3. Work, Power and Energy

[This material relates predominantly to modules ELP034, ELP035]

3.1 Work done by a constant force

When the point at which a force acts moves, the force is said to have done work.

When the force is constant, the work done is defined as the product of the force and distance moved.

$$\text{work done} = \text{force} \times \text{distance moved in direction of force}$$

Consider the example in Figure 3.1, a force F acting at the angle θ moves a body from point A to point B.

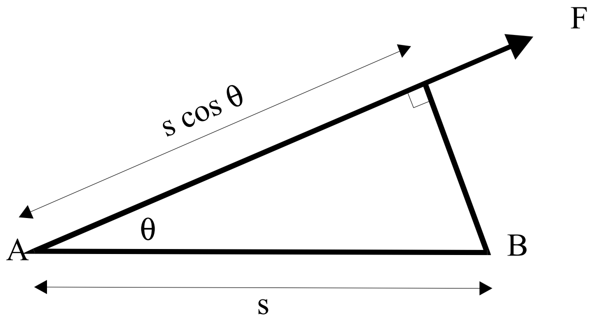

Figure 3.1: Notation for work done by a force

The distance moved in the direction of the force is given by

$$\text{Distance in direction of force} = s\cos\theta$$

So the work done by the force F is

$$\text{Work done} = F\,s\cos\theta$$

Equation 3.1

If the body moves in the same direction as the force the angle is 0.0 so

$$\text{Work done} = Fs$$

When the angle is 90 then the work done is zero.

The SI units for work are Joules J (with force, F, in Newton's N and distance, s, in metres m).

Worked Example 3.1

How much work is done when a force of 5 kN moves its point of application 600mm in the direction of the force.

Solution

$$\begin{aligned}\text{work done} &= \left(5\times10^{3}\right)\times\left(600\times10^{-3}\right)\\ &= 3000\,J\\ &= 3\,kJ\end{aligned}$$

Worked Example 3.2

Find the work done in raising 100 kg of water through a vertical distance of 3m.

Solution

The force is the weight of the water, so

$$\begin{aligned}\text{work done} &= (100\times9.81)\times3\\ &= 2943\,J\end{aligned}$$

3.2 Energy

A body which has the capacity to do work is said to possess energy.

For example , water in a reservoir is said to possesses energy as it could be used to drive a turbine lower down the valley. There are many forms of energy e.g. electrical, chemical heat, nuclear, mechanical etc.

The SI units are the same as those for work, Joules J.

In this module only purely mechanical energy will be considered. This may be of two kinds, **potential** and **kinetic**.

Potential Energy

There are different forms of potential energy two examples are: i) a pile driver raised ready to fall on to its target possesses gravitational potential energy while (ii) a coiled spring which is compressed possesses an internal potential energy.

Only gravitational potential energy will be considered here. It may be described as energy due to position relative to a standard position (normally chosen to be he earth's surface.)

The potential energy of a body may be defined as the amount of work it would do if it were to move from the its current position to the standard position.

Formulae for gravitational potential energy

A body is at rest on the earth's surface. It is then raised a vertical distance h above the surface. The work required to do this is the force required times the distance h.

Since the force required is it's weight, and weight, $W = mg$, then the work required is mgh. The body now possesses this amount of energy - stored as potential energy - it has the capacity to do this amount of work, and would do so if allowed to fall to earth.

Potential energy is thus given by:

$$\text{potential energy} = mgh$$

Equation 3.3

where h is the height above the earth's surface.

Worked example 3.3

What is the potential energy of a 10kg mass:

a) 100m above the surface of the earth
b) at the bottom of a vertical mine shaft 1000m deep.

Solution

a)

$$\begin{aligned}\text{potential energy} &= mgh \\ &= 10 \times 9.81 \times 100 \\ &= 9810J \\ &= 9.81kJ\end{aligned}$$

b)

$$\begin{aligned}\text{potential energy} &= mgh \\ &= 10\times 9.81\times(-1000) \\ &= -98100\,J \\ &= -98.1\,kJ\end{aligned}$$

Kinetic energy

Kinetic energy may be described as energy due to motion.

The kinetic energy of a body may be defined as the amount of work it can do before being brought to rest.

For example when a hammer is used to knock in a nail, work is done on the nail by the hammer and hence the hammer must have possessed energy.

Only linear motion will be considered here.

Formulae for kinetic energy

Let a body of mass *m* moving with speed *v* be brought to rest with uniform deceleration by a constant force *F* over a distance *s*.

Using Equation 1.4

$$\begin{aligned}v^2 &= u^2 + 2as \\ 0 &= u^2 + 2as \\ s &= \frac{v^2}{2a}\end{aligned}$$

And work done is given by

$$\begin{aligned}\text{work done} &= \text{force}\times\text{distance} \\ &= Fs \\ &= F\frac{v^2}{2a}\end{aligned}$$

The force is $F = ma$ so

$$\begin{aligned}\text{work done} &= ma\frac{v^2}{2a} \\ &= \frac{1}{2}mv^2\end{aligned}$$

Thus the kinetic energy is given by

$$\text{kinetic energy} = \frac{1}{2}mv^2$$

Equation 3.4

Kinetic energy and work done

When a body with mass m has its speed increased from u to v in a distance s by a constant force F which produces an acceleration a, then from Equation 1.3 we know

$$v^2 = u^2 + 2as$$
$$\frac{1}{2}v^2 - \frac{1}{2}u^2 = as$$

multiplying this by m give an expression of the increase in kinetic energy (the difference in kinetic energy at the end and the start)

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

Thus since $F = ma$

$$\text{increase in kinetic energy} = Fs$$

but also we know

$$Fs = \text{work done}$$

So the relationship between kinetic energy can be summed up as

Work done by forces acting on a body = change of kinetic energy in the body

Equation 3.5

This is sometimes known as the work-energy theorem.

Worked example 3.4

A car of mass 1000 kg travelling at 30m/s has its speed reduced to 10m/s by a constant breaking force over a distance of 75m.
Find:
a) The cars initial kinetic energy
b) The final kinetic energy
c) The breaking force

Solution

a)

$$\begin{aligned}\text{Initial kinetic energy} &= \frac{1}{2}mv^2 \\ &= 500 \times 30^2 \\ &= 450000\,J \\ &= 450\,kJ\end{aligned}$$

b)

$$\begin{aligned} \text{Final kinetic energy} &= \frac{1}{2}mv^2 \\ &= 500\times 10^2 \\ &= 50000\,J \\ &= 50\,kJ \end{aligned}$$

c)

$$\text{Change in kinetic energy} = 400\ kJ$$

By Equation 3.5 work done = change in kinetic energy so

$$\begin{aligned} Fs &= \text{change in kinetic energy} \\ \text{Breaking force}\times 75 &= 400\,000 \\ \text{Breaking force} &= 5333\,N \end{aligned}$$

3.3 Conservation of energy

The principle of conservation of energy state that the total energy of a system remains constant. Energy cannot be created or destroyed but may be converted from one form to another.

Take the case of a crate on a slope. Initially it is at rest, all its energy is potential energy. As it accelerates, some of it potential energy is converted into kinetic energy and some used to overcome friction. This energy used to overcome friction is not lost but converted into heat. At the bottom of the slope the energy will be purely kinetic (assuming the datum for potential energy is the bottom of the slope.)

If we consider a body falling freely in air, neglecting air resistance, then **mechanical energy is conserved**, as potential energy is lost and equal amount of kinetic energy is gained as speed increases.
If the motion involves friction or collisions then the principle of conservation of energy is true, but conservation of mechanical energy is not applicable as some energy is converted to heat and perhaps sound.

Worked Example 3.5

A cyclist and his bicycle has a mass of 80 kg. After 100m he reaches the top of a hill, with slope 1 in 20 measured along the slope, at a speed of 2 m/s. He then free wheels the 100m to the bottom of the hill where his speed has increased to 9m/s.
How much energy has he lost on the hill?

Solution

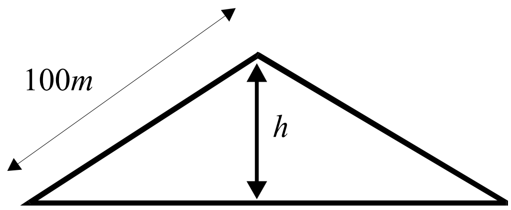

Figure 3.3: Dimensions of the hill in worked example 3.5

If the hill is 100m long then the height is:

$$h = 100\frac{1}{20} = 5m$$

So potential energy lost is

$$mgh = 80 \times 9.81 \times 5 = 3924J$$

Increase in kinetic energy is

$$\begin{aligned}\frac{1}{2}mv^2 - \frac{1}{2}mu^2 &= \frac{1}{2}m\left(v^2 - u^2\right)\\ &= 40(81-4)\\ &= 3080J\end{aligned}$$

By the principle of conservation of energy

$$\text{Initial energy} = \text{Final energy} + \text{loss of energy(due to friction etc.)}$$

$$\text{loss of energy (due to friction etc.)} = 3924 - 3080 = 844J$$

3.4 Power

Power is the rate at which work is done, or the rate at which energy is used transferred.

$$\text{Power} = \frac{\text{work done}}{\text{time taken}}$$

Equation 3.6

The SI unit for power is the watt W.
A power of $1W$ means that work is being done at the rate of $1J/s$.
Larger units for power are the kilowatt kW ($1kW = 1000\ W = 10^3\ W$) and
the megawatt MW ($1\ MW = 1000000\ W = 10^6\ W$).

If work is being done by a machine moving at speed v against a constant force, or resistance, F, then since work doe is force times distance, work done per second is Fv, which is the same as power.

$$\text{Power} = Fv$$

Equation 3.7

Worked Example 3.6

A constant force of 2kN pulls a crate along a level floor a distance of 10 m in 50s.
What is the power used?

Solution

$$\begin{aligned}\text{Work done} &= \text{force} \times \text{distance} \\ &= 2000 \times 10 \\ &= 20000\,J\end{aligned}$$

$$\begin{aligned}\text{Power} &= \frac{\text{work done}}{\text{time taken}} \\ &= \frac{20000}{50} \\ &= 400\,W\end{aligned}$$

Alternatively we could have calculated the speed first

$$\begin{aligned}v &= \frac{\text{distance}}{\text{time}} \\ &= \frac{10}{50} = 0.2\,m/s\end{aligned}$$

and then calculated power

$$\begin{aligned}\text{Power} &= \text{Force} \times \text{Speed} \\ &= Fv \\ &= 2000 \times 0.2 \\ &= 400\,W\end{aligned}$$

Worked Example 3.7

A hoist operated by an electric motor has a mass of 500 kg. It raises a load of 300 kg vertically at a steady speed of 0.2 m/s. Frictional resistance can be taken to be constant at 1200 N.
What is the power required?

Solution

$$\begin{aligned}\text{Total mass} &= m = 800\,kg \\ \text{Weight} &= 800 \times 9.81 \\ &= 7848\,N\end{aligned}$$

$$\text{Total force} = 7848 + 1200$$
$$= 9048\,N$$

From Equation 3.7

$$\text{Power} = \text{force} \times \text{speed}$$
$$= 9048 \times 0.2$$
$$= 1810\,W$$
$$= 1.81\,kW$$

<u>Worked example 3.8</u>

A car of mass 900 kg has an engine with power output of 42 kW. It can achieve a maximum speed of 120 km/h along the level.

a) What is the resistance to motion?

b) If the maximum power and the resistance remained the same what would be the maximum speed the car could achieve up an incline of 1 in 40 along the slope?

Solution

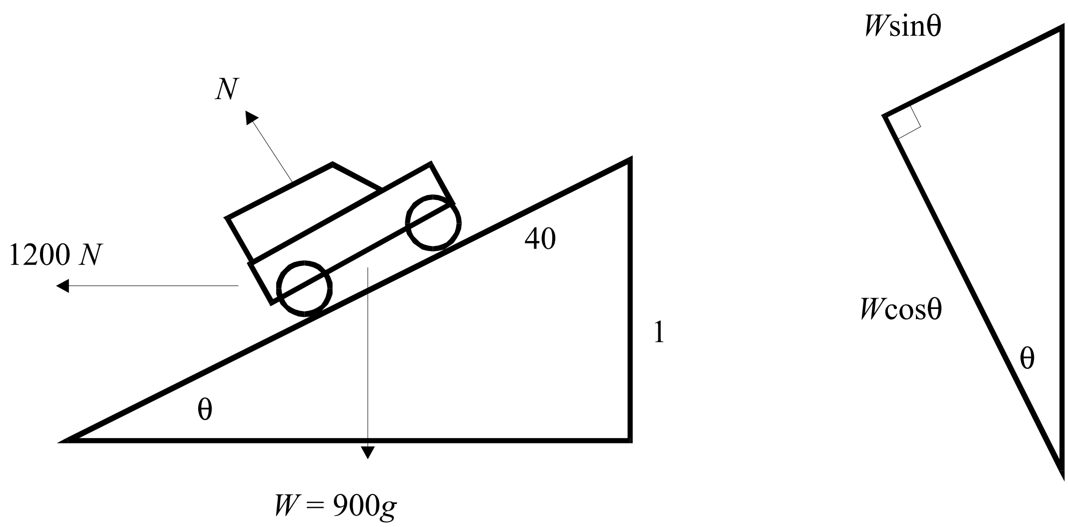

Figure 3.4: Forces on the car on a slope in Worked Example 3.8

First get the information into the correct units:

$$120\,km/h = \frac{120 \times 1000}{3600}$$
$$= 33.33\,m/s$$

a) Calculate the resistance

$$\begin{aligned}\text{Power} &= \text{Force} \times \text{speed}\\ &= \text{Resistance} \times \text{speed}\\ 42000 &= \text{Resistance} \times 33.33\\ \text{Resistance} &= \frac{42000}{33.33} = 1260\,N\end{aligned}$$

b)

$$\begin{aligned}\text{Total force down the incline} &= \text{frictional force} + \text{component of weight down incline}\\ &= 1260 + mg\sin\theta\\ &= 1260 + 900 \times 9.81\frac{1}{40}\\ &= 1260 + 221\\ &= 1481\,N\end{aligned}$$

$$\begin{aligned}\text{Power} &= \text{force} \times \text{speed}\\ \text{Speed} &= \frac{\text{Power}}{\text{force}}\\ &= \frac{42000}{1481}\\ &= 28.4\,m/s\end{aligned}$$

Or in km/h

$$\begin{aligned}\text{Speed} &= 28.4\frac{3600}{1000}\\ &= 102\,km/h\end{aligned}$$

3.5 Moment, couple and torque

The moment of a force F about a point is its turning effect about the point.
It is quantified as the product of the force and the perpendicular distance from the point to the line of action of the force.

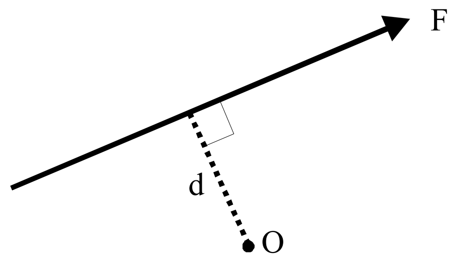

Figure 3.4: Moment of a force

In Figure 3.5 the moment of F about point O is

$$\text{moment} = Fd$$

Equation 3.8

A **couple** is a pair of equal and parallel but opposite forces as shown in Figure 3.6:

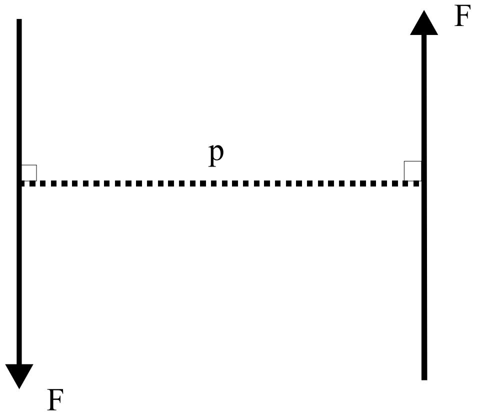

Figure 3.6: A couple

The moment of a couple about any point in its plane is the product of one force and the perpendicular distance between them:

$$\text{Moment of couple} = Fp$$

Equation 3.9

Example of a couple include turning on/off a tap, or winding a clock.

The SI units for a moment or a couple are Newton metres, Nm.

In engineering the moment of a force or couple is know as **torque**. A spanner tightening a nut is said to exert a torque on the nut, similarly a belt turning a pulley exerts a torque on the pulley.

Work done by a constant torque

Let a force F turn a light rod OA with length r through an angle of θ to position OB, as shown in Figure 3.7.

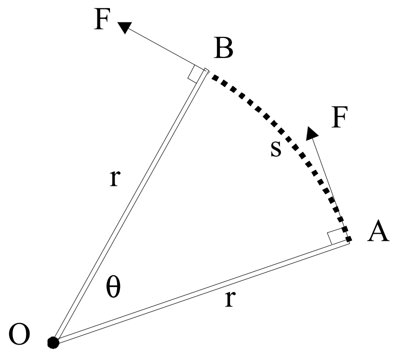

Figure 3.7: Work done by a constant torque

The torque T_Q exerted about O is force times perpendicular distance from O.

$$T_Q = Fr$$

Equation 3.10

Now work done by F is

$$\text{work done} = Fs$$

s is the arc of the circle, when θ is measure in radians

$$s = r\theta$$

$$\text{work done} = Fr\theta$$
$$\text{work done} = T_Q\theta$$

Equation 3.11

The work done by a constant torque T_Q is thus the product of the torque and the angle through which it turns (where the angle is measured in radians.)

As the SI units for work is Joules, T_Q must be in Nm

Power transmitted by a constant torque

Power is rate of doing work. It the rod in Figure 3.7 rotates at n revolutions per second, then in one second the angle turned through is

$$\theta = 2\pi n$$

radians, and the work done per second will be, by Equation 3.11

$$\text{work done per second} = \text{power} = T_Q 2\pi n$$

as angular speed is

$$\omega = 2\pi n$$

then

$$\text{power} = 2\pi n T_Q$$

$$\text{power} = \omega T_Q$$

Equation 3.12

The units of power are Watts, *W*, with n in *rev/s*, ω in *rad/s* and T_Q in *Nm*.

Worked Example 3.9

A spanner that is used to tighten a nut is 300mm long. The force exerted on the end of a spanner is 100 N.

a) What is the torque exerted on the nut?
b) What is the work done when the nut turns through 30°?

Solution

a)
Calculate the torque by Equation 3.10

$$\begin{aligned} T_Q &= Fr \\ &= 100 \times \left(300 \times 10^{-3}\right) \\ &= 30\,Nm \end{aligned}$$

b)
Calculate the work done by Equation 3.11

$$\begin{aligned} \text{work done} &= T_Q \theta \\ &= 30 \times \frac{\pi}{6} \\ &= 15.7\,J \end{aligned}$$

Worked Example 3.10

An electric motor is rated at 400 W. If its efficiency is 80%, find the maximum torque which it can exert when running at 2850 rev/min.

Solution

Calculate the speed in rev/s using Equation 3.12

$$\text{power} = 2\pi n T_Q$$

$$n = \frac{\text{power}}{2\pi T_Q}$$

$$n = \frac{2850}{60} = 47.5\,rev/s$$

Calculate the power as the motor is 80% efficient

$$\text{power} = 400 \times \frac{80}{100} = 320\,W$$

$$\text{power} = 2\pi n T_Q$$

$$\begin{aligned} T_Q &= \frac{\text{power}}{2\pi n} \\ &= \frac{320}{2\pi 47.5} \\ &= 1.07\,Nm \end{aligned}$$

Work done by a variable torque

In practice the torque is often variable. In this case the work done cannot be calculated by Equation 3.11, but must be found in a similar way to that used for a variable force (see earlier.)

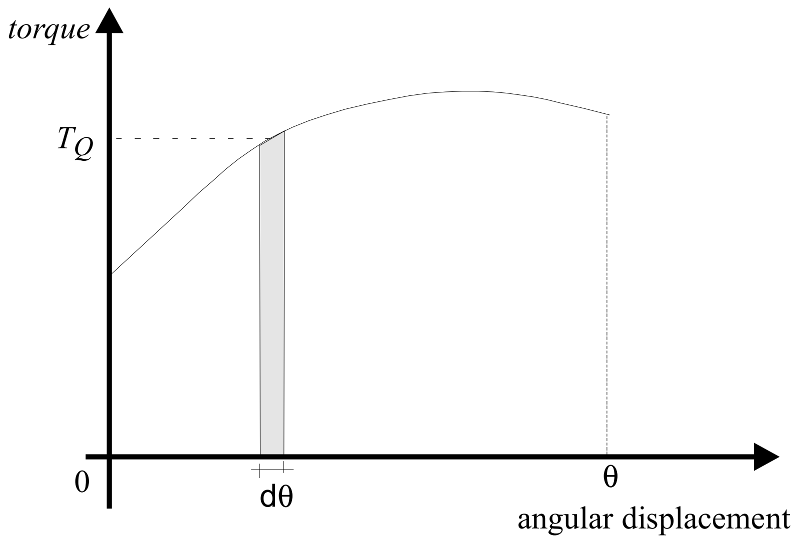

Figure 3.8: Work done by a variable torque

The work done when angular displacement is $d\theta$ is $T_Q\ d\theta$. This is the area of the shaded strip in Figure 3.8. the total work done for the angular displacement θ is thus the area under the torque/displacement graph.

For variable torque

$$\text{work done} = \text{area under torque/angular displacement graph}$$

Equation 3.13

As with variable forces, in general you must uses some special integration technique to obtain the area under a curve. Three common techniques are the trapezoidal, mid-ordinate and Simpson's rule. They are not detailed here but may be found in many mathematical text book.

Worked Example 3.12

A machine requires a variable torque as shown in Figure 3.9, Find:

a) The work done per revolution
b) The average torque over one revolution
c) The power required if the machine operates at 30 rev/min

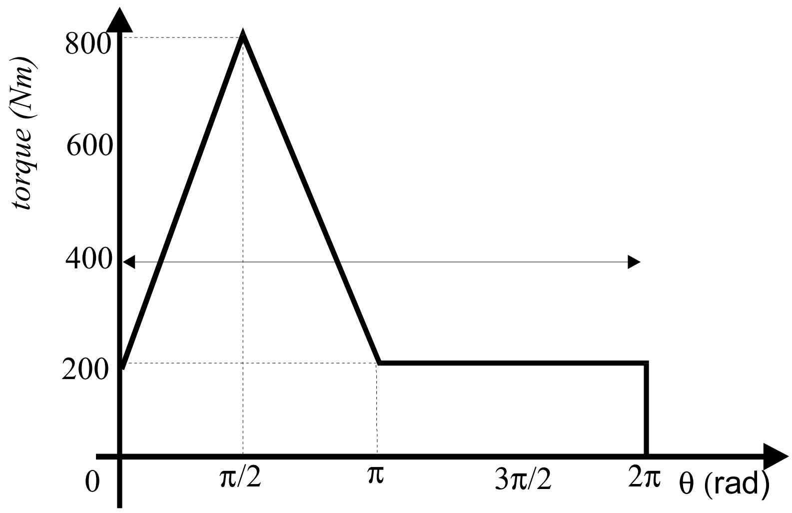

Figure 3.9: Torque requirement for Worked Example 3.12

Solution

a)
From Equation 3.13

$$\begin{aligned} \text{work done} &= \text{area under torque}/\theta \text{ graph} \\ &= \text{area of triangle ABC} + \text{area of rectangle ADEO} \\ &= \frac{1}{2}(\pi \times 600) + 2\pi 200 \\ &= 2200\,J \end{aligned}$$

for one revolution

b)
Average torque is the average height of figure OABCDE = area /2π

$$\text{Average torque} = \frac{2200}{2\pi} = 350\,Nm$$

c)

$$\begin{aligned}\text{power} &= 2\pi n \times \text{average torque} \\ &= 2\pi\left(\frac{30}{60}\right)350 \\ &= 1100W\end{aligned}$$