2. Mass, Force and Acceleration

[This material relates predominantly to modules ELP034, ELP035]

2.1 Newton's first law of motion

This law is really only a definition of force. It says that when a body is acted upon by an external resultant force it will accelerate.

If the resultant force is zero, then the body either remains at rest or else it will continue to move at constant velocity (that is with constant speed in a straight line).

Consider a parachutist who jumps from a plane travelling horizontally. His initial vertical speed is zero. He will immediately be acted upon by his weight acting vertically downwards and since the external resultant force is not zero he will accelerate.

He will not increase in speed indefinitely because as his speed increases so does his frictional drag – this opposes the down force of his weight. Eventually the frictional drag will increase until it equals his weight at this point the resultant force will be zero and he will stop accelerating but continue to fall at a constant (or terminal) speed. For the human body this is about 45-55 m/s or 100-120 mph.

When the parachutist releases his parachute the frictional drag is suddenly increased to be greater that his weight and the resultant force is upwards – he will start to decelerate. This will continue with the drag force reducing until the forces are again equal and the speed of fall constant. At this point this terminal speed should be only a few m/s.

In summary Newton's first law can be defined as *A body not acted upon by an external resultant force moves with constant velocity or is at rest.*

Mass

The mass of a body is sometime said to be the quantity of matter in a body. This is quite vague because of the use of the word *quantity*.

The mass of a body is a number assigned to it to distinguish it from another which may appear identical. It determines the behaviour of the body when acted upon by a force which causes it to change its motion. The mass can then be considered to be a measure of the resistance to change of motion. The resistance to change in motion is known as **inertia**. A body with a large mass is said to have a large inertia. Newton's first law is sometime knows as the *law of inertia*.

2.2 Newton's second law of motion

Before stating this law we will first define linear momentum, or simply momentum.

Momentum will be denoted p and is defined as

$$\text{Momentum } \; p = \text{mass} \times \text{velocity}$$
$$p = mv$$

Equation 2.1

Momentum is a vector quantity and is expressed in SI units by *kg m/s* or *Ns* (note that these units are in fact equivalent).

The second law states that the force causing acceleration is proportional to the rate of change of momentum with time and acts in the direction of the change.

If a force F changes the velocity of a body with constant mass m uniformly from u to v in time t, then Newton's second law states

$$F \propto \frac{mv - mu}{t}$$
$$F \propto \frac{m(v-u)}{t}$$
$$F = k\,ma$$

since acceleration, $a = (v-u)/t$.

k is the constant of proportionality. By definition the SI unit of force, the Newton N, causes an acceleration of 1 m/s^2 of a mass of 1 kg. This conveniently gives k =1 and the second law of motion may be summarised as

$$\text{Force} = \text{mass} \times \text{acceleration}$$

$$F = ma$$

Equation 2.2

Newton's first law is a special case of his second law with the force equal to zero.

Worked Example 2.1

A railway engine pulls a wagon of mass 10 000 kg along a straight track at a steady speed. The pull force in the couplings between the engine and wagon is 1000 N.

a) What is the force opposing the motion of the wagon?
b) If the pull force is increased to 1200 N and the resistance to movement of the wagon remains constant, what would be the acceleration of the wagon?

Solution

a)
When the speed is steady, by Newton's first law, the resultant force must be zero. The pull on the wagon must equal the resistance to motion. So the force resisting motion is 1000 N.

b)
The resultant force on the wagon is 1200 – 1000 = 200 N
By Equation 2.2

$$F = ma$$
$$200 = 10000a$$
$$a = 0.02\, m/s^2$$

Worked Example 2.2

a) Find the acceleration of a 20 kg crate along a horizontal floor when it is pushed with a resultant force of 10 N parallel to the floor.
b) How far will the crate move in 5s (starting from rest)?

Solution
a)

$$F = ma$$
$$10 = 20a$$
$$a = 0.5m/s^2$$

b) Distance travelled is given by Equation 1.3

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}0.5 \times 5^2$$
$$s = 6.25\, m$$

Mass and weight

The **weight** of a body may be defined as the **force** with which it is attracted to earth. When a body fall freely to earth (strictly speaking in a vacuum – but approximately in air) its acceleration is constant at $g = 9.81$ m/s^2. Since force is mass × acceleration, then weight

$$W = mg$$

Equation 2.3

Weight is a force – so in SI units it is in Newton's N.

(Do not confuse weight with mass – this is a very common mistake)

Worked Example 2.3

A 1kg stone fall freely from rest from a bridge.
a) What is the force causing it to accelerate?
b) What is its speed 4s later?
c) How far has it fallen in this time?

Solution

a)
The force causing it to fall is its weight. As it is falling with acceleration due to gravity

$$F = ma$$
$$F = 1 \times 9.81$$
$$F = 9.81\, m/s^2$$

b)
From Equation 1.1

$$v = u + at$$
$$v = 0 + 9.81 \times 4$$
$$v = 39.2\, m/s$$

c)
From Equation 1.3

$$s = ut + \frac{1}{2}at^2$$
$$s = 0 + \frac{1}{2}9.81 \times 4^2$$
$$s = 78.5\, m$$

2.3 Newton's third law of motion

This law states that is a body, A, exerts a force, F, on body B then body B exerts an equal but opposite force on body A. This law applies bodies both at rest or in motion.

The law is sometimes summarised as *Every action has an equal and opposite reaction*.

A block of mass M resting on a table exerts a downward force Mg on the table. By Newton's third law the table exerts an equal force in the vertically upward direction on the block.

<u>*Worked Example 2.4*</u>

A lift with its load has a mass of 2000 kg. It is supported by a steel cable. Find the tension in he cable when it:

a) is at rest
b) accelerates upwards uniformly at 1m/s^2
c) move upwards at a steady speed of 1 m/s
d) moves downwards at a steady speed of 1 m/s
e) accelerates downwards with uniform acceleration of 1 m/s^2

Solution

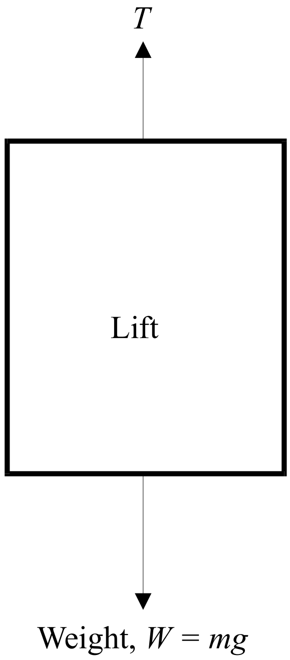

Figure 2.1 Lift and force direction for Worked Example 2.4

a)
When at rest we can use Newton's first law which says that the resultant force on the lift is zero.

Force acting down is the lifts weight, the force acting up is the tension in the cable. These two must be equal and opposite to give a resultant force of zero. So,

$$\text{Weight} \quad W = mg = 2000 \times 9.81$$
$$W = 19600N$$
$$T = W$$

This T acts vertically upwards.

b)
As the lift is accelerating upwards so T must *exceed* the weight *mg*. So the resultant acceleration force

$$F = T - mg$$

by Newton second law, $F = ma$, so

$$T - mg = ma$$
$$T - 2000g = 2000 \times 1$$
$$T = 2000 + 19600$$
$$T = 19600N$$

c)

As in (a), by Newton's first law, the resultant force on the lift must be zero, so

$$T = mg = 19600\,N$$

d)
As in (c) the tension in the cable will still equal *mg* since the change in direction of motion does not alter the fact that there is no acceleration.

$$T = mg = 19600\,N$$

e)
If the lift accelerates downwards, then *mg* must exceed the tension *T*. So the resultant accelerating force is

$$F = mg - T$$

By Newton's second law $F = ma$, so

$$\begin{aligned} mg - T &= ma \\ 2000g - T &= 2000 \times 1 \\ T &= 17600\,N \end{aligned}$$

2.4 Friction

The force which prevents or tries to prevent the slipping or sliding of two surfaces in contact is called friction.

Several rules (or laws) have been developed from experiment and experience (rather than by theory). These, described below, apply only to dry surfaces.

i. The frictional resistance between two sliding surfaces is directly proportional to the force pressing the two surfaces together.

$$F_S \propto N$$

F_S = frictional force resisting sliding motion. *N* is the force pressing the two surfaces together

This law fails when the force pressing the surface together is very small or very large. E.g. when very large the surfaces may tend to seize together.

ii. The frictional resistance depends on the nature of the roughness of the surfaces involved

iii. The frictional resistance is independent of the area of the surface in contact.

This law fails when the area of the surface is so small that damage to the surface occurs leading to increased friction.

iv. The frictional resistance is independent of speed of sliding.

This law fails when the speed of sliding is very high or very low. When the speed is very high, the temperature of the surfaces may increase and change the frictional properties.

Frictional resistance is greatest when the speed of sliding is zero - when motion is about to commence.

Clearly the above laws must be applied with caution. At extremes of force and speed careful thought must be given to decide whether the results is acceptable.

The laws for lubricated surface are considerably different to those above. We will not be going into those in this module.

Coefficient of friction

Consider a block, as shown in Figure 2.4, with mass *m* resting on a horizontal surface.

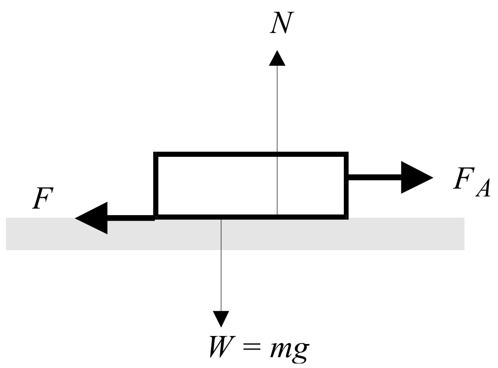

Figure 2.4: A block on a rough surface with frictional resistance to sliding

Let the force between the surfaces be N (known often as the *normal reaction*.) N will equal the weight, by Newton's third law.

Also let the external force applied to the block be F_A . As F_A increases from zero, the frictional resistance to motion F will also increase from zero. When F reaches the maximum value F_S the block will be on the point of moving. At this point $F_A = F_S$ and by law (*i*)

$$F_S \propto N$$
$$F_S = \mu_S N$$

Equation 2.6

μ_S is the **coefficient of static friction**.

From experiments it has been shown that once the block starts to move, the applied force required to keep it moving steadily is less than F_S. That is the frictional resistance F when sliding is less than F_S. Or that the force required to move an object from a stationary position is greater than to that required to keep it moving steadily.

This then means the we need to have a **coefficient of sliding friction** (μ) for when the block is moving

$$F \propto N$$

$$F = \mu N$$

Equation 2.7

The values of μ *and* μ_S depend on the surface in contact. Some typical values are given below:

Steel on steel	$\mu_S = 0.58$
Masonry on rock	$\mu_S = 0.6\text{-}0.7$
Masonry on clay	$\mu_S = 0.30$
Wood on brick	$\mu_S = 0.6$
Rubber sliding on bitumen at 100m/min	$\mu = 1.07$

Worked example 2.6

A crate with mass 50 kg will just slide with uniform speed down a rough ramp at 30° to the horizontal.
What is the coefficient of (static) friction ?

Solution

The force acting on the crate are:
- Its weight mg
- The normal reaction of the ramp N
- The frictional resistance F

These are shown in Figure 2.5

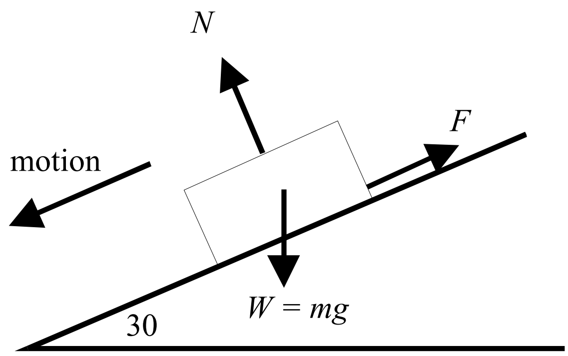

Figure 2.5: A crate sliding down a slope. Worked Example 2.5

Since there is no acceleration down the ramp (the crate isn't moving!) the resultant force parallel to the ramp must be zero.

Resolving forces parallel to the ramp gives

$$F - \text{component of weight down the ramp} = 0$$

$$F - mg\cos 60 = 0$$

$$F = 50 \times 9.81 \times \cos 60$$

$$F = 245\,N$$

Also, since there is no acceleration at right angles to the ramp the resultant force at right angles must be zero. Resolving forces at right angles gives

$$N = mg\cos 30$$

$$N = 50 \times 9.81 \times \cos 30$$

$$N = 425\,N$$

For the coefficient of friction, as

$$F = \mu N$$

$$\mu = \frac{F}{N}$$

$$\mu = \frac{245}{425} = 0.576$$

2.5 Circular motion - banked roads or tracks

In section one the acceleration a of a body moving round a circle, radius r, moving with uniform speed v, was shown to be (Equation 1.13)

$$a = \frac{v^2}{r}$$

and that this acts towards the centre of the circle.

By Newton's second law the force F producing this acceleration on a mass m is

$$F = ma = \frac{mv^2}{r}$$

Equation 2.8

In this case the force is called the **centripetal force**.
The force may be provided in many ways, for example for a car travelling round a curved level road the force is the friction between the tyres and the road.

When the curved tract is banked downwards toward the centre of motion then the frictional forces in the case of the car would be less than if the track were level. The reduced dependence on friction to provide a centripetal force gives greater comfort and safety.

Consider the four wheeled vehicle travelling round a track banked at an angle θ to the horizontal, as in Figure 2.6. Let the conditions be such that when the vehicle is moving with a speed of v there is no sideways force.

The forces acting are then weight acting at the centre of gravity G of the vehicle, and the normal reaction R_1 and R_2 at the wheels.

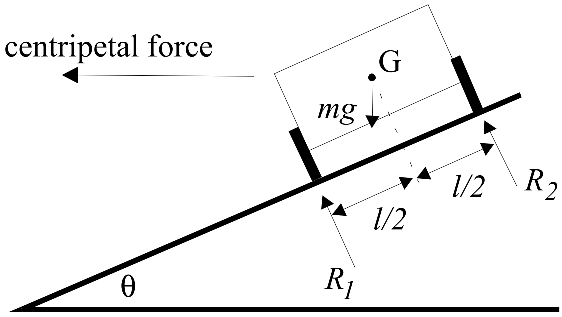

Figure 2.6: Forces on a vehicle on a banked track

Taking moments about G gives

$$R_1 \frac{l}{2} = R_2 \frac{l}{2}$$

i.e. $R_1 = R_2$ - the reaction forces are the same, we will call them both R from now on.

Vertically there is no motion so no acceleration so resolving forces in the vertical direction gives

$$2R\cos\theta = mg$$

Equation 2.9

Horizontally, by Equation 2.8

$$F = ma = \frac{mv^2}{r}$$

so resolving forces horizontally gives

$$\frac{mv^2}{r} = 2R\sin\theta$$

Equation 2.10

Dividing Equation 2.10 by 2.9 gives

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{mv^2}{mgr}$$

$$\tan\theta = \frac{v^2}{gr}$$

Equation 2.11

This is the condition for no sideways force on the vehicle.