1. Heat Transfer Processes

1. Introduction

Energy takes many different forms. These are more closely related than their appearance suggests. Some commonly encountered forms are

- Heat
- Electromagnetic
- Gravitation
- Chemical
- Nuclear
- Mechanical (stress/strain)
- Kinetic

Energy can be loosely defined as the potential for doing work. Of course this begs the question of what is work. Perhaps, to avoid circularity we should agree, in our context, that work is anything useful that is produced by a process. It can always be related by analogy to its simplest form: mechanical work where it is the product of Force and Distance.

Careful examination of the list of energy forms reveals two distinct categories: energy associated with the fundamental forces of nature (gravitational, electromagnetic (weak), nuclear (strong) etc), and kinetic energy on a macroscopic or microscopic level (e.g. heat). There are links and complexities surrounding the topics of statistical physics (order, entropy etc) but we need not be too concerned by these at this stage.

2. Heat transfer processes

Heat is energy transfer between bodies as a result of temperature difference. This lecture deals with the elementary fundamentals of heat transfer. Heat is transferred by the following modes:

i) mass transport
ii) conduction
iii) convection
v) radiation

In fact convection, both forced and natural, is really a combination of conduction and mass transport. It has however been found convenient to designate and characterise it independently.

2.1 Mass transport
This is the simplest means of moving thermal energy from one location to another. This energy is associated with a mass, usually of a moving fluid. For ease of description we will assume the other heat transport mechanisms to be negligible. Take for example water moving in a highly insulated pipe of cross sectional area, A, with a velocity, *V*.

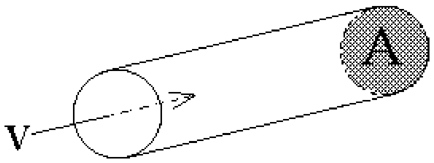

Figure 1: Schematic illustration of heat transfer by mass transport

If C_p is the specific heat capacity[1] of the fluid, the flux of heat passing through any section of the pipe is given by $\dot{m} C_p \Delta T_f$ where ΔT_f is the relevant temperature difference for the fluid and $\dot{m}$ is the mass flow rate given by ρAV; ρ being the fluid density.

2.2 Conduction
Conduction is the transfer of heat from one part of a body to another with no macroscopic movement within body. It is described adequately for our purposes by a linear theory.

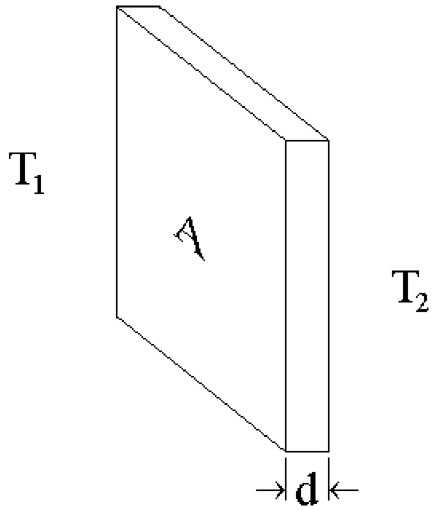

Figure 2: Schematic illustration of conductive heat transfer

Ignoring end effects, the heat conducted between the two faces of the material, assumed to be at temperatures of T_1 and T_2 respectively is given by

$$Q = A\frac{k}{d}|T_1 - T_2|$$

1 Specific heat of a material is the heat required to raise the temperature of a unit mass of the substance by one degree.

where A is the cross sectional area of the sample and *d* its thickness. *k* is the conductivity of the sample material. Its units are $W/m^2\,{}^\circ C/m = W/m^\circ C$

Often for simplicity of term *k/d* is combined for a given thickness of material and is called a U value with units

$$W/m^2\,{}^oC$$

(NB this has the form of a heat transfer coefficient)

Conductive heat loss/gain is then simply given by $Q_{cond} = AU\Delta T$

where $\Delta T = |T_1 - T_2|$ •

The same basic form is used for all linear heat transfer. In general h denotes a heat transfer coefficient (U is just a particular example); and $Q = Ah\Delta T$.

2.3 Convection

Convection is divided into two categories: natural convection where the fluids move under their own buoyancy forces and forced convection where the fluid is driven past the surfaces of interest. In both cases the concern is with the transfer of heat from a solid surface to a fluid or vice versa, or both.

2.3.1 Forced convection

The efficiency of heat transfer from a surface to a fluid will depend on the nature of the flow, in particular whether the flow is laminar or turbulent. Clearly turbulent flow is much more effective at moving heat away from a surface. The relations describing the heat transfer can thus be expected to involve the *Reynolds number*, *Re*, which describes the transition from laminar to turbulent flow. The other dimensionless number involved is the *Nusselt number*, *Nu*. For parallel plates distance L apart the Nusselt number is defined as

$$Nu = [(L/k) / (1/h)]$$

where h is the heat transfer coefficient. It can be seen to be the ratio of conduction resistance to convection resistance. *Nu* = 1 represents pure conduction.

An example of an empirical relation giving the heat transfer coefficient is stated here for turbulent air flow between flat plates where one plate is heated.

$$Nu = 0.0158\, Re^{0.8}$$

Fluid properties are evaluated at the mean fluid temperature. Other published relations are similar and sometimes, depending on the physical circumstances, also involve the Prandtl number, *Pr*.

The heat transfer coefficient has units of $W/m^2\,{}^0C$

2.3.2 Natural convection between parallel plates

Natural convection involves buoyancy forces arising from differential expansion and is limited by viscous forces. Consequently yet another dimensionless number is required. This is the Grashof number, Gr, defined as follows:

$$Gr = \frac{g\beta\Delta T\, L^3}{v^2}$$

where β = volumetric coefficient of expansion
($^1/_T$ for an ideal gas and T is absolute temperature)

g = gravitational constant

ΔT = temperature difference (between plates)

v = kinematic viscosity

The relation between Nu and Gr has been found to depend on the inclination of plates. For horizontal plates a commonly used fit gives:

$$Nu = 0.152\,(Gr)^{0.281}$$

For vertical plates

$$Nu = 0.062\,(Gr)^{0.327} \text{ for } (1.5 \times 10^5 < Gr < 10^7)$$

and

$$Nu = 0.033\,(Gr)^{0.381} \text{ for } (1.5 \times 10^4 < Gr < 1.5 \times 10^5)$$

For other angles of tilt; s, a useful empirical formula (for air as the fluid) is

$$Nu = [0.06 - 0.017\,(s/90)]\,(Gr)^{1/3}$$

These equations can be used for looking at the optimal spacing for double glazing.

Once the Nusselt number is computed, the convective heat transfer coefficient; h_{conv}, is calculated from

$$h_{conv} = \frac{Nu\,k}{L}$$

and the convective heat transfer; Q_{conv} is then given by $Q_{conv} = Ah_{conv}\,\Delta T$
where A is the area of plate concerned.

It can often be simpler to work in terms of heat transport per unit area, ie $\frac{Q_{conv}}{A} = h_{conv}\,\Delta T$

2.3.3 Combined natural and forced convection

The dominance of a heat transfer mode is governed by the fluid velocity associated with that mode. The criterion is that for natural convection to dominate:

$$Gr/Re^2 > 10$$

2.3 Radiative heat transfer*

Thermal radiation is electromagnetic radiation with wavelengths in the range approximately 0.2 to 100 μ*m*. The energy associated with photon of thermal radiation is given (as for all electromagnetic radiation) by

$$E = h\nu$$

where *h* is Planck's' constant (6.6256×10^{-34} *J sec*) and *v* is the frequency of the radiation. The wavelength is related to the frequency by the velocity of light, *c* as

$$\lambda = c/\nu$$

2.4.1 Black body radiation

Every body emits electromagnetic radiation. The quantity and the spectral distribution of this radiation depends on the temperature of the body. The term 'black body' relates to a perfect absorber (and emitter) that is in thermal equilibrium with its environment. If a body has a higher temperature than its surroundings it will try to obtain thermal equilibrium by emitting radiation. If a body is in thermal equilibrium with its surrounding it will still be emitting radiation. However, in this case the total amount of energy absorbed will equal the total amount emitted.

The energy of the photons emitted from a black body gives rise to the characteristic spectral radiation and intensity, known as 'black body radiation'. The dependence of intensity on the temperature and wavelength is given by Planck's law given by

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(\frac{hc}{\lambda kT}) - 1}$$

where λ is the wavelength of radiation, c is the speed of the light ($3 \times 10^8 ms^{-1}$), h is Planks constant (6.63×10^{-34} Js) and k is the Boltzmann constant (1.38×10^{-23} JK^{-1}).

Figure 3 shows the intensity of radiation emitted from a black body plotted as a function of wavelength for various temperatures. It can be seen that as the temperature increases the spectrum becomes more peaked and shifts to shorter wavelengths. In addition the total energy emitted increases strongly with temperature. By integrating equation above over all wavelengths we obtain the total energy emitted by the body. It is found that there is a simple relationship between the total energy and the temperature and is given by the Stefan-Boltzmann-Law:

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* When evaluating a radiation heat transfer rate temperature must be expressed in degrees Kelvin.

$$E = \sigma T^4$$

Where σ is the Stephan-Boltzmann constant (5.65 x 10^{-8} JK^{-4})

The spectral distribution of emission from a black body for a range of temperatures in given in figure 3.

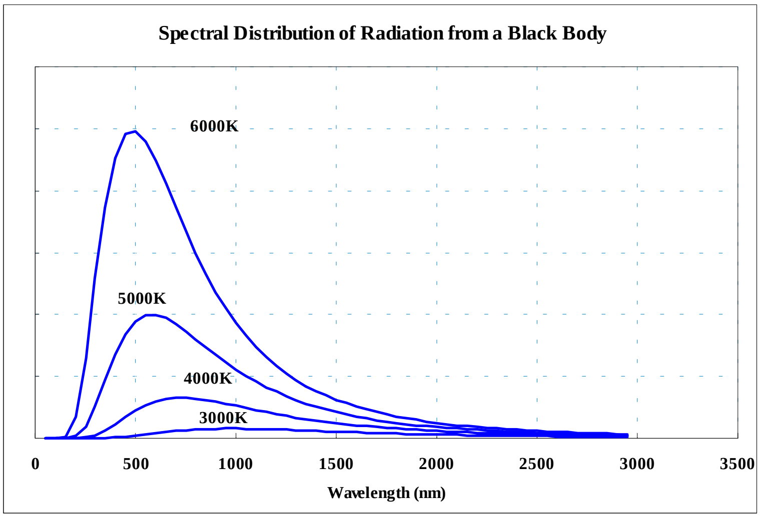

Figure 3: The spectral distribution of emission from a black body for a range of temperatures

For non-black bodies the total energy emitted is given by

$$E = \varepsilon \sigma T^4$$

Where ε is called the emissivity and is a characteristic of the material.

2.4.2 Absorptance and emittance

Directional absorptance, $\alpha(\mu,\varnothing)$ is the fraction of all radiation from a given direction which is absorbed by the surface. A total (hemispherical) absorptance, α is the fraction of all radiation form all directions absorbed by a surface. Corresponding terms $\varepsilon(\mu,\varnothing)$ and ε are defined as directional and total emittance. For two bodies in thermal equilibrium $\varepsilon = \alpha$.

2.4.3 Radiative exchange between 2 surfaces

We state, without derivation the following equation for heat transfer between two surfaces at temperatures T_1 and T_2 with areas A_1 and A_2 having emittances ε_1 and ε_2.

$$Q_1 = -Q_2 = \frac{\sigma(T_2^4 - T_1^4)}{(1-\varepsilon_1)/\varepsilon_1 A_1 + 1/A_1 F_{12} + (1-\varepsilon_2)/\varepsilon_2 A_2}$$

where F_{12} is the view factor. For unrestricted viewing F = 1.

Hence for two infinite, parallel plates

$$Q/A = \frac{\sigma(T_2^4 - T_1^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

A second useful case is for a small enclosed object surrounded by a large enclosure. A_1/A_2 tends to zero, F = 1 and so

$$Q_1 = \varepsilon A_1 \sigma (T_2^4 - T_1^4)$$

where the subscript 1 refers to the small enclosed object.

2.5 Overall Heat Transfer Coefficient

Most practical heat transfer problems involve more than one heat transfer mode, and therefore the problem needs to be analysed from an overall point of view. Two values need to be determined, the overall heat transfer coefficient; U, and the overall temperature difference, $\Delta T_{overall}$.

The overall heat transfer coefficient can be determined by establishing a thermal resistance network analogous to the electrical case; whereby the thermal resistance is the ratio of the driving potential (temp. diff.) to the transfer rate (heat) per unit area. A very common case is that of a composite wall (figure 3). For this case, the elements are in series and the net resistance; R, is simply the sum of the individual resistances.

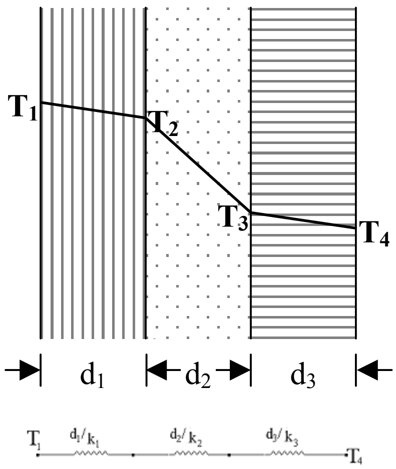

Figure 4: Equivalent thermal network for a composite wall

The overall heat transfer is then $Q=AU\Delta T_{overall}$

where A is the appropriate area for heat flow path, $\Delta T_{overall} = T_1 - T_4$, and U=1/R.