

FB-DC5 Electric Circuits: Capacitors

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1. Electric fields and capacitance

Whenever an electric voltage exists between two separated conductors, an electric field is present within the space between those conductors. In basic electronics, we study the interactions of voltage, current, and resistance as they pertain to circuits, which are conductive paths through which electrons may travel. When we talk about fields, however, we're dealing with interactions that can be spread across empty space.

Admittedly, the concept of a "field" is somewhat abstract. At least with electric current it isn't too difficult to envision tiny particles called electrons moving their way between the nuclei of atoms within a conductor, but a "field" doesn't even have mass, and need not exist within matter at all.

Despite its abstract nature, almost every one of us has direct experience with fields, at least in the form of magnets. Have you ever played with a pair of magnets, noticing how they attract or repel each other depending on their relative orientation? There is an undeniable force between a pair of magnets, and this force is without "substance." It has no mass, no colour, no odour, and if not for the physical force exerted on the magnets themselves, it would be utterly insensible to our bodies. Physicists describe the interaction of magnets in terms of *magnetic fields* in the space between them. If iron filings are placed near a magnet, they orient themselves along the lines of the field, visually indicating its presence.

The subject of this chapter is *electric* fields (and devices called *capacitors* that exploit them), not *magnetic* fields, but there are many similarities. Most likely you have experienced electric fields as well. Unit FB1 began with an explanation of static electricity, and how materials such as wax and wool -- when rubbed against each other -- produced a physical attraction. Again, physicists would describe this interaction in terms of *electric fields* generated by the two objects as a result of their electron imbalances. Suffice it to say that whenever a voltage exists between two points, there will be an electric field manifested in the space between those points.

Fields have two measures: a field *force* and a field *flux*. The field *force* is the amount of "push" that a field exerts over a certain distance. The field *flux* is the total quantity, or effect, of the field through space. Field force and flux are roughly analogous to voltage ("push") and current (flow) through a conductor, respectively, although field flux can exist in totally empty space (without the motion of particles such as electrons) whereas current can only take place where there are free electrons to move. Field flux can be opposed in space, just as the flow of electrons can be opposed by resistance. The amount of field flux that will develop in space is proportional to the amount of field force applied, divided by the amount of opposition to flux. Just as the type of conducting material dictates that conductor's specific resistance to electric current, the type of insulating material separating two conductors dictates the specific opposition to field flux.

Normally, electrons cannot enter a conductor unless there is a path for an equal amount of electrons to exit (remember the marble-in-tube analogy?). This is why conductors must be connected together in a circular path (a circuit) for continuous current to occur. Oddly enough, however, extra electrons can be "squeezed" into a conductor without a path to exit if an electric field is allowed to develop in space relative to another conductor. The

number of extra free electrons added to the conductor (or free electrons taken away) is directly proportional to the amount of field flux between the two conductors.

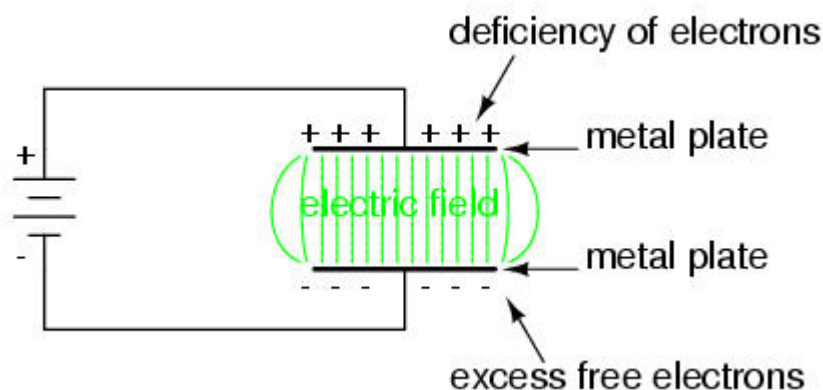
Capacitors are components designed to take advantage of this phenomenon by placing two conductive plates (usually metal) in close proximity with each other. There are many different styles of capacitor construction, each one suited for particular ratings and purposes. For very small capacitors, two circular plates sandwiching an insulating material will suffice. For larger capacitor values, the "plates" may be strips of metal foil, sandwiched around a flexible insulating medium and rolled up for compactness. The highest capacitance values are obtained by using a microscopic-thickness layer of insulating oxide separating two conductive surfaces. In any case, though, the general idea is the same: two conductors, separated by an insulator.

The schematic symbol for a capacitor is quite simple, being little more than two short, parallel lines (representing the plates) separated by a gap. Wires attach to the respective plates for connection to other components. An older, obsolete schematic symbol for capacitors showed interleaved plates, which is actually a more accurate way of representing the real construction of most capacitors:

Capacitor symbols



When a voltage is applied across the two plates of a capacitor, a concentrated field flux is created between them, allowing a significant difference of free electrons (a charge) to develop between the two plates:



As the electric field is established by the applied voltage, extra free electrons are forced to collect on the negative conductor, while free electrons are "robbed" from the positive conductor. This differential charge equates to storage of energy in the capacitor,

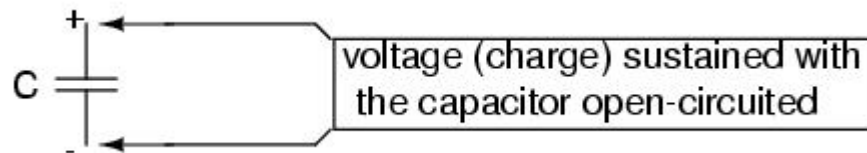
representing the potential charge of the electrons between the two plates. The greater the difference of electrons on opposing plates of a capacitor, the greater the field flux, and the greater "charge" of energy the capacitor will store.

Because capacitors store the potential energy of accumulated electrons in the form of an electric field, they behave quite differently than resistors (which simply dissipate energy in the form of heat) in a circuit. Energy storage in a capacitor is a function of the voltage between the plates, as well as other factors which we will discuss later in this chapter. A capacitor's ability to store energy as a function of voltage (potential difference between the two leads) results in a tendency to try to maintain voltage at a constant level. In other words, capacitors tend to resist *changes* in voltage drop. When voltage across a capacitor is increased or decreased, the capacitor "resists" the *change* by drawing current from or supplying current to the source of the voltage change, in opposition to the *change*.

To store more energy in a capacitor, the voltage across it must be increased. This means that more electrons must be added to the (-) plate and more taken away from the (+) plate, necessitating a current in that direction. Conversely, to release energy from a capacitor, the voltage across it must be decreased. This means some of the excess electrons on the (-) plate must be returned to the (+) plate, necessitating a current in the other direction.

Just as Isaac Newton's first Law of Motion ("an object in motion tends to stay in motion; an object at rest tends to stay at rest") describes the tendency of a mass to oppose changes in velocity, we can state a capacitor's tendency to oppose changes in voltage as such: "A charged capacitor tends to stay charged; a discharged capacitor tends to stay discharged."

Hypothetically, a capacitor left untouched will indefinitely maintain whatever state of voltage charge that it's been left it. Only an outside source (or drain) of current can alter the voltage charge stored by a perfect capacitor:



Practically speaking, however, capacitors will eventually lose their stored voltage charges due to internal leakage paths for electrons to flow from one plate to the other. Depending on the specific type of capacitor, the time it takes for a stored voltage charge to self-dissipate can be a *long* time (several years with the capacitor sitting on a shelf!).

When the voltage across a capacitor is increased, it draws current from the rest of the circuit, acting as a power load. In this condition the capacitor is said to be *charging*, because there is an increasing amount of energy being stored in its electric field.

Conversely, when the voltage across a capacitor is decreased, the capacitor supplies current to the rest of the circuit, acting as a power source. In this condition the capacitor is said to be *discharging*. Its store of energy -- held in the electric field -- is decreasing now as energy is released to the rest of the circuit.

If a source of voltage is suddenly applied to an uncharged capacitor (a sudden increase of voltage), the capacitor will draw current from that source, absorbing energy from it, until the capacitor's voltage equals that of the source. Once the capacitor voltage reached this

final (charged) state, its current decays to zero. Conversely, if a load resistance is connected to a charged capacitor, the capacitor will supply current to the load, until it has released all its stored energy and its voltage decays to zero. Once the capacitor voltage reaches this final (discharged) state, its current decays to zero. In their ability to be charged and discharged, capacitors can be thought of as acting somewhat like secondary-cell batteries.

The choice of insulating material between the plates, as was mentioned before, has a great impact upon how much field flux (and therefore how much charge) will develop with any given amount of voltage applied across the plates. Because of the role of this insulating material in affecting field flux, it has a special name: *dielectric*. Not all dielectric materials are equal: the extent to which materials inhibit or encourage the formation of electric field flux is called the *permittivity* of the dielectric.

The measure of a capacitor's ability to store energy for a given amount of voltage drop is called *capacitance*. Not surprisingly, capacitance is also a measure of the intensity of opposition to changes in voltage (exactly how much current it will produce for a given rate of change in voltage). Capacitance is symbolically denoted with a capital "C," and is measured in the unit of the Farad, abbreviated as "F."

Convention, for some odd reason, has favoured the metric prefix "micro" in the measurement of large capacitances, and so many capacitors are rated in terms of confusingly large microFarad values: for example, one large capacitor I have seen was rated 330,000 microFarads!! Why not state it as 330 milliFarads? I don't know.

Section Review:

- Capacitors react against changes in voltage by supplying or drawing current in the direction necessary to oppose the change.
- When a capacitor is faced with an increasing voltage, it acts as a *load*: drawing current as it absorbs energy (current going in the negative side and out the positive side, like a resistor).
- When a capacitor is faced with a decreasing voltage, it acts as a *source*: supplying current as it releases stored energy (current going out the negative side and in the positive side, like a battery).
- The ability of a capacitor to store energy in the form of an electric field (and consequently to oppose changes in voltage) is called *capacitance*. It is measured in the unit of the *Farad* (F).

2. Capacitors and calculus

Capacitors do not have a stable "resistance" as conductors do. However, there is a definite mathematical relationship between voltage and current for a capacitor, as follows:

"Ohm's Law" for a capacitor

$$i = C \frac{dv}{dt}$$

Where,

i = Instantaneous current through the capacitor

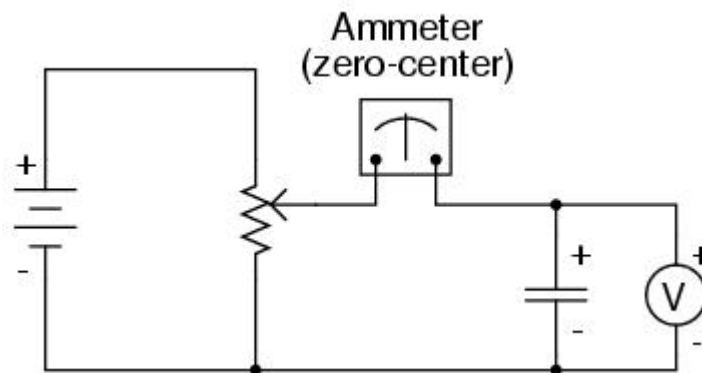
C = Capacitance in Farads

$\frac{dv}{dt}$ = Instantaneous rate of voltage change
(volts per second)

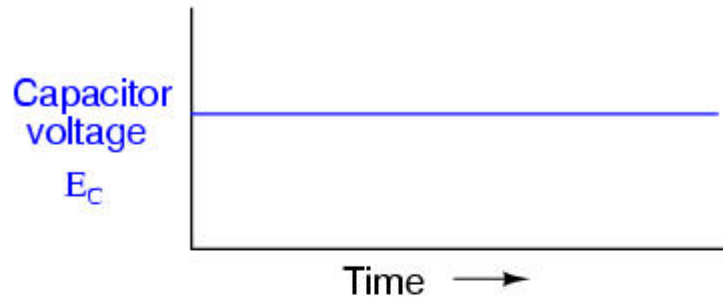
The lower-case letter "i" symbolizes *instantaneous* current, which means the amount of current at a specific point in time. This stands in contrast to constant current or average current (capital letter "I") over an unspecified period of time.

In this equation we see something novel to our experience thus far with electric circuits: the variable of *time*. When relating the quantities of voltage, current, and resistance to a resistor, it doesn't matter if we're dealing with measurements taken over an unspecified period of time ($V=IR$), or at a specific moment in time ($v=ir$). The same basic formula holds true, because time is irrelevant to voltage, current, and resistance in a component like a resistor.

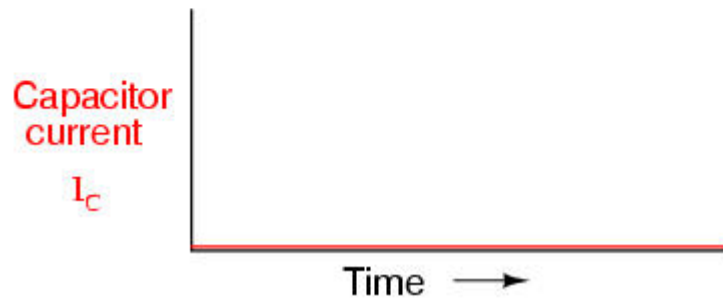
In a capacitor, however, time is an essential variable, because current is related to how *rapidly* voltage changes over time. To fully understand this, a few illustrations may be necessary. Suppose we were to connect a capacitor to a variable-voltage source, constructed with a potentiometer and a battery:



If the potentiometer mechanism remains in a single position (wiper is stationary), the voltmeter connected across the capacitor will register a constant (unchanging) voltage, and the ammeter will register 0 amps. In this scenario, the instantaneous rate of voltage change (dv/dt) is equal to zero, because the voltage is unchanging. The equation tells us that with 0 volts per second change for a dv/dt , there must be zero instantaneous current (i). From a physical perspective, with no change in voltage, there is no need for any electron motion to add or subtract charge from the capacitor's plates, and thus there will be no current.

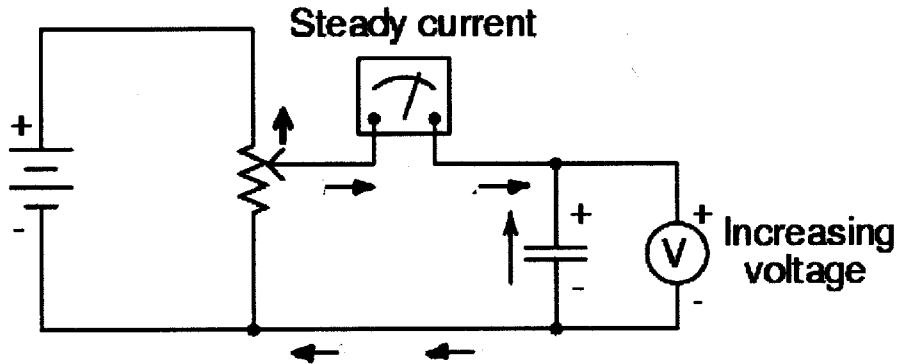


Potentiometer wiper not moving

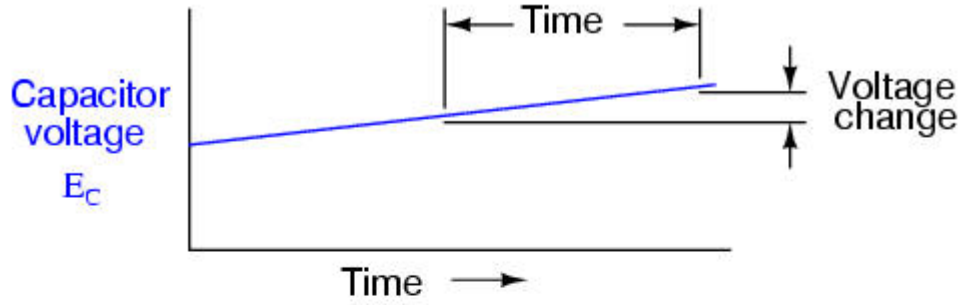


Now, if the potentiometer wiper is moved slowly and steadily in the "up" direction, a greater voltage will gradually be imposed across the capacitor. Thus, the voltmeter indication will be increasing at a slow rate:

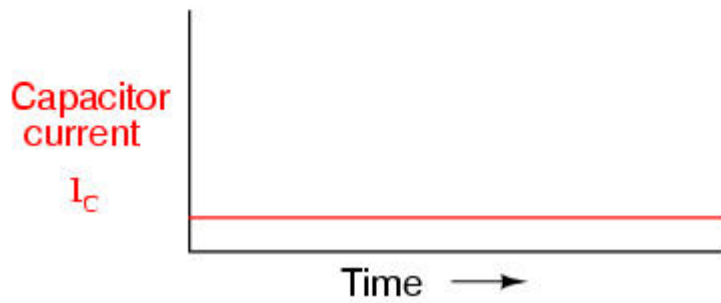
Potentiometer wiper moving slowly in the "up" direction



If we assume that the potentiometer wiper is being moved such that the *rate* of voltage increase across the capacitor is steady (for example, voltage increasing at a constant rate of 2 volts per second), the dv/dt term of the formula will be a fixed value. According to the equation, this fixed value of dv/dt , multiplied by the capacitor's capacitance in Farads (also fixed), results in a fixed current of some magnitude. From a physical perspective, an increasing voltage across the capacitor demands that there be an increasing charge differential between the plates. Thus, for a slow, steady voltage increase rate, there must be a slow, steady rate of charge building in the capacitor, which equates to a slow, steady flow rate of electrons, or current. In this scenario, the capacitor is acting as a *load*, with electrons entering the negative plate and exiting the positive, accumulating energy in the electric field.

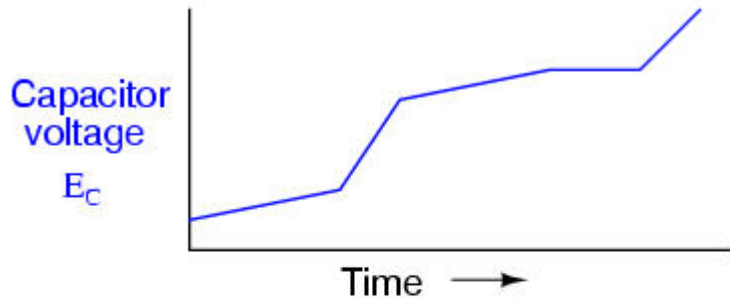


Potentiometer wiper moving slowly "up"

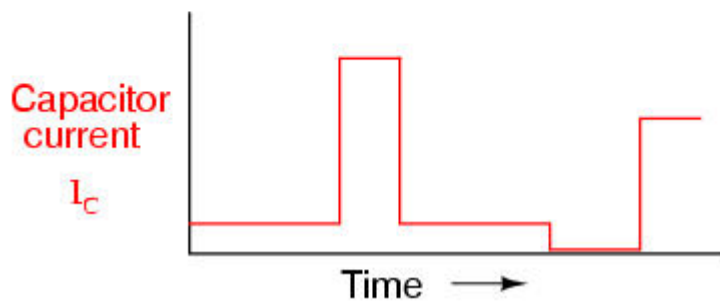


If the potentiometer is moved in the same direction, but at a faster rate, the rate of voltage change (dv/dt) will be greater and so will be the capacitor's current:

If we were to move the potentiometer's wiper in the same direction as before ("up"), but at varying rates, we would obtain graphs that looked like this:



Potentiometer wiper moving "up" at different rates



Note how that at any given point in time, the capacitor's current is proportional to the rate-of-change, or *slope* of the capacitor's voltage plot. When the voltage plot line is rising quickly (steep slope), the current will likewise be great. Where the voltage plot has a mild slope, the current is small. At one place in the voltage plot where it levels off (zero slope, representing a period of time when the potentiometer wasn't moving), the current falls to zero.

If we were to move the potentiometer wiper in the "down" direction, the capacitor voltage would *decrease* rather than increase. Again, the capacitor will react to this change of voltage by producing a current, but this time the current will be in the opposite direction. A decreasing capacitor voltage requires that the charge differential between the capacitor's plates be reduced, and that only way that can happen is if the current reverses its direction of flow, the capacitor discharging rather than charging. In this condition the capacitor will act as a *source*, like a battery, releasing its stored energy to the rest of the circuit.

Again, the amount of current through the capacitor is directly proportional to the rate of voltage change across it. The only difference between the effects of a *decreasing* voltage and an *increasing* voltage is the *direction* of electron flow. For the same rate of voltage change over time, either increasing or decreasing, the current magnitude (amps) will be the same. Mathematically, a decreasing voltage rate-of-change is represented as a negative quantity for dv/dt . Following the formula $i = C(dv/dt)$, this will result in a current figure that is likewise negative in sign, indicating a direction of flow corresponding to discharge of the capacitor.

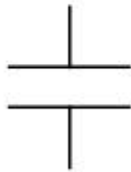
3. Factors affecting capacitance

There are three basic factors of capacitor construction determining the amount of capacitance created. These factors all dictate capacitance by affecting how much electric field flux (relative difference of electrons between plates) will develop for a given amount of magnetic field force (voltage between the two plates):

PLATE AREA: All other factors being equal, greater plate gives greater capacitance; less plate gives less capacitance.

Explanation: Larger plate area results in more field flux (charge collected on the plates) for a given field force (voltage across the plates).

less capacitance



more capacitance

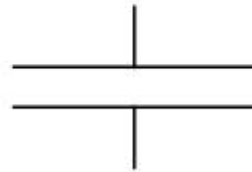
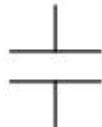


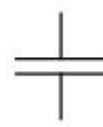
PLATE SPACING: All other factors being equal, further plate spacing gives less capacitance; closer plate spacing gives greater capacitance.

Explanation: Closer spacing results in a greater field force (voltage across the capacitor divided by the distance between the plates), which results in a greater field flux (charge collected on the plates) for any given voltage applied across the plates.

less capacitance

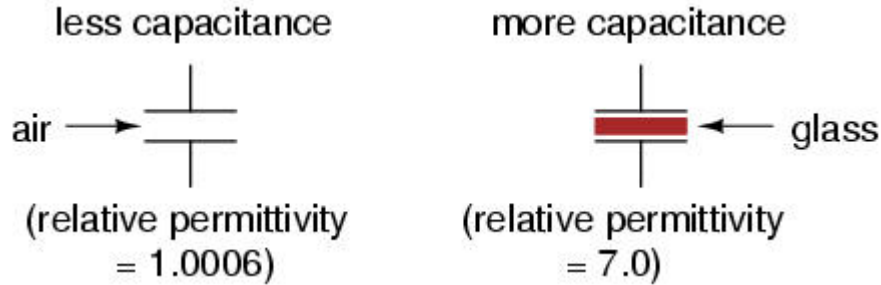


more capacitance



DIELECTRIC MATERIAL: All other factors being equal, greater permittivity of the dielectric gives greater capacitance; less permittivity of the dielectric gives less capacitance.

Explanation: Although it's complicated to explain, some materials offer less opposition to field flux for a given amount of field force. Materials with a greater permittivity allow for more field flux (offer less opposition), and thus a greater collected charge, for any given amount of field force (applied voltage).



"Relative" permittivity means the permittivity of a material, relative to that of a pure vacuum. The greater the number, the greater the permittivity of the material. Glass, for instance, with a relative permittivity of 7, has seven times the permittivity of a pure vacuum, and consequently will allow for the establishment of an electric field flux seven times stronger than that of a vacuum, all other factors being equal.

An approximation of capacitance for any pair of separated conductors can be found with this formula:

$$C = \frac{\epsilon A}{d}$$

Where,

C = Capacitance in Farads

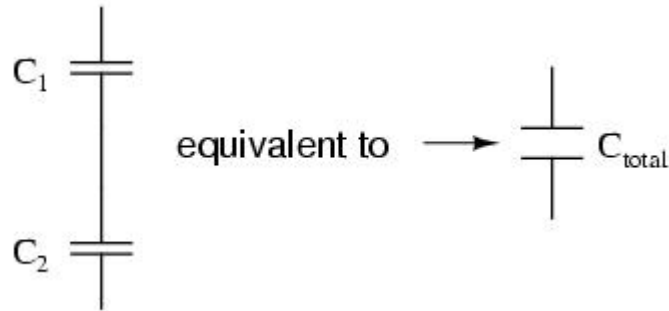
ϵ = Permittivity of dielectric (absolute, not relative)

A = Area of plate overlap in square meters

d = Distance between plates in meters

4. Series and parallel capacitors

When capacitors are connected in series, the total capacitance is less than any one of the series capacitors' individual capacitances. If two or more capacitors are connected in series, the overall effect is that of a single (equivalent) capacitor having the sum total of the plate spacings of the individual capacitors. As we've just seen, an increase in plate spacing, with all other factors unchanged, results in decreased capacitance.

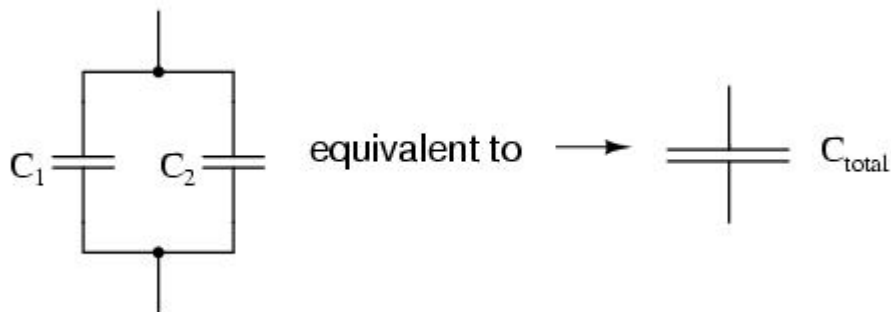


Thus, the total capacitance is less than any one of the individual capacitors' capacitances. The formula for calculating the series total capacitance is the same form as for calculating parallel resistances:

Series Capacitances

$$C_{\text{total}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}}$$

When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitors' capacitances. If two or more capacitors are connected in parallel, the overall effect is that of a single equivalent capacitor having the sum total of the plate areas of the individual capacitors. As we've just seen, an increase in plate area, with all other factors unchanged, results in increased capacitance.



Thus, the total capacitance is more than any one of the individual capacitors' capacitances. The formula for calculating the parallel total capacitance is the same form as for calculating series resistances:

Parallel Capacitances

$$C_{\text{total}} = C_1 + C_2 + \dots + C_n$$

As you will no doubt notice, this is exactly opposite of the phenomenon exhibited by resistors. With resistors, series connections result in additive values while parallel connections result in diminished values. With capacitors, it's the reverse: parallel connections result in additive values while series connections result in diminished values.

Section Review:

- Capacitances diminish in series.
- Capacitances add in parallel.