4. Normal Stress, Shear Stress and Bending Moment

[This material relates predominantly to module ELP034]

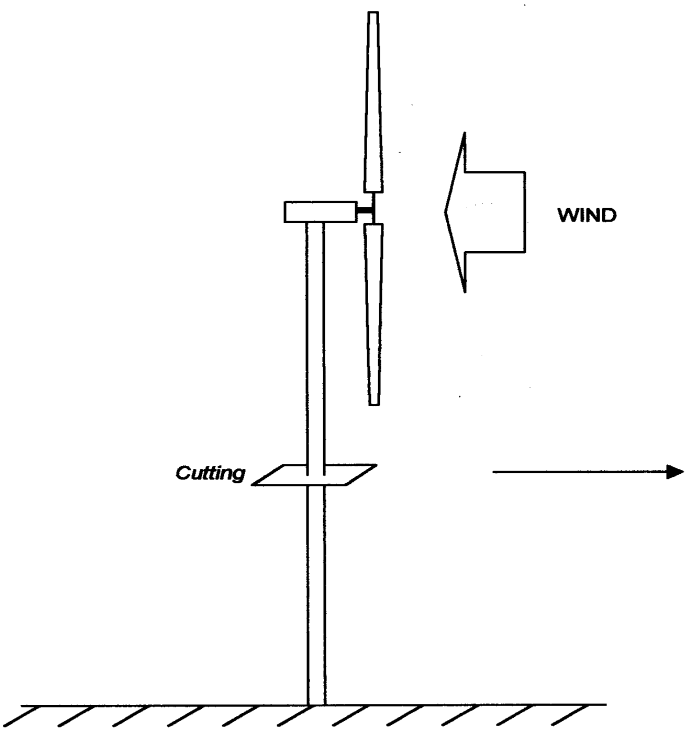
4.1 Bending Moment

4.2 Horizontal Reaction Force

4.3 Vertical Reaction Force

The concepts of Normal Stress, shear Stress and Bending Moment will be described through a practical example that refers to a wind turbine that experiences a drag force in the direction of the wind velocity.

Consider Fig. 4.1. If the mast of the turbine is cut in two and the upper part is kept floating in the air, what will be the forces that will maintain it fixed in the air, i.e. in a state of equilibrium?



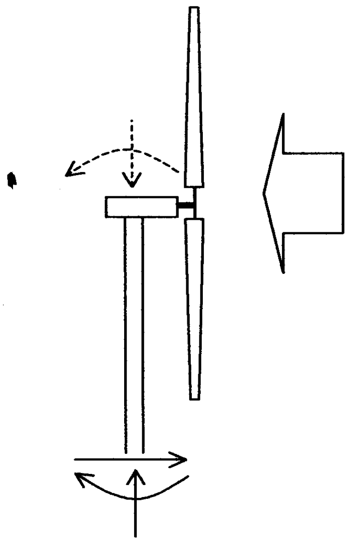


Figure 4.1: a wind turbine subjected to wind loading

Figure 4.2: Forces maintaining the half in the air (plain lines) turbine

4.1 Bending Moment

There are two forces and a bending moment. First we need a resisting *bending moment*. What is a bending moment?
A bending moment occurs whenever a force is applied to a material and when the point considered on the material is not the point of application of the force, see fig.4.3

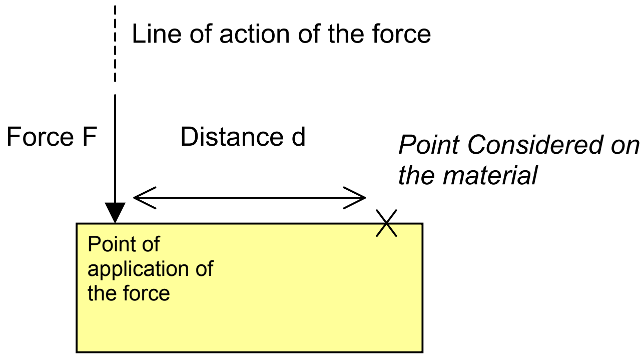


Figure 4.3: Force applied to a material

From fig. 4.3, the magnitude of the bending moment *at point x* on the material equals Fd, i.e. applied force × distance from the point to the line of action of the force (measured at right angle).
From this definition it is clear that the magnitude of the bending moment is different for points on the material located at different distances from the point of application of the force. At the point of application of the force, the bending moment is zero.

Now imagine that there is a rod passing through point x and through the material, and is fixed to the material, as shown below.

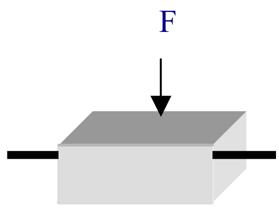


Figure 4.4: Rod through the material

Then suppose that you hold each end of the rod in your hands, and the force F is applied. If you want to stop the material rotating about the rod, you need to resist with your hands the rotating movement of the material. That resistance that you provide is called *resisting bending moment*. If you manage to maintain the material still, then the material is said to be in *equilibrium* and the magnitude of the bending moment you have applied exactly equals Fd.
The *direction* of the resisting bending moment that you have applied is clockwise, i.e. in the opposite direction as the bending moment provided by the force F, as shown in fig. 4.5.

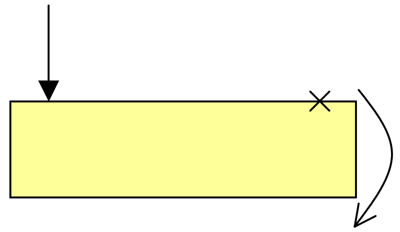


Figure 4.5: Direction of the resisting bending moment

To summarise, a bending moment is always measured at a point. It has both magnitude with units Nm and direction (clockwise or anti-clockwise).

Now, coming back to the wind turbine, the resistive bending moment that needs to be applied at the point of the cutting in order to maintain the turbine still in the air equals:
a) In magnitude: the distance from the cutting to the line of action of the force of the wind acting on the hub of the turbine, multiplied by the magnitude of that force (if the force acts over an area, we consider the resultant point force).
b) In direction: clockwise, as drawn on fig. 4.2 (plain line)

4.2 Horizontal Reaction Force

If you only provide the required resisting bending moment at the base of the half-turbine to resist the wind force, it will indeed prevent the turbine to rotate but the turbine will move to the left. Therefore you also need to apply at the base a horizontal force of magnitude equals to the wind force and of opposite direction to the wind force, as illustrated on figure 4.2.

This force is a shear force because the line of action of the force is parallel to the object (in fact it is on the object in this case). It is common to speak in terms of *shear stress* rather than shear force. Shear stress is simply the shear force divided by the area on which the force acts (in this case the cross-sectional area of the turbine mast).

4.3 Vertical Reaction Force

If you only provide the horizontal force and bending moment at the base of the half turbine, you will not prevent the turbine falling to the ground! Therefore, quite obviously, if you want to maintain the turbine floating in the air you need, at the base of the half turbine, a vertical upward force whose magnitude equals the weight of the turbine.

This force is a normal force because the line of action of the force is at right angles to the object. It is common to speak in terms of *normal stress* rather than normal force. Normal stress is simply the normal force divided by the area on which the force acts (in this case the cross-sectional area of the turbine mast).

The Full Turbine

The three elements required to maintain the half-turbine still are in fact the same that act at the base of a full turbine. The only difference is the magnitude of the bending moment: as we have mentioned, the bending moment is different at different points along the mast and is greatest at the base of the mast (since the distance to the force is the greatest). These three elements are provided by the foundations.