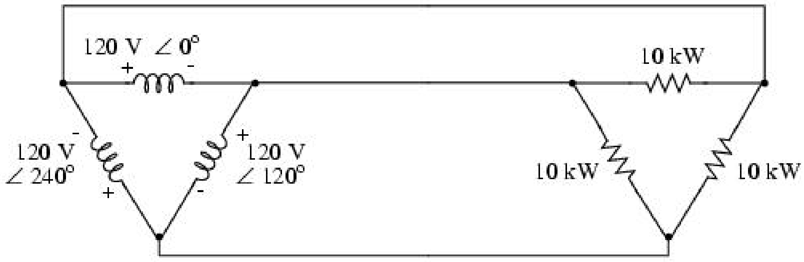


Fig. 23

With each load resistance receiving 120 volts from its respective phase winding at the source, the current in each phase of this circuit will be 83.33 amps:

$$I = \frac{P}{E}$$

$$I = \frac{10\ \text{kW}}{120\ \text{V}}$$

$$I = 83.33\ \text{A} \quad \textit{(for each load resistor and source winding)}$$

$$I_{line} = \sqrt{3}\ I_{phase}$$

$$I_{line} = \sqrt{3}\ (83.33\ \text{A})$$

$$I_{line} = 144.34\ \text{A}$$

One distinct characteristic of a delta-connected system is its lack of a neutral wire. With a Y-connected system, a neutral wire may be needed if load imbalances are expected. This is not necessary (or even possible!) in a delta-connected circuit. With each load phase element directly connected across a respective source phase winding, the phase voltage will be constant regardless of imbalances in the load elements.

Perhaps the greatest advantage of the delta-connected source is its fault tolerance. It is possible for one of the windings in a delta-connected three-phase source to fail open without affecting load voltage or current (Fig. 23).

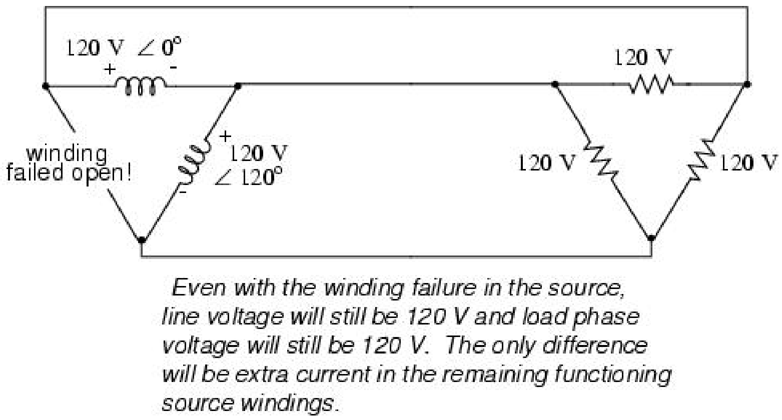


Fig 23

There are other reasons for the adoption of delta connected systems which are beyond the terms of reference of these notes.

Section Review

- The conductors connected to the three points of a three-phase source or load are called *lines*.
- The three components comprising a three-phase source or load are called *phases*.
- *Line voltage* is the voltage measured between any two lines in a three-phase circuit.
- *Phase voltage* is the voltage measured across a single component in a three-phase source or load.
- *Line current* is the current through any one line between a three-phase source and load.
- *Phase current* is the current through any one component comprising a three-phase source or load.
- In balanced "Y" circuits, line voltage is equal to phase voltage times the square root of 3, while line current is equal to phase current.

  For "Y" circuits:

  $$E_{line} = \sqrt{3}\ E_{phase}$$

  $$I_{line} = I_{phase}$$
- In balanced delta circuits, line voltage is equal to phase voltage, while line current is equal to phase current times the square root of 3.

  For Δ ("delta") circuits:

  $$E_{line} = E_{phase}$$

  $$I_{line} = \sqrt{3}\ I_{phase}$$
- Delta-connected three-phase voltage sources give greater reliability in the event of winding failure than Y-connected sources. However, Y-connected sources can deliver the same amount of power with less line current than delta-connected sources.

9. Power in three-phase systems
ELP032

The total power in a balanced three-phase load is the sum of three equal phase powers or

$$P_{\text{total}} = 3P_p = 3V_pI_p \cos\theta$$

where $\cos\theta$ is the power factor of the load or θ is the angle between phase voltage V_p and phase current I_p.

It is easier to measure line quantities, however, so an expression for total power in terms of V_l and I_l is useful.

In a Δ load, $V_l = V_p$ and $I_l = \sqrt{3}\,I_p$; therefore,

$$P_\Delta = 3V_pI_p\cos\theta = 3V_l\frac{I_l}{\sqrt{3}}\cos\theta = \sqrt{3}\,V_lI_l\cos\theta$$

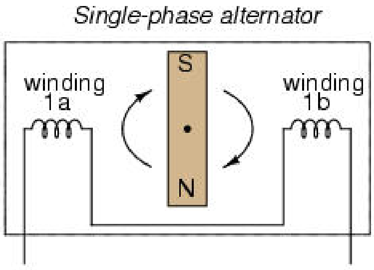
In a Y load, $V_l = \sqrt{3}\,V_p$ and $I_l = I_p$; therefore, the power is

$$P_Y = 3V_pI_p\cos\theta = 3\frac{V_l}{\sqrt{3}}I_l\cos\theta = \sqrt{3}\;V_lI_l\cos\theta$$

The same expression holds for power in a Δ- or a Y-connected load.

10. Generation of three-phase voltages

One question remains to be answered: how in the world do we get three AC voltage sources whose phase angles are exactly 120° apart? The phase-shifted voltage sources necessary for a polyphase power system are created in alternators with multiple sets of windings as shown in Fig. 24 . These winding sets are spaced around the circumference of the rotor's rotation at the desired angles, 120° in the three-phase system.



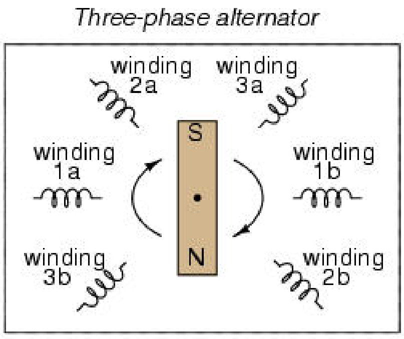


Fig. 24

Together, the six windings of a three-phase alternator are connected to comprise three winding pairs, each pair producing AC voltage with a phase angle 120^o shifted from either of the other two winding pairs. The interconnections between pairs of windings of the same phase have been omitted from the three-phase alternator drawing for simplicity.

We have established that three-phase are superior to single-phase systems in terms of efficient utilisation of transmission lines. Alternators built for three phase systems are also far superior to those for single-phase in terms of the efficient use of magnetic (iron) and electrical (copper) materials that constitute the generator. Additionally, three-phase systems can create a 'rotating magnetic field'(to be discussed in the next Unit), a property that is the basis of operation of perhaps 95% of all the industrial motors in the world. Finally, it can be shown that the power generated or absorbed by a three-phase system and the torque produced by three-phase motors are completely steady, i.e. without any ripple or harmonic content. It is not surprising then that three-phase has been adopted throughout the world for the transmission of electrical power.

Section Review

- The phase-shifted voltage sources necessary for a polyphase power system are created in alternators with multiple sets of windings. These winding sets are spaced around the circumference of the rotor's rotation at the desired angles.